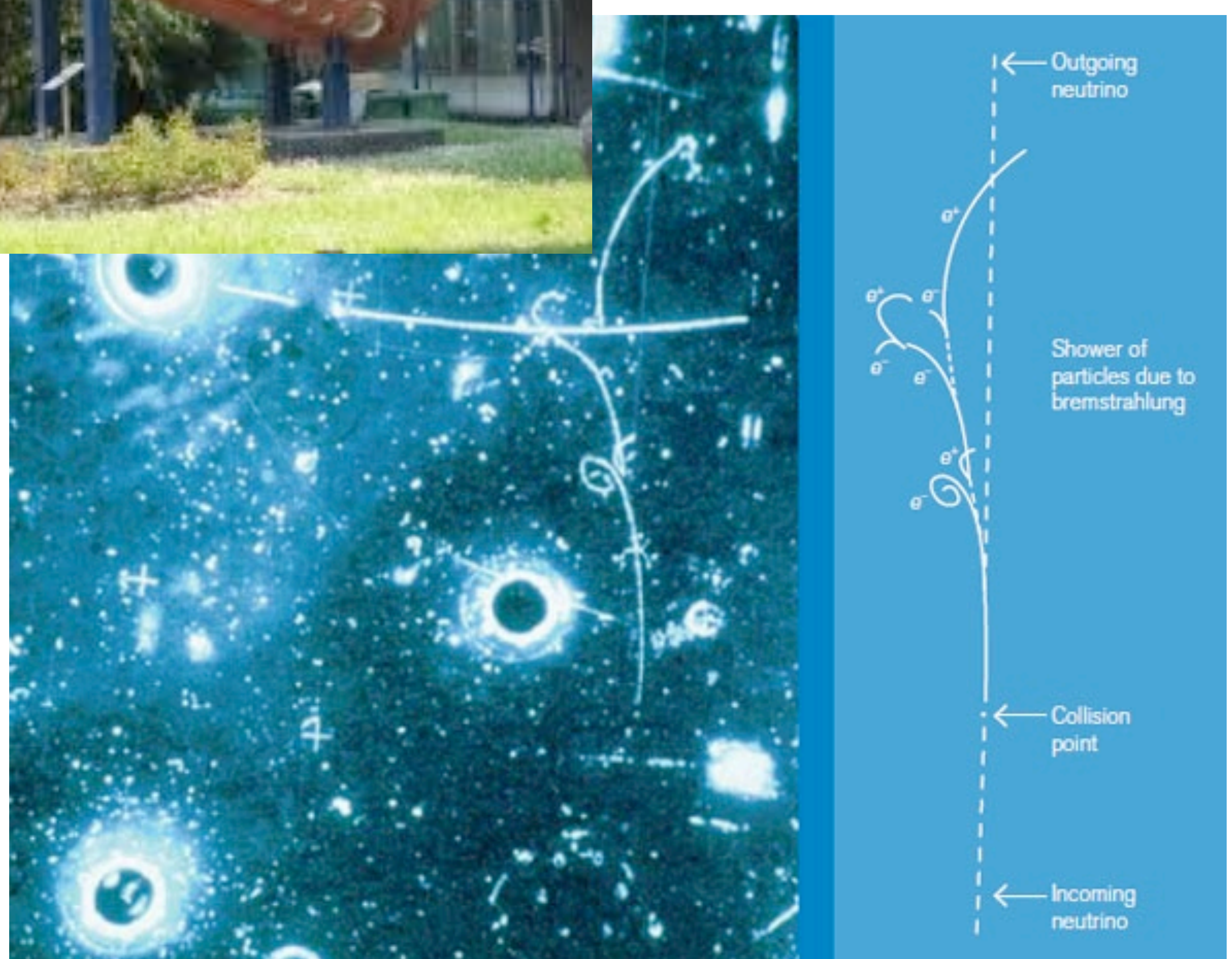
Particle Physics

Dr Victoria Martin, Spring Semester 2012

Lecture 7: Renormalisation and Weak Force



- ★ Renormalisation in QED
- ★ Weak Charged Current
- ★ $V-A$ structure of W-boson interactions
- ★ Muon decay
- ★ Beta decay
- ★ Weak Neutral Current
- ★ Neutrino scattering

Reminders

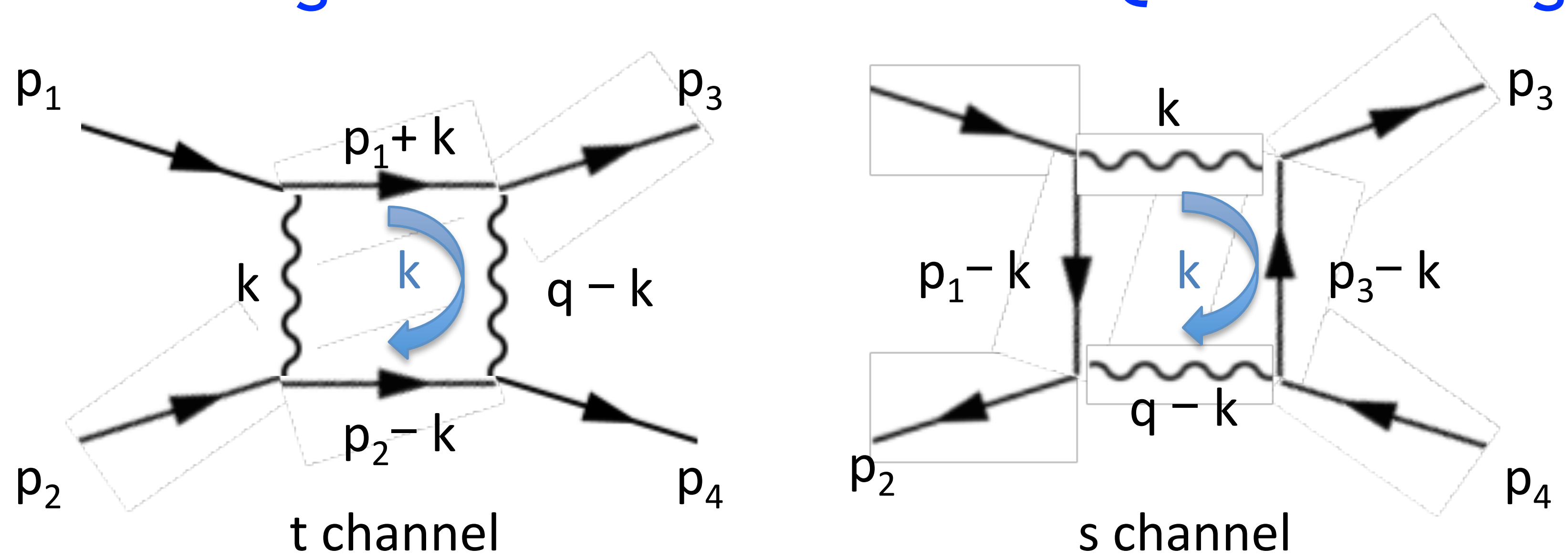
- No lecture on Friday.

- Next tutorial: Monday 11th February.
  - New tutorial sheet will be posted to webpage. Paper copies available at tutorial

- Next lecture: Tuesday 12th February!

Higher Order QED

Two photon "box" diagrams also contribute to QED scattering



Two extra vertices ⇒ contribution is suppress by a factor of $\alpha = 1/137$

- The four momentum must be conserved at each vertex.

- However, four momentum k flowing round the loop can be anything!

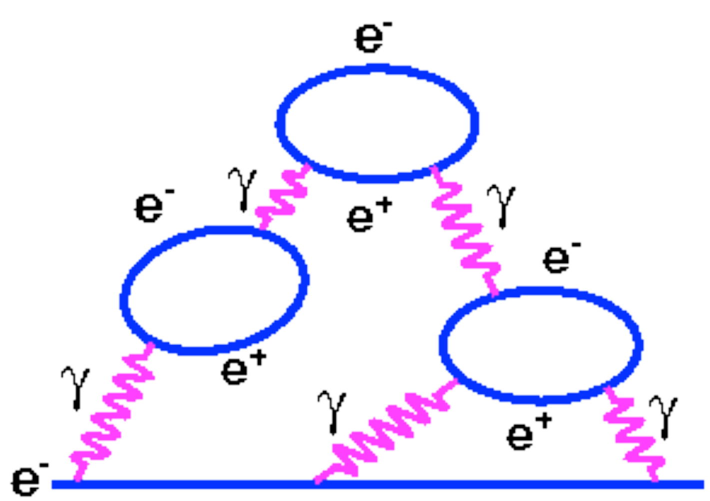
- In calculating $\mathcal{M}$ integrate over all possible allowed momentum configurations: $\int f(k)\, d^4k \sim \ln(k)$ leads to a divergent integral!
- This is solved by **renormalisation** in which the infinities are "miraculously swept up into redefinitions of mass and charge" (Aitchison & Hey P.51)

Renormalisation

- Impose a "cutoff" mass M, do not allow the loop four momentum to be larger than M. Use $M^2 >> q^2$, the momentum transferred between initial and final state.
  - ➡ This can be interpreted as a limit on the shortest range of the interaction
  - ➡ Or interpreted as possible substructure in pointlike fermions
  - ➡ Physical amplitudes should not depend on choice of M
- Find that $\ln(M^2)$ terms appear in the $\mathcal{M}$
- Absorb $\ln(M^2)$ into **redefining fermion masses and vertex couplings**
  - ➡ Masses $m(q^2)$ and couplings $\alpha(q^2)$ are now functions of q^2

- *e.g.* Renormalisation of electric charge (considering only effects from one type of fermion):

$$e_R = e\sqrt{1 - \frac{e^2}{12\pi^2}\ln\left(\frac{M^2}{q^2}\right)}$$



- Can be interpreted as a "screening" correction due to the production of electron/positron pairs in a region round the primary vertex
- e_R is the effective charge we actually measure!

Running Coupling Constant

- Renormalise α, and correct for all possible fermion types in the loop:

$$\alpha(q^2) = \alpha(0)\left(1 + \frac{\alpha(0)}{3\pi} z_f \ln(\frac{-q^2}{M^2})\right)$$

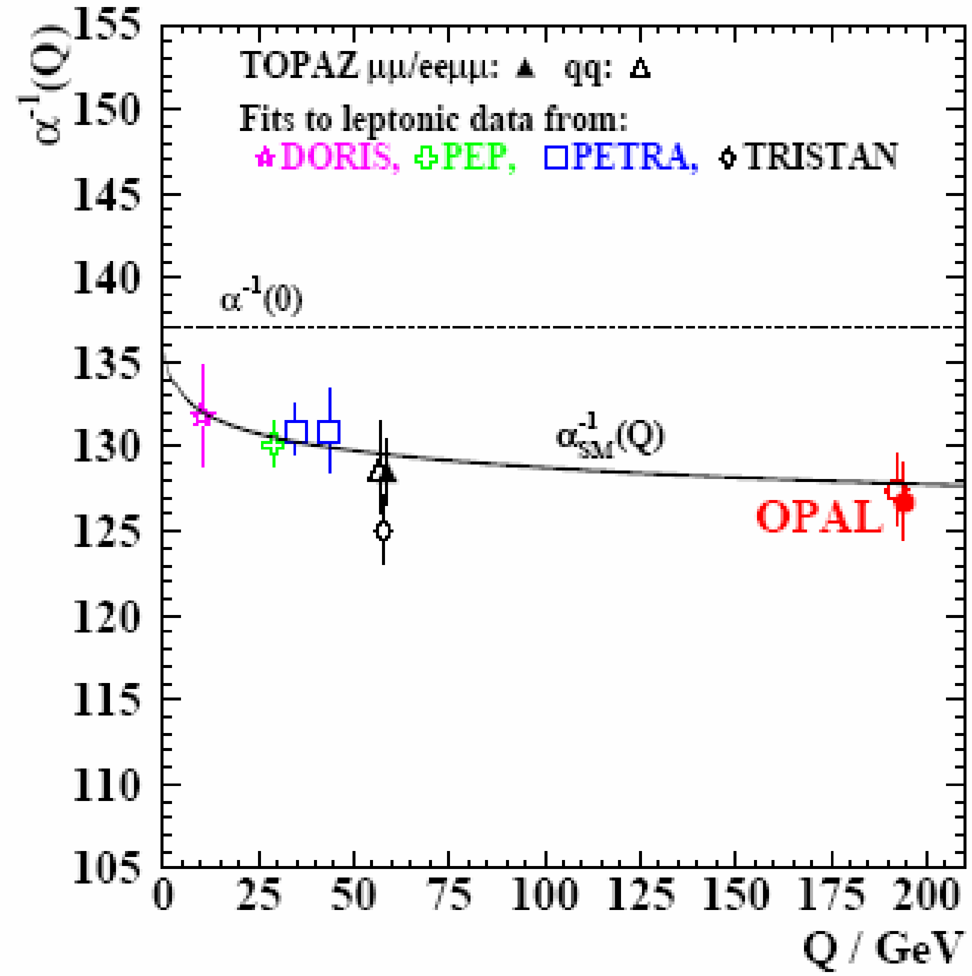
- z_f is the sum of charges over all possible fermions in the loop
  - ➡ At $q^2 \sim 1$ **MeV** only electron, $z_f = 1$
  - ➡ At $q^2 \sim 100$ **GeV**, $f = e, \mu, \tau, u, d, s, c, b$ $z_f = 38/9$

$$z_f = \sum_f Q_f^2$$

- Instead of using M^2 dependence, replace with a reference value μ^2:

$$\alpha(q^2) = \alpha(\mu^2)\left(1 - \frac{\alpha(\mu^2)}{3\pi} z_f \ln(\frac{q^2}{\mu^2})\right)^{-1}$$

- Usual choices for μ are **1 MeV** or $m_Z \sim 91$ **GeV.**
  - ➡ $\alpha(\mu^2 = 1\ \mathrm{MeV}^2) = 1/137$
  - ➡ $\alpha(\mu^2 = (91\ \mathrm{GeV})^2) = 1/128$
- We choose a value of μ where make a initial measurement of α, but once we do the evolution of the values of α are determined by the above eqn.

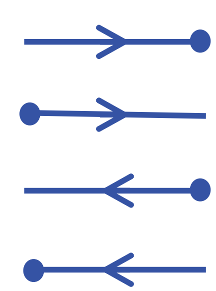


QED Summary

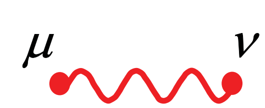
- For full QED calculations use spinors to define fermion current in Feynman diagrams.
- Fermion current are $\bar{\psi}\gamma^{\mu}\psi$ where $\bar{\psi}$ is the adjoint spinor: $\bar{\psi} \equiv \psi^{\dagger}\gamma^{0}$
- Spin-1 bosons are described by polarisation vectors, $\varepsilon^{\mu}(s)$
- To calculate the cross section for an unpolarised process need to average over initial helicities and sum over all possible final states.
- Fermion masses, charges and the coupling constant α evolve as a function of momentum transfer.

spin 1/2

incoming particle	$u(p)$	
outgoing particle	$\bar{u}(p)$	
incoming antiparticle	$\bar{v}(p)$	
outgoing antiparticle	$v(p)$	

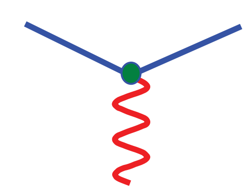


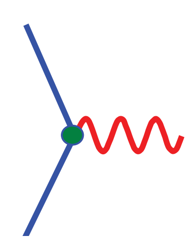
spin 1 **photon** $\dfrac{g_{\mu\nu}}{q^2}$



- **Vertex Factors**

spin 1/2 **fermion (charge -|e|)** $e\gamma^{\mu}$

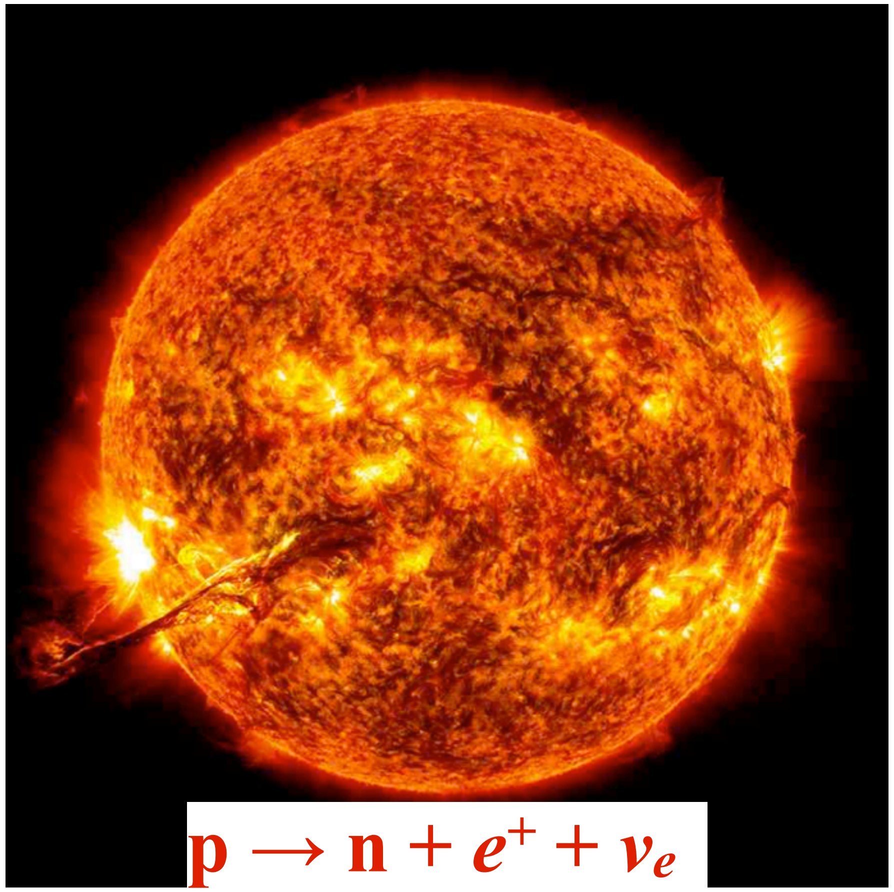


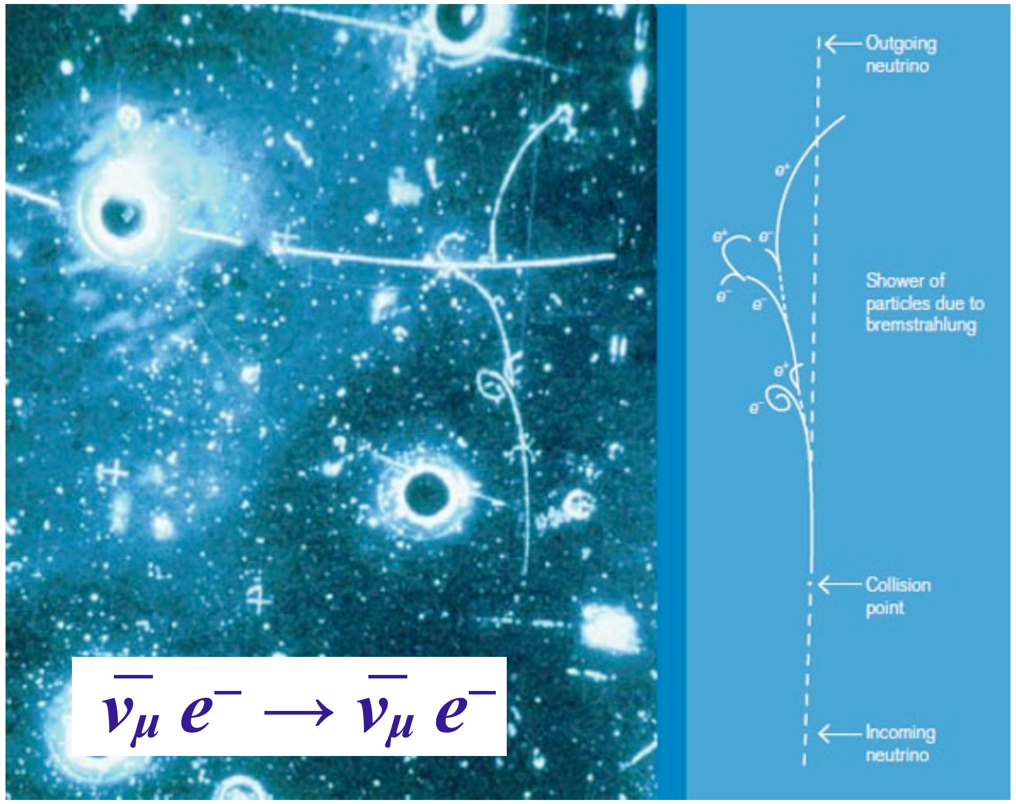


The Weak Force

- Exchange of **massive** W and Z bosons.
  - $\mathbf{m_W = 80.385 \pm 0.015}$ **GeV**
  - $\mathbf{m_Z = 91.1876 \pm 0.0021}$ **GeV**

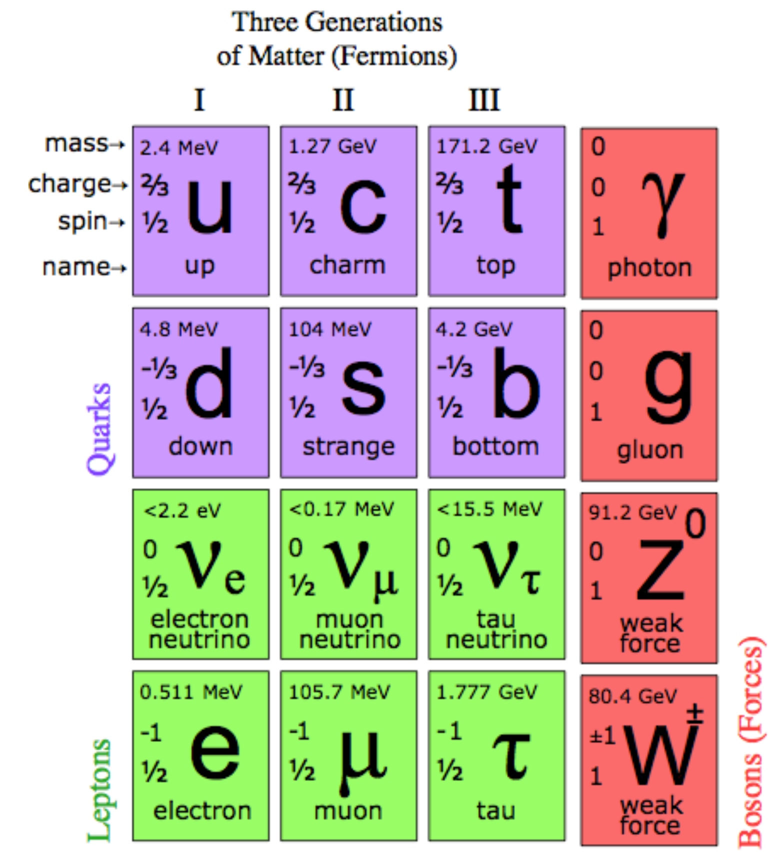
- Responsible for:
  - Beta decay
  - Fusion
  - Neutrino interactions





Review: Baryon and Lepton Number; Chirality

- **Lepton number** is the number of leptons minus the number of anti-leptons:
  - ➡ Electron number: $L_e = N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e)$
  - ➡ Muon number: $L_\mu = N(\mu^-) - N(\mu^+) + N(\nu_\mu) - N(\bar{\nu}_\mu)$
  - ➡ Tau-lepton number: $L_\tau = N(\tau^-) - N(\tau^+) + N(\nu_\tau) - N(\bar{\nu}_\tau)$

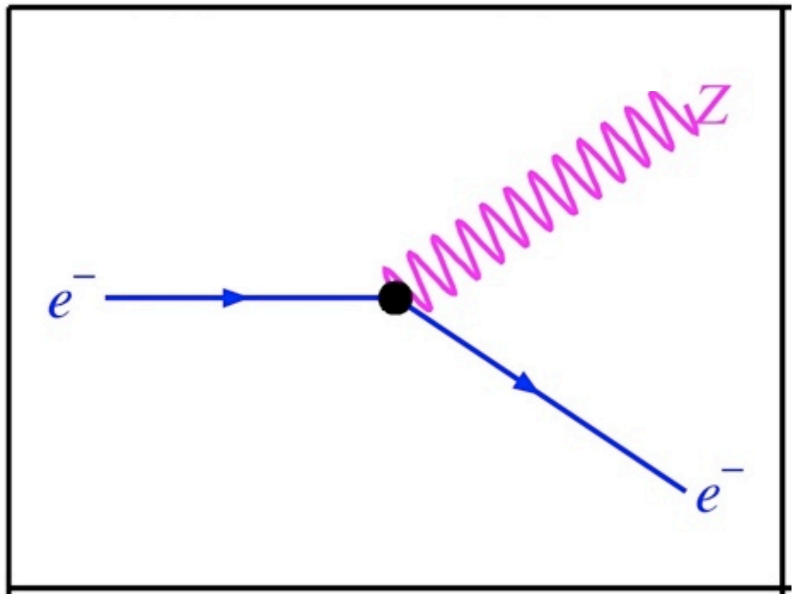


- **Baryon number** is a measure of the net number of quarks:
  $\mathcal{B} = \frac{1}{3}(N(q) - N(\bar{q}))$

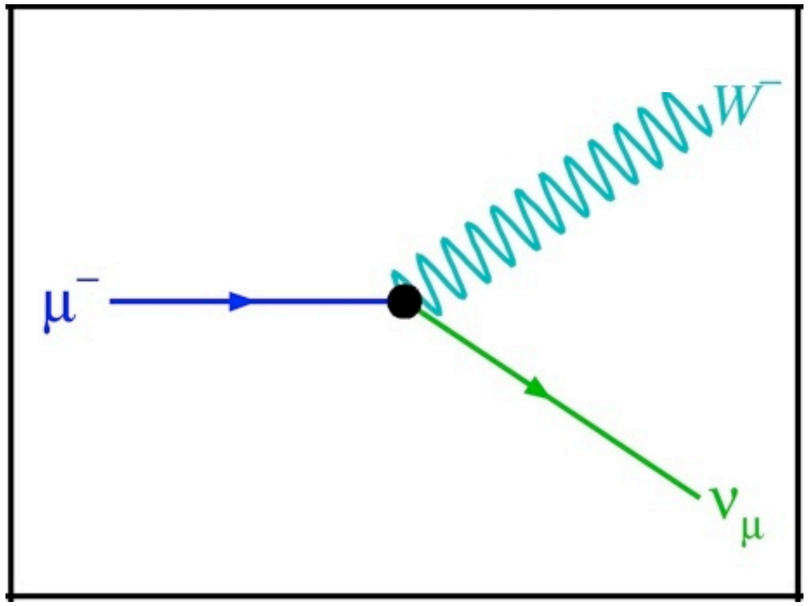
- LH projection operator $P_L = (1 - \gamma^5)/2$ projects out left-handed chiral state
- RH projection operator $P_R = (1 + \gamma^5)/2$ projects out right-handed chiral state
  - Massive fermions have both left-handed and right-handed chiral components
  - Massless fermions would have only left-handed components
  - Massless anti-fermions would have only right-handed components
  - To date, only **left-handed neutrinos** and **right-handed anti-neutrinos** have been observed

W and Z boson interactions

- Any fermion (quark, lepton) may emit or absorb a Z-boson.
  - ➡ That fermion will remain the same flavour.
  - ➡ Very similar to QED, but neutrinos can interact with a Z boson too.

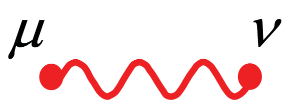
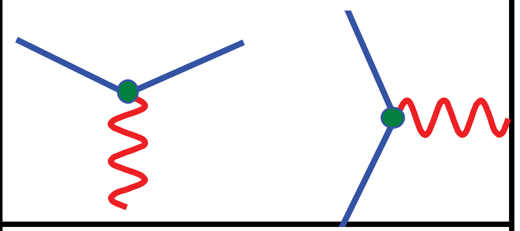


- Any fermion (quark, lepton) may emit or absorb a W-boson.
  - ➡ To conserve electric charge that fermion **must** change flavour!
  - ➡ To conserve lepton number $e \leftrightarrow \nu_e$, $\mu \leftrightarrow \nu_\mu$, $\tau \leftrightarrow \nu_\tau$
  - ➡ To conserve baryon number $(\mathbf{d, s, b}) \leftrightarrow (\mathbf{u, c, t})$



down-type quark ↔ up-type quark

Feynman Rules for Charged Current

	propagator 	interaction vertex 
***W*-boson**	$\dfrac{g_{\mu\nu}}{q^2 - m_W^2}$	$\frac{1}{2\sqrt{2}} g_W \, \gamma^\mu (1 - \gamma^5)$
photon, γ	$\dfrac{g_{\mu\nu}}{q^2}$	$e \, \gamma^\mu$

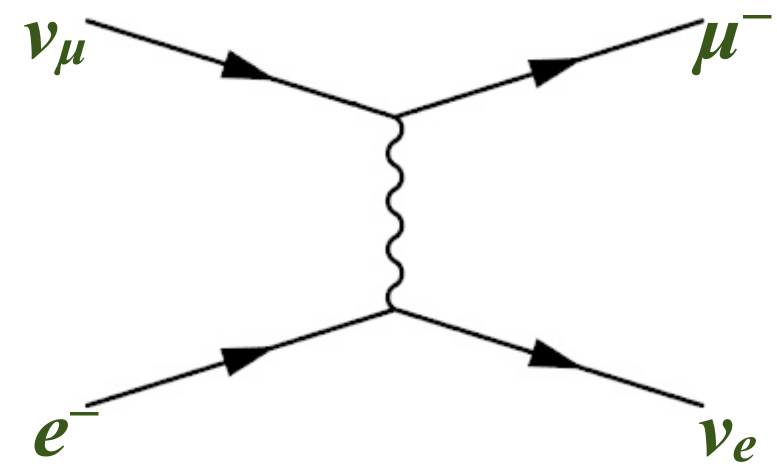
- Left-handed interactions are also known as $V\!-\!A$ theory
  - ➡ γ^μ gives a vector current (V)
  - ➡ $\gamma^\mu\gamma^5$ gives an axial vector current (A)
- Photon interactions are purely vector

- Key differences w.r.t QED.
  - ➡ $q^2 - m_W^2$ as denominator of propagator
  - ➡ The $\frac{1}{2}(1-\gamma^5)$ term: this is observed experimentally.
- The overall factor of $1/\sqrt{8}$ is conventional
- Recall $P_L = (1-\gamma^5)/2$ is the Left Handed projection operator
  - ➡ W-boson interactions only act on **left-handed chiral components** of fermions

- For low energy interactions with $q \ll m_W$: effective propagator is $g_{\mu\nu}/m_W^2$

"Inverse Muon Decay"

- Start with a calculation of the process $\nu_\mu\, e^- \to \mu^-\, \nu_e$
- Not an easy process to measure experimentally, but easy to calculate!



$$\mathcal{M} = \frac{g_W^2}{8}\, \bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)\, \frac{g_{\mu\nu}}{q^2 - m_W^2}\, u(\nu_\mu)\gamma^\nu(1-\gamma^5)u(\mu)$$

$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 \left[\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)\right]^2 \left[\bar{u}(\mu)\gamma^\mu(1-\gamma^5)u(\nu_\mu)\right]^2$$

- Usually we would average over initial spin and sum over final spin states:
  - However the neutrinos are only left handed
  - The equation can be solved as (see Griffiths section 9.1):

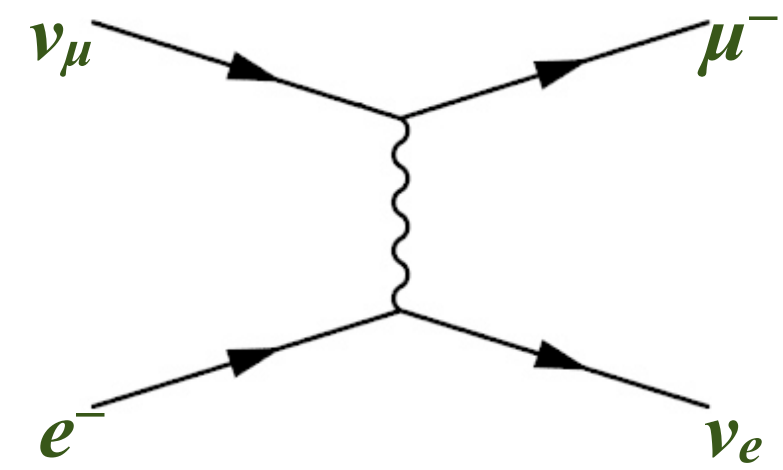
$$|\mathcal{M}|^2 = 2\left(\frac{g_W^2}{m_W^2}\right)^2 \left(p^\mu(e)\cdot p^\mu(\nu_\mu)\right)\left(p^\mu(\mu)\cdot p^\mu(\nu_e)\right)$$

  - In the CM frame, where E is energy of initial electron or neutrino, and m_e neglected as $m_e << E$:

$$|\mathcal{M}|^2 = 8E^4\left(\frac{g_W^2}{m_W^2}\right)^2\left(1 - \frac{m_\mu^2}{2E^2}\right)^2$$

“Inverse Muon Decay” Cross Section

- Cross section = $|\mathcal{M}|^2 \rho$, substituting for ρ (see problem sheet 1):



$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|\vec{p}^{\,*}_f|}{|\vec{p}^{\,*}_i|}$$

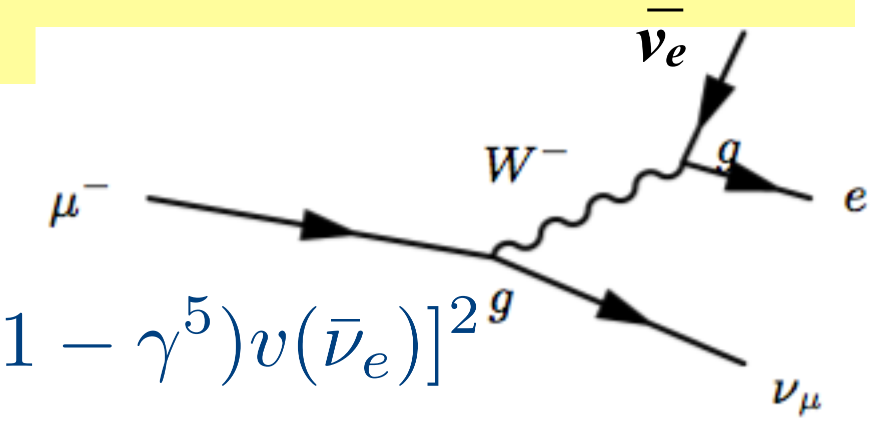
- Substitute:
  - ➡ centre of mass energy, $(E_1+E_2)^2=4E^2$
  - ➡ For elastic scattering particle $|p^*_f|=|p^*_i|$
  - ➡ $S=1$ as no identical particles in final state

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{32\pi^2}\left(\frac{g_W^2}{m_W^2}\right)^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

  - ➡ Fermi coupling constant $G_F = \sqrt{2} g_W^2/8m_W^2$
  - ➡ Unlike electromagnetic interaction, no angular dependence
  - ➡ Integral over 4π solid angle

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = \frac{4}{\pi}E^2 G_F^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

Muon Decay



- Muon decay: $\mu^- \to e^- \bar{\nu}_e \, \nu_\mu$ (Griffiths 9.2):

$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 \left[\bar{u}(\nu_\mu)\gamma^\mu(1-\gamma^5)u(\mu)\right]^2\left[\bar{u}(e)\gamma^\mu(1-\gamma^5)v(\bar{\nu}_e)\right]^2$$

$$= 2\left(\frac{g_W^2}{m_W^2}\right)^2 \left(p^\mu(e)\cdot p^\mu(\nu_\mu)\right)\left(p^\mu(\mu)\cdot p^\mu(\nu_e)\right)$$

- The phase space, ρ, for a 1 → 3 decay is, (Griffiths equation 6.21):

$$\frac{d\Gamma}{dE_e} = \frac{1}{4\pi^3}\left(\frac{\sqrt{2}g_W^2}{8M_W^2}\right)^2 m_\mu^2 E_e^2 \left(1 - \frac{4E_e}{3m_\mu^2}\right)$$

- Integrate over allowed values of E_e:

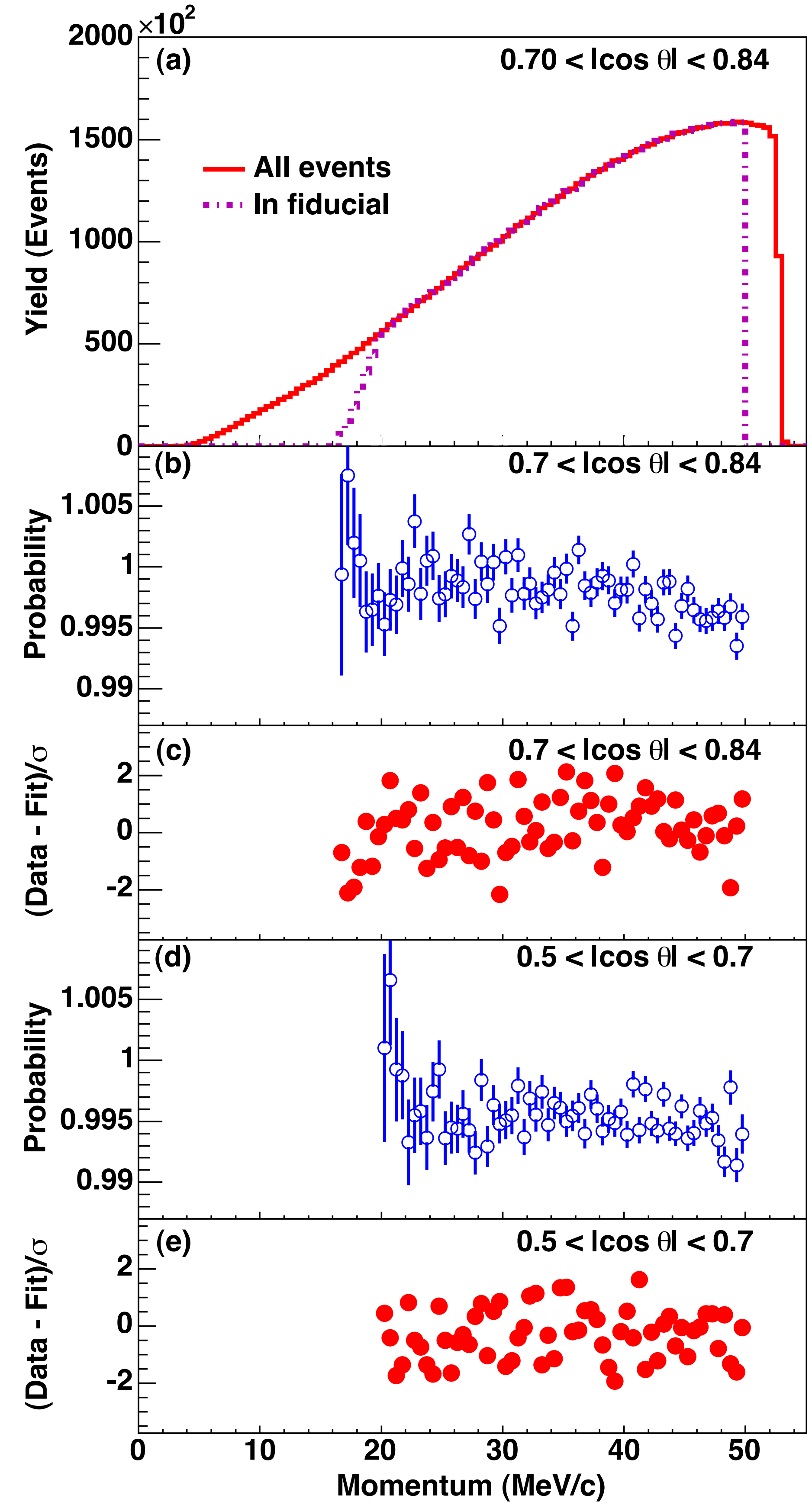
$$\Gamma = \int_0^{m_\mu/2} \frac{d\Gamma}{dE_e} dE_e = \frac{G_F^2 m_\mu^2}{4\pi^3}\int_0^{m_\mu/2} E_e^2\left(1 - \frac{4E_e}{3m_\mu^2}\right)dE_e = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- Only muon decay mode for muons **BR($\mu^- \to e^- \bar{\nu}_e \, \nu_\mu$) ≈ 100%**, only one decay mode contributes to lifetime

$$\tau \equiv \frac{1}{\Gamma} = \frac{192\pi^3}{G_F^2 m_\mu^5} = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4}$$

Muon Decay Measurements

- TWIST experiment at TRIMF in Canada measures $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ decay spectrum.
- Excellent agreement between data and prediction!
- Measurements of muon lifetime and mass used to define a value for G_F (values from PDG 2010)
  - ➡ $\tau = (2.19703 \pm 0.00002) \times 10^6$ s
  - ➡ $m = 105.658367 \pm 0.000004$ MeV
- Applying small corrections for finite electron mass and second order effects
  - ➡ $G_F = 1.166364(5) \times 10^{-5}\ \mathrm{GeV}^{-2}$
- Implies $g_W = 0.653$, $\alpha_W = g_W^2/4\pi = 1/29.5$
- $\alpha_W >> \alpha_{EM}$, the weak force not intrinsically weak, just appears so due to mass of W-boson



J.R. Musser et al., Physical Review Letters 94, 101805 (2005)

Summary

Renormalisation

- QED: Photons and fermions can appear in loops. Effectively modifying coupling (effective charge) and masses of fermions.
- This leads to running coupling α as a function of energy of scattering q.
- This happens in QED, Weak and QCD

Weak Charged Current

- Carried by the massive W-boson: acts on all quarks and leptons. $\frac{g_{\mu\nu}}{q^2 - m_W^2}$
- A W-boson interaction changes the flavour of the fermion.
- Acts only on the left-handed components of the fermions: $V\!-\!A$ structure.

$$\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)$$

- At low energy, responsible for muon & tau decay, beta decay...

Weak Neutral Current

- Carried by the massive Z-boson: acts on all quarks and leptons. $\frac{g_{\mu\nu}}{q^2 - m_Z^2}$
- No flavour changes observed.

$$\bar{u}(e)\gamma^\mu(c_V^e - c_A^e\gamma^5)u(e)$$

- At low energies, responsible for neutrino scattering