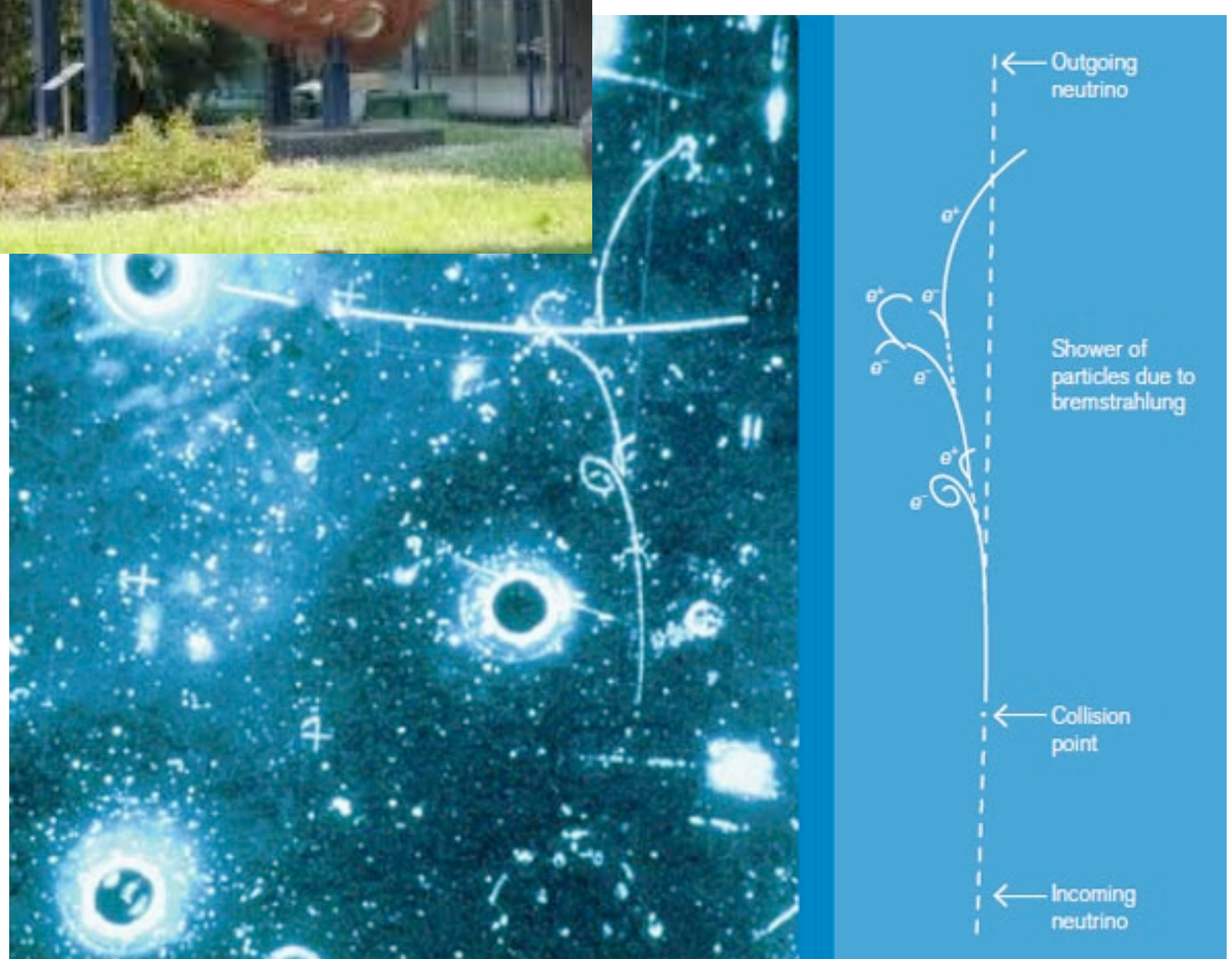
Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 8: Calculating the Weak Force and Symmetries

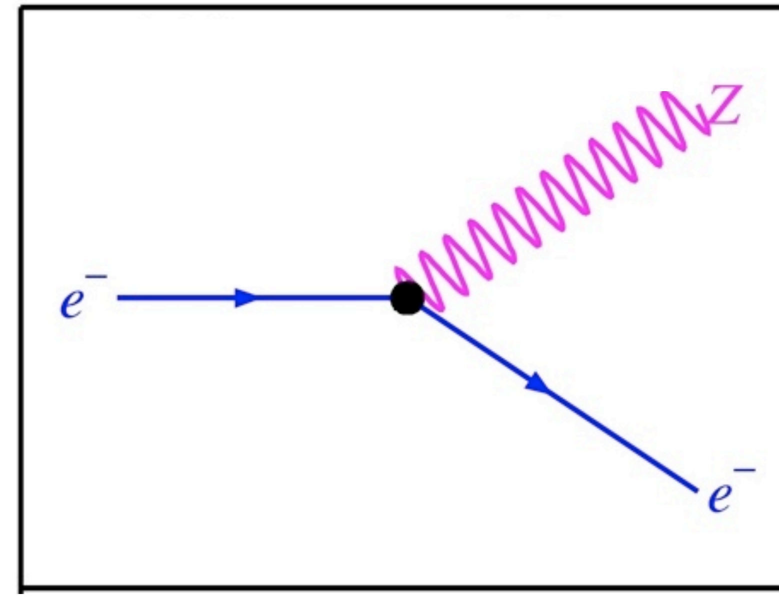


- ★Muon decay
- ★Beta decay
- ★Weak Neutral Current
- ★Neutrino scattering

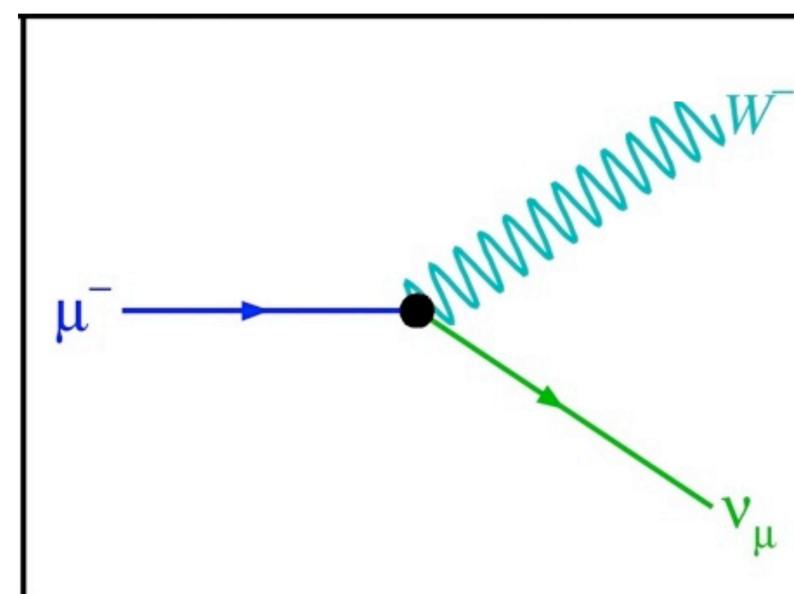
- ★Symmetries in PP

W and Z boson interactions

- Any fermion (quark, lepton) may emit or absorb a Z-boson.
  - ➡ That fermion will remain the same flavour.
  - ➡ Very similar to QED, but neutrinos can interact with a Z boson too.

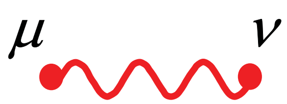
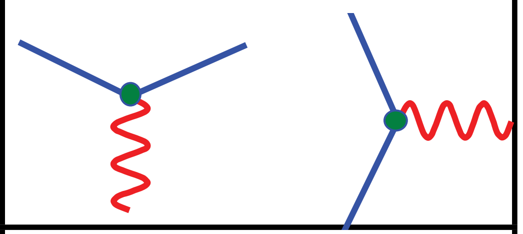


- Any fermion (quark, lepton) may emit or absorb a W-boson.
  - ➡ To conserve electric charge that fermion **must** change flavour!
  - ➡ To conserve lepton number $e \leftrightarrow \nu_e$, $\mu \leftrightarrow \nu_\mu$, $\tau \leftrightarrow \nu_\tau$
  - ➡ To conserve baryon number **(d, s, b) ↔ (u, c, t)**



down-type quark ↔ up-type quark

Feynman Rules for Charged Current

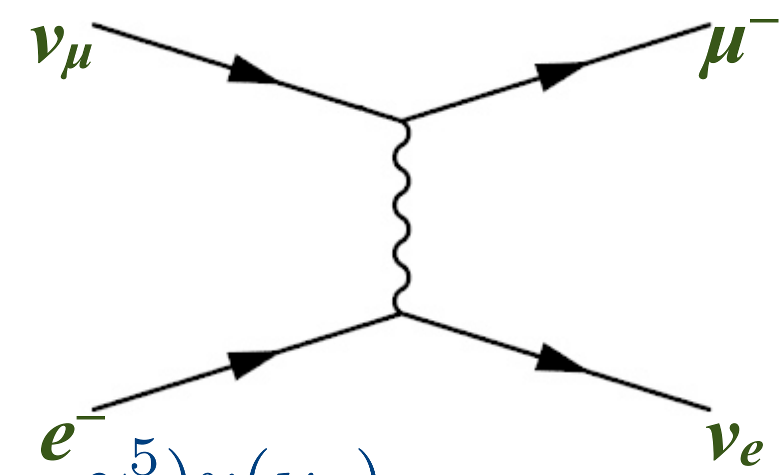
	propagator 	interaction vertex 
***W*-boson**	$\dfrac{g_{\mu\nu}}{q^2 - m_W^2}$	$\frac{1}{2\sqrt{2}} g_W \, \gamma^\mu (1 - \gamma^5)$
photon, γ	$\dfrac{g_{\mu\nu}}{q^2}$	$e \, \gamma^\mu$

- Left-handed interactions are also known as $V\!-\!A$ theory
  - ➡ γ^μ gives a vector current (V)
  - ➡ $\gamma^\mu\gamma^5$ gives an axial vector current (A)
- Photon interactions are purely vector

- Key differences w.r.t QED.
  - ➡ $q^2 - m_W^2$ as denominator of propagator
  - ➡ The $\frac{1}{2}(1-\gamma^5)$ term: this is observed experimentally.
- The overall factor of $1/\sqrt{8}$ is conventional
- Recall $P_L = (1-\gamma^5)/2$ is the Left Handed projection operator
  - ➡ W-boson interactions only act on **left-handed chiral components** of fermions

- For low energy interactions with $q \ll m_W$: effective propagator is $g_{\mu\nu}/m_W^2$

"Inverse Muon Decay"



- Start with a calculation of the process $\nu_\mu \, e^- \rightarrow \mu^- \, \nu_e$
- Not an easy process to measure experimentally, but easy to calculate!

$$\mathcal{M} = \frac{g_W^2}{8} \, \bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-) \, \frac{g_{\mu\nu}}{q^2 - m_W^2} \, \bar{u}(\mu)\gamma^\nu(1-\gamma^5)u(\nu_\mu)$$

$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 \left[\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)\right]^2 \left[\bar{u}(\mu)\gamma^\mu(1-\gamma^5)u(\nu_\mu)\right]^2$$

- Usually we would average over initial spin and sum over final spin states:
  - However the neutrinos are only left handed
  - The equation can be solved as (see Griffiths section 9.1):

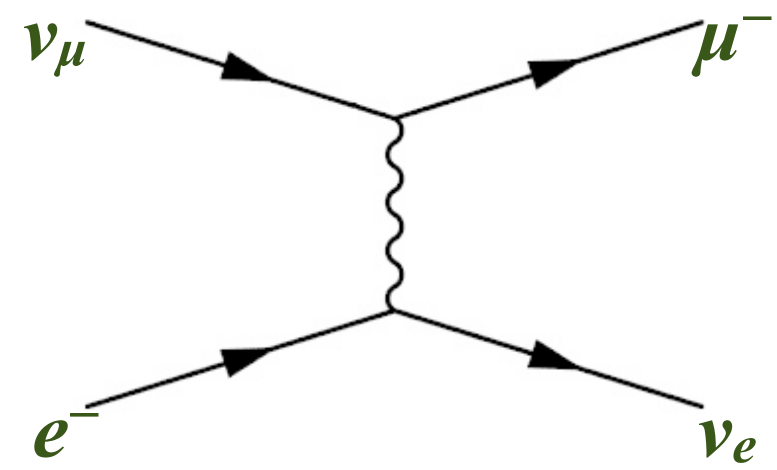
$$|\mathcal{M}|^2 = 2\left(\frac{g_W^2}{m_W^2}\right)^2 \left(p^\mu(e) \cdot p^\mu(\nu_\mu)\right)\left(p^\mu(\mu) \cdot p^\mu(\nu_e)\right)$$

  - In the CM frame, where E is energy of initial electron or neutrino, and m_e neglected as $m_e << E$:

$$|\mathcal{M}|^2 = 8E^4 \left(\frac{g_W^2}{m_W^2}\right)^2 \left(1 - \frac{m_\mu^2}{2E^2}\right)^2$$

"Inverse Muon Decay" Cross Section

- Cross section = $|\mathcal{M}|^2 \rho$, substituting for ρ (see problem sheet 1):



$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|\vec{p}^{\,*}_f|}{|\vec{p}^{\,*}_i|}$$

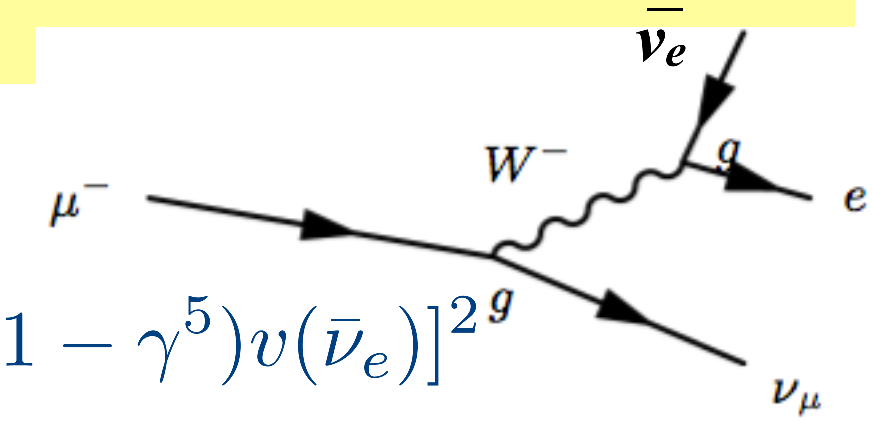
- Substitute:
  - ➡ centre of mass energy, $(E_1+E_2)^2=4E^2$
  - ➡ For elastic scattering particle $|p^*_f|=|p^*_i|$
  - ➡ $S=1$ as no identical particles in final state

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{32\pi^2}\left(\frac{g_W^2}{m_W^2}\right)^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

  - ➡ Fermi coupling constant $G_F = \sqrt{2} g_W^2/8m_W^2$
  - ➡ Unlike electromagnetic interaction, no angular dependence
  - ➡ Integral over 4π solid angle

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = \frac{4}{\pi}E^2 G_F^2\left(1-\frac{m_\mu^2}{2E^2}\right)^2$$

Muon Decay



- Muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ (Griffiths 9.2):

$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 [\bar{u}(\nu_\mu)\gamma^\mu(1-\gamma^5)u(\mu)]^2[\bar{u}(e)\gamma^\mu(1-\gamma^5)v(\bar{\nu}_e)]^2$$

$$= 2\left(\frac{g_W^2}{m_W^2}\right)^2 (p^\mu(e)\cdot p^\mu(\nu_\mu))\,(p^\mu(\mu)\cdot p^\mu(\nu_e))$$

- The phase space, ρ, for a 1 → 3 decay is, (Griffiths equation 6.21):

$$\frac{d\Gamma}{dE_e} = \frac{1}{4\pi^3}\left(\frac{\sqrt{2}g_W^2}{8M_W^2}\right)^2 m_\mu^2 E_e^2 \left(1 - \frac{4E_e}{3m_\mu^2}\right)$$

- Integrate over allowed values of E_e:

$$\Gamma = \int_0^{m_\mu/2} \frac{d\Gamma}{dE_e} dE_e = \frac{G_F^2 m_\mu^2}{4\pi^3}\int_0^{m_\mu/2} E_e^2\left(1-\frac{4E_e}{3m_\mu^2}\right)dE_e = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- Only muon decay mode for muons $\mathbf{BR}(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \approx \mathbf{100\%}$, only one decay mode contributes to lifetime

$$\tau \equiv \frac{1}{\Gamma} = \frac{192\pi^3}{G_F^2 m_\mu^5} = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4}$$

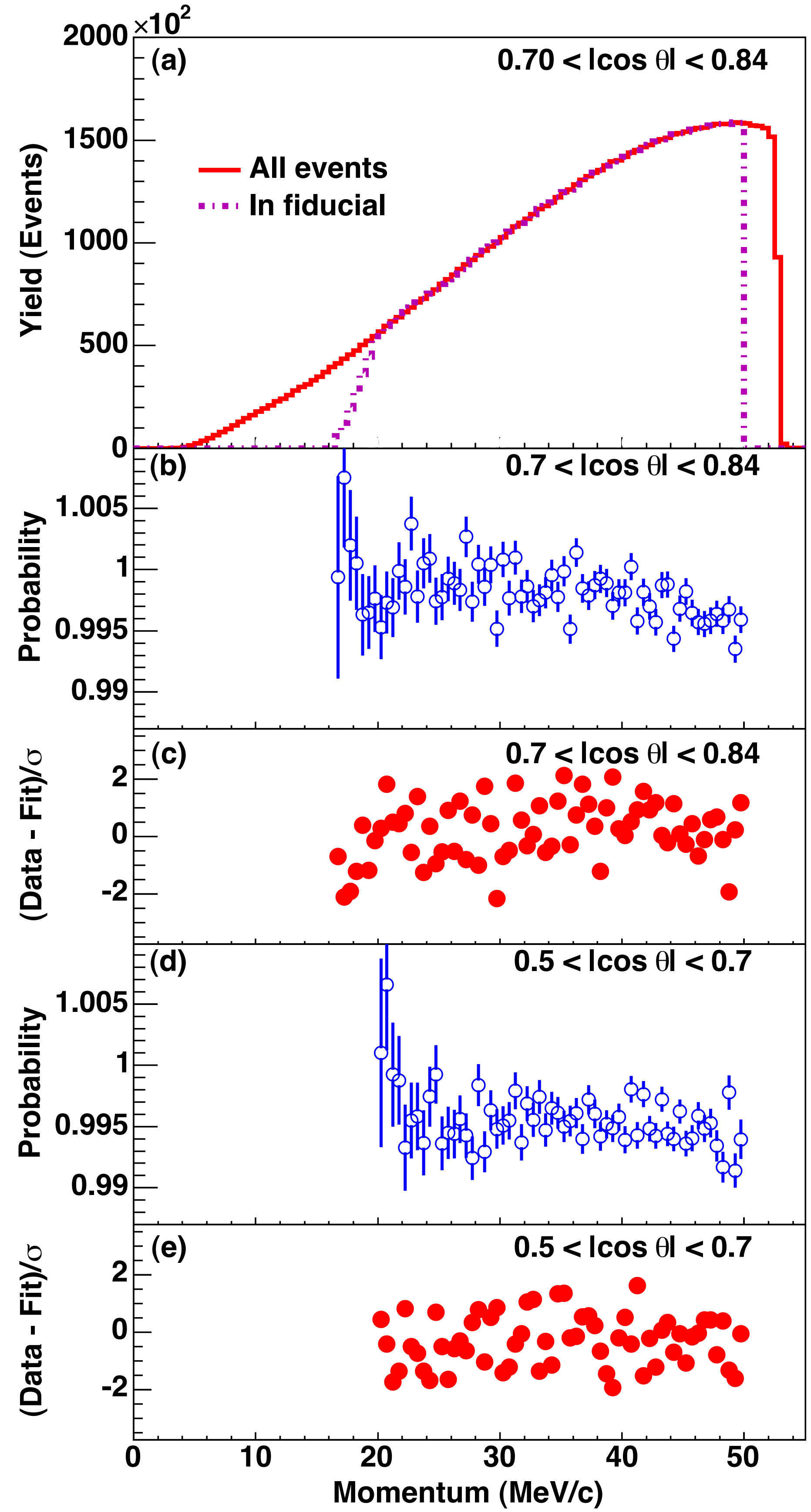
Muon Decay Measurements

- TWIST experiment at TRIMF in Canada measures $\mu^+ \to e^+ \nu_e \overline{\nu}_\mu$ decay spectrum.
- Excellent agreement between data and prediction!

- Measurements of muon lifetime and mass used to define a value for G_F (values from PDG 2010)
  - ➡ $\tau = (2.19703 \pm 0.00002) \times 10^{-6}$ s
  - ➡ $m = 105.658367 \pm 0.000004$ MeV

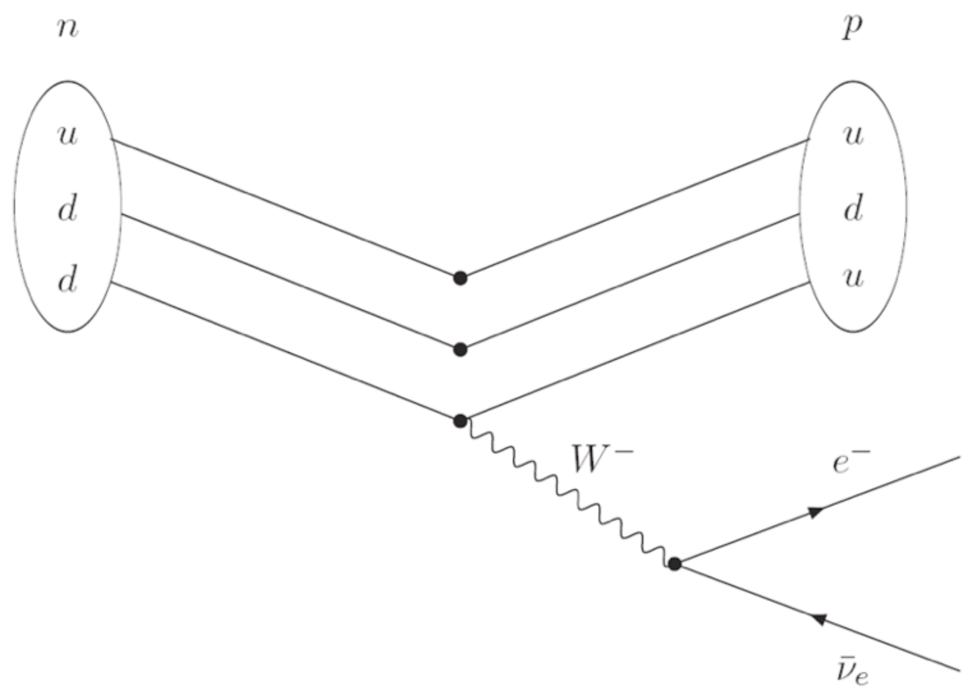
- Applying small corrections for finite electron mass and second order effects
  - ➡ $G_F = 1.166364(5) \times 10^{-5}$ GeV^{-2}

- Implies $g_W = 0.653$, $\alpha_W = g_W^2/4\pi = 1/29.5$
- $\alpha_W \gg \alpha_{EM}$, the weak force not intrinsically weak, just appears so due to mass of W-boson



J.R. Musser et al., Physical Review Letters 94, 101805 (2005)

Beta Decay



- W boson is responsible for beta decay.

Quark level

- $\mathbf{u} \rightarrow \mathbf{d}\ e^+\ \nu_e$ or $\mathbf{d} \rightarrow \mathbf{u}\ e^-\ \overline{\nu_e}$ with coupling $g_W V_{ud}$, $V_{ud} = 0.974$
- not directly observable because no free quarks

$$\mathcal{M} = \frac{V_{\rm ud} g_W^2}{8}\, \bar{u}(\mathrm{d})\gamma^\nu(1-\gamma^5)u(\mathrm{u})\, \frac{g_{\mu\nu}}{q^2 - m_W^2}\, \bar{v}(\bar{\nu}_e)\gamma^\mu(1-\gamma^5)u(e)$$

Hadron level

- $\mathbf{n} \rightarrow \mathbf{p}\ e^-\ \overline{\nu_e}$ is allowed (free neutron lifetime $\tau_n = 886\mathrm{s}$)
- $\mathbf{p} \rightarrow \mathbf{n}\ e^+\ \nu_e$ is forbidden $m_p < m_n$ (free proton stable)
- Hadronic interactions (form factors) play a role in decay rate/lifetime

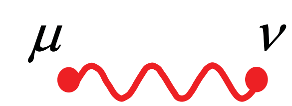
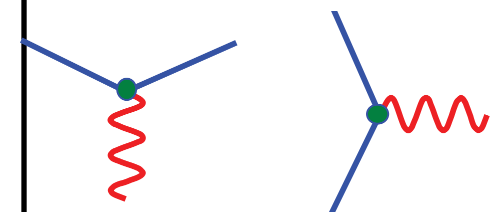
Nuclear level

- $\beta+$ decay e.g. $^{22}\mathrm{Na} \rightarrow {}^{22}\mathrm{Ne}^*\ e^-\ \overline{\nu_e}$
- $\beta-$ decay e.g. $^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni}^*\ e^-\ \overline{\nu_e}$
- which type occurs depends on the energy available (Q)

Neutral Current Interactions

- Exchange of massive Z-bosons, $m_Z = 91.1897(21)$ GeV
- Couples to all quarks and all leptons (including neutrinos)
- No allowed flavour changes!

- Coupling to Z-boson depends on the flavour of the fermion (f): c^f_V, c^f_A
- Both vector ($c^f_V \gamma^\mu$) and axial vector contributions ($c^f_A \gamma^\mu \gamma^5$).

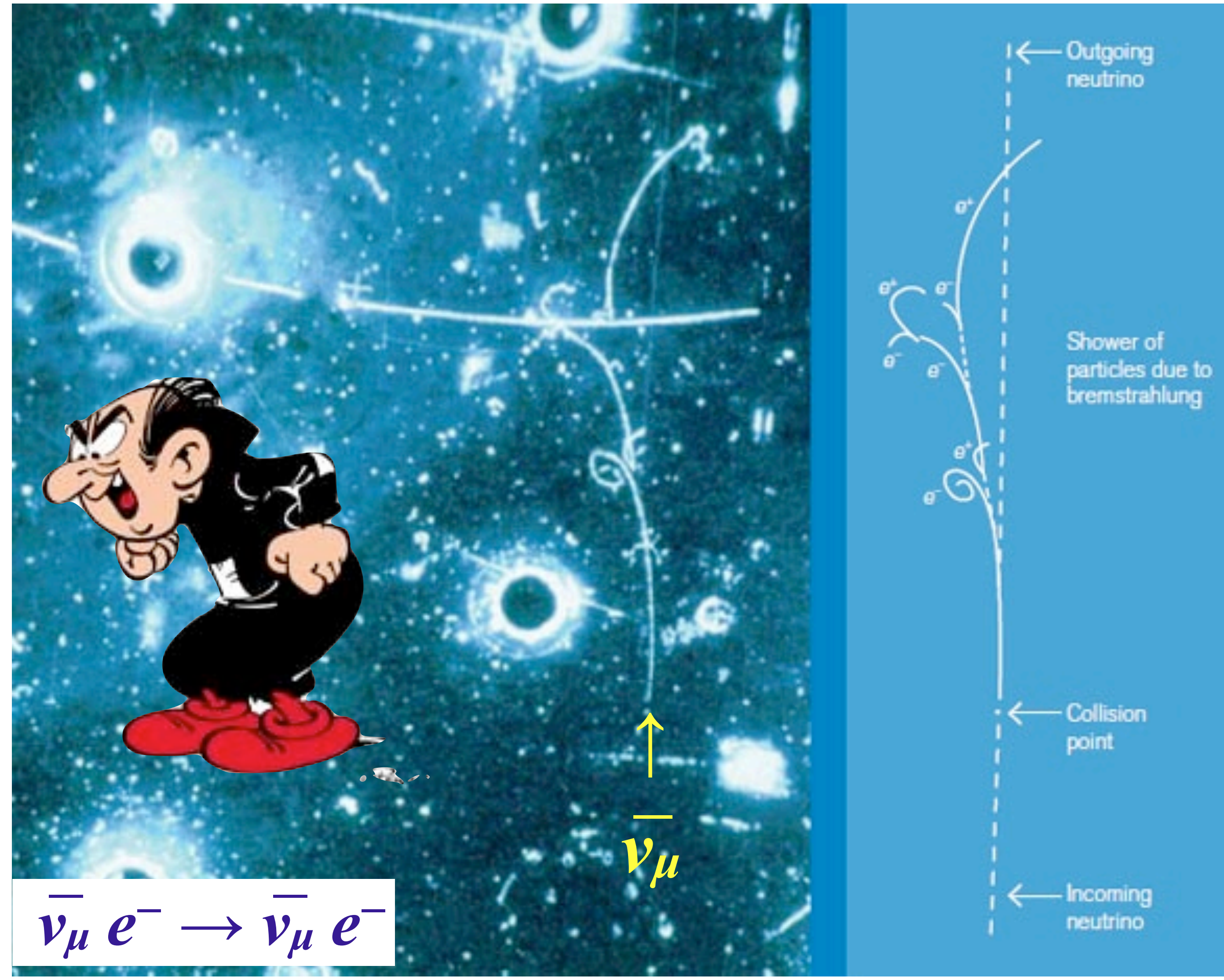
	propagator 	interaction vertex 
W-boson	$\frac{g_{\mu\nu}}{q^2 - m_W^2}$	$\frac{1}{2\sqrt{2}} g_W \gamma^\mu (1-\gamma^5)$
Z-boson	$\frac{g_{\mu\nu}}{q^2 - m_Z^2}$	$\frac{1}{2} g_Z \gamma^\mu (c_V^f - c_A^f \gamma^5)$
photon, γ	$\frac{g_{\mu\nu}}{q^2}$	$e \gamma^\mu$

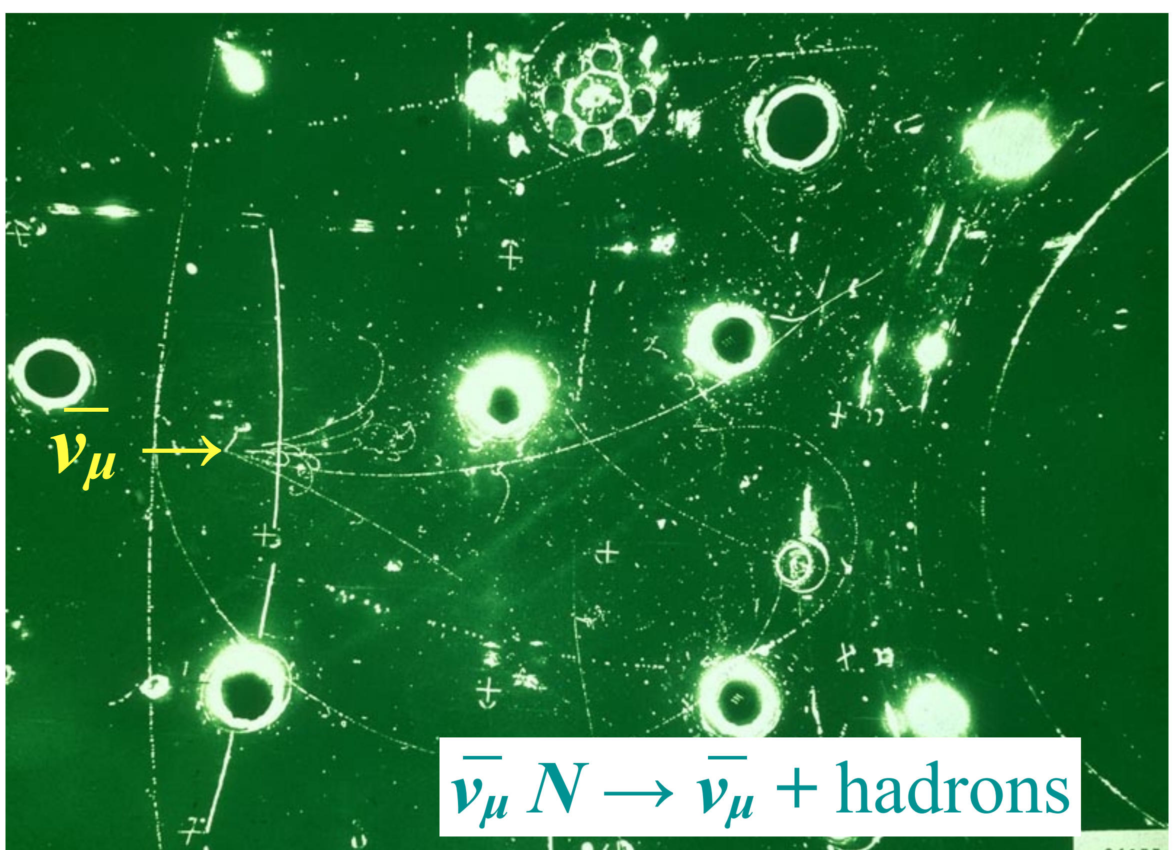
Lepton	c^f_V	c^f_A	Quark	c^f_V	c^f_A
ν_e, ν_μ, ν_τ	½	½	u, c, t	0.19	½
e, μ, τ	-0.03	−½	d, s, b	-0.34	− ½

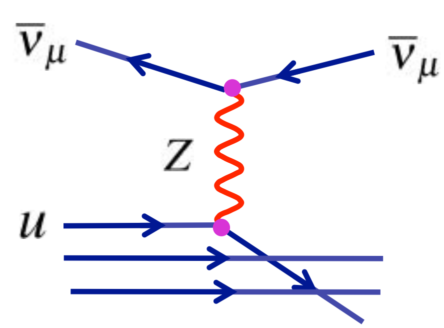
- g_Z coupling is related to g_W: $g_Z = g_W\, m_Z / m_W$
    - Neutral weak current for electron: $\bar{u}(e)\gamma^\mu (c_V^e - c_A^e \gamma^5) u(e)$

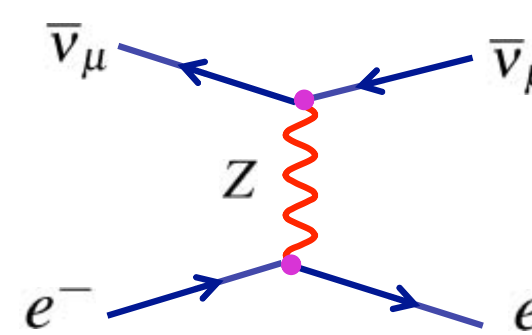
Weak Neutral Current

- At low energy, the main effect of Z-boson exchange is neutrino scattering. (All other Z-boson phenomena can also due to γ exchange.)
- Z-boson exchange first observed in the Gargamelle bubble chamber in 1973.
- Interaction of muon neutrinos produce a final state muon.









Symmetries in Particle Physics

- The EM, Weak and Strong forces all display a property known as Gauge Symmetry.
- In QM, a symmetry is present if **physical observables** (e.g. cross section, decay widths) are invariant under the following change in the wavefunction:

$$\psi \rightarrow \psi' = \hat{U}\psi$$

- e.g. in electromagnetism, the physical observable fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are independent of the value of the EM potential, $\boldsymbol{A_\mu}$:

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \chi \qquad A_\mu = (V, \vec{A}) \text{ with } \vec{B} = \vec{\nabla} \times \vec{A}$$

- The conditions on $\boldsymbol{U}$ are that $\boldsymbol{U}$ is unitary, and commutes with the Hamiltonian:

$$\hat{U}^\dagger \hat{U} = \mathbf{1} \qquad [\hat{U}, \hat{H}] = 0$$

- e.g. for EM, $\hat{U} = e^{i\phi}$ where $\boldsymbol{\phi}$ is an arbitrary phase: $\psi \rightarrow \psi' = e^{i\phi}\psi$

Symmetries in QED

- Instead of a global phase transformation $e^{i\phi}$ imagine a local phase transformation, where the phase $\phi \sim q\,\chi$ is a function of x^μ: $\chi(x^\mu)$.
  - q is a constant (will be electric charge)

$$\psi \to \psi' = \hat{U}\psi = e^{iq\chi(x^\mu)}\psi$$

- Substitute into Dirac Equation $(i\gamma^\mu\partial_\mu - m)\psi = 0$

$$\begin{aligned}
(i\gamma^\mu\partial_\mu - m)\psi' &= 0 \\
(i\gamma^\mu\partial_\mu - m)e^{iq\chi(x)}\psi &= 0 \\
i\gamma^\mu(e^{iq\chi(x)}\psi + iq\partial_\mu\chi - m)e^{iq\chi(x)}\psi &= 0
\end{aligned}$$

- An interaction term $-q\gamma^\mu\partial_\mu\chi\psi$ term appears in the Dirac Equation.
- To cancel this, modify the Dirac Equation for interacting fermions:

$$(i\gamma^\mu\partial_\mu + iqA_\mu - m)\psi = 0$$

- With A^μ transforming as:
- $A_\mu \to A'_\mu = A_\mu - \partial_\mu\chi$ to cancel interaction term

Gauge Symmetry in QED

- Demanding that QED is invariant by a local phase shift:

$$\psi \rightarrow \psi' = \hat{U}\psi = e^{iq\chi(x^\mu)}\psi$$

- Tells us that fermions interact with the photon field as:

$$q\gamma^\mu A_\mu \psi$$

- This local phase shift is know as a **local U(1) gauge symmetry**.

- Next lecture we will see a similar effect in QCD.