Particle Physics Study Guide

This is a guide of what to revise for the exam. Other material we covered in the course may appear in questions but in conjunction with what's given below. Remember that, in an exam, the masses of the particles are provided on the constant sheet.

At the end of this summary a list of the equations you need to remember, and a summary of the forces and Feynman rules.

Quarks, Leptons and Quantum Numbers

- The quarks and leptons (a.k.a. the fermions) and their anti-particles. Which fermions interact under which forces.
- That fermions are described, quantum mechanically, by "spinors". That a spinor is a four-component object, a function of the fermion's four-momentum.
- That for each flavour of fermion there are four possible spinors: e.g. for electrons - the four solutions represent: left-handed electrons, right-handed electrons, left-handed positrons and right-handed positrons.
- The definition of the following quantum numbers and charges, and the value of them for the fundamental particles.
  - Electric charge, Q.
  - Total quark number, N_{q}.
  - Lepton numbers: L_e, L_μ, L_τ
  - Spin: total spin S and third component: S_Z.
  - The individual quark flavour quantum numbers: N_{u}, N_{d}, N_{s}, N_{c}, N_{b}, N_{t}.
  - Colour charge, in so far as the quarks always carry a colour charge, there are three colour charges, and the net colour charge is conserved.
  - Weak Hypercharge: total T and third component T_3.
  - Weak Hypercharge $Y = 2(Q - T_3)$.
  - The definition of helicity.
  - The definition of left-handed and right-handed chirality.

Forces

There is a summary of the forces at the end of this guide. For the strong, electromagnetic and weak forces:

- The boson(s) responsible for the force.
- The rough mass of each boson (remember exact masses of massive particles are on the data sheets).
- The coupling constants, and what charge each of the forces couple to.
- The allowed flavour changes for W-boson interactions. (The flavour of the quarks and leptons is conserved in interactions with the other bosons.)
- Running of α and α_S as a function of the boson momentum transfer, q^2.
- The vertex terms for bosons in Feynman diagrams.

Scattering and Decay

These are the two main processes we can measure in particle physics to investigate the properties of the fermions and the interactions. You should revise:

- Elastic collision and inelastic collisions.
- The definition of the Mandelstam variables, s, t and u.
- The definition of total width for a particle (equation (5)). The definition of partial width and branching ratio for a decay (equations (7) and (6)).
- What conditions must be satisfied for decays to occur.
- Fermi's Golden Rule.

Hadrons

- What hadrons, mesons and baryons are, and why quarks are confined to hadrons.
- What a parton is, the consequences of the parton model, and definition of the associated quantity x.
- What jets are and how they are produced.
- The evidence for gluons.

Relativistic Dynamics

- Conservation of four-momentum! This is especially important for calculating the boson four momentum, q, in the boson propagator terms, and in decays and scattering.

Natural Units

- We set $\hbar = c = 1$. The most important implication of this is that mass, momentum and energy are all measured in units of energy, usually MeV or GeV.

Feynman Diagrams and Feynman Rules

- How to draw simple Feynman diagrams.
- How to write down the fermion spinors, vertex and boson propagator terms (including CKM matrix elements) to calculate simple matrix elements $\mathcal{M}$ for simple diagrams with just one boson.
- How to relate $\mathcal{M}$ to the cross sections, σ, (equation (9)) and partial decay widths, Γ (equation (7)) through Fermi's Golden Rule.
- That higher order diagrams (with more vertices and more boson exchange) are suppressed by factors of the coupling constant, and are therefore less likely to happen.

Important Experimental Results

- **Muon decay measurements**: what the muon decays to and how its lifetime can be used to measure the Fermi coupling constant: $G_F \propto g_W^2/m_W^2$.
- **The ratio, R**:

$$R(E_{\text{CM}}) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- **Higgs physics**: How the Higgs boson behaves at the LHC: how it can be produced in LHC collisions and how it decays and can be detected.

C, P and T

- The definition of the charge conjugation (**C**: changes the sign of all the charges), parity (**P**: $\vec{x} \to -\vec{x}$) and time reversal (**T**: $t \to -t$) operators.
- When **P**, **C**, **CP** are conserved symmetries, when they are nearly conserved symmetries, and when the are completely useless symmetries ("maximally violated").
- How neutral matter–anti-matter pairs of mesons can "mix" with each other.
- The CKM matrix and what this has to do with **CP** violation.

Equations

This is a list of equations you need to know! Note some of the equations use specific collisions and decays as examples; the equations, however, apply to all types of collisions and decays.

- The number of time a particular process occurs, N, is related to the cross section of a process, σ, and the integrated luminosity, $\int \mathcal{L}dt$:

$$N = \sigma \cdot \int \mathcal{L}dt \tag{1}$$

- The four momentum of a particle, in natural units is:

$$p^\mu = (E, p_x, p_y, p_z) = (E, \vec{p}) \tag{2}$$

  where E is the energy, and $\vec{p}$ is the three momentum.

- The square of the four momentum of any *initial* or *final state* particle is its mass squared:

$$\underline{\underline{p}}^2 = E^2 - \vec{p} \cdot \vec{p} = m^2 \tag{3}$$

  This is not necessarily true for intermediate particles such as bosons propagating an interaction: if $\underline{\underline{q}}^2_{\text{boson}} \neq m^2_{\text{boson}}$ we say the boson is *virtual.*

- The Lorentz Transformations in natural units are:

$$\gamma = E/m \;\Rightarrow\; E = \gamma m \qquad \beta = |\vec{p}|/E \qquad \gamma\beta = |\vec{p}|/m \tag{4}$$

- The total width of a particle (Γ) and the lifetime of the particle (τ) are related as:

$$\Gamma = \hbar/\tau \tag{5}$$

- The branching ratios of a decay is, *e.g.*:

$$\mathrm{BR}(K^0 \to \pi^+\pi^-) = \frac{\Gamma(K^0 \to \pi^+\pi^-)}{\Gamma_{K^0}} \tag{6}$$

  where $\Gamma(K^0 \to \pi^+\pi^-)$ is the width of individual decay $K^0 \to \pi^+\pi^-$.

- The width of individual decay modes (called partial widths) are proportional to $\mathcal{M}^2$, *e.g.*

$$\Gamma(K^0 \to \pi^+\pi^-) = \frac{2\pi}{\hbar}\big(\mathcal{M}(K^0 \to \pi^+\pi^-)\big)^2\rho \tag{7}$$

  where ρ is the phase space, which is completely determined by kinematics. The total width of a particle is the sum of the partial width of all possible decay modes:

$$\Gamma_{K^0} = \Gamma(K^0 \to \pi^+\pi^-) + \Gamma(K^0 \to \pi^0\pi^0) \tag{8}$$

- The cross section, σ, of a scattering is proportional to the matrix element squared: $\mathcal{M}^2$, *e.g.*

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{2\pi}{\hbar}\big(\mathcal{M}(e^+e^- \to \mu^+\mu^-)\big)^2\rho \tag{9}$$

  where again ρ is the phase space, which is completely determined by kinematics.

- The definition of the Mandelstam variables, for a decay $12 \to 34$:

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - p_3)^2 \\ s &= (p_1 - p_4)^2 \end{aligned} \tag{10}$$

  In a collision (at a collider or fixed target) the centre of mass energy is $\sqrt{s}$.

- The CKM matrix:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \tag{11}$$

  and what it means!

- How to use the Dirac Equation which represents the wavefunctions of spin-1/2 particles.

$$\left(i\gamma^0\frac{\partial}{\partial t} + i\vec{\gamma}\cdot\vec{\nabla} - m\right)\psi = 0 \qquad (i\gamma^\mu\partial_\mu - m)\,\psi = 0 \tag{12}$$

  In the final equation the repeated Lorentz index $\mu = 0, 1, 2, 3$ is summed over; $\partial_\mu = \partial/\partial x_\mu$.

What you don't have to remember

If you need any of these, they will be given!

- The masses of all the particles; in an exam they are given on the constant sheet.
- The lifetimes of all the particles.
- The quark content of all the hadrons.
- The names of the accelerators and experiments, which particles and energies they use.
- Long and complicated or derived equations. However you may need to know how to use them, or derive them again with guidance.

QED	QCD	Weak Neutral Current	Weak Charged Current
quantum theory of EM interactions	quantum theory of strong interactions	quantum theory of weak interactions	
mediated by exchange of virtual photons	mediated by exchange of gluons	mediated by exchange of Z bosons	mediated by exchange of W bosons
acts on all charged particles	acts on quarks only	acts on **all** quarks and leptons	
couples to electric charge	couples to colour charge	does not change quark or lepton flavour	changes quark and leptons flavours
coupling strength $\propto$ fermion charge $\propto e \propto \sqrt{\alpha}$	coupling strength $\propto g_S \propto \sqrt{\alpha_S}$ (equal for all quark flavours)	coupling strength $\propto g'_W$	coupling strength $\propto g_W \propto \sqrt{\alpha_W}$ at lepton vertex: g_w at quark vertex: $g_w V_{qq'}$
photon propagator: $1/\underline{\underline{q}}^2$	gluon propagator: $1/\underline{\underline{q}}^2$	Z-boson propagator: $1/(\underline{\underline{q}}^2 - m_Z^2)$	W-boson propagator: $1/(\underline{\underline{q}}^2 - m_W^2)$