4 Checkpoint 4

Aim of Checkpoint

This checkpoint explores the behaviour of a damped simple harmonic oscillator for a range of damping coefficients. It involves writing a medium sized JAVA program to trace out the amplitude against time for a damped SHM oscillator under a range on conditions.

From a computing viewpoint this checkpoint demonstrates the use of loops, calling standard `Math` mathematical methods and the `SimpleGraph` class combined with the `Display` class. From a Physics 2A viewpoint it illustrates the results from the vibration lectures that you will have / are about to, complete.

This checkpoint is worth **30%** of the course mark.

This is also the starting point for more experienced JAVA*programmers.*

Submission Dates

It is expected that this checkpoint is completed during the **fourth** or **fifth** laboratory session.

Final submission date for this checkpoint is: **5.00 pm, Thursday 3rd November**

Damped SHM

As discussed in detail in the Physics 2A lectures, the damped simple harmonic oscillator satisfies the second order differential equation,

$$m\ddot{x} + b\dot{x} + kx = 0$$

where m is the mass of the oscillator, b is the coefficient of damping, and k is the harmonic force constant. Defining new constants $\gamma = b/m$, and $\omega_0^2 = k/m$, we can re-write this as:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

where ω_0 is the *natural frequency* of the undamped oscillator.

The solutions to this equation has the following forms:

$$x = \exp(-\gamma t/2)\,(a\cosh(pt) + b\sinh(pt)) \qquad \text{when } \gamma > 2\omega_0 \text{ with } p^2 = \left(\gamma^2/4\right) - \omega_0^2$$
$$x = \exp(-\gamma t/2)(a + bt) \qquad \text{when } \gamma = 2\omega_0$$
$$x = \exp(-\gamma t/2)\,(a\cos(\omega t) + b\sin(\omega t)) \qquad \text{when } \gamma < 2\omega_0 \text{ with } \omega^2 = \omega_0^2 - \left(\gamma^2/4\right)$$

where these three conditions are known as "*over damped*" ,"*critically damped*" and "*under damped*" respectively.

Read these equations very carefully and watch where the brackets are.

With the initial conditions that $x = 1$ and $\dot{x} = 0$ at $t = 0$ the above constants, after some manipulation, become,

$$a = 1 \quad b = \gamma/2p \quad \text{when } \gamma > 2\omega_0$$
$$a = 1 \quad b = \gamma/2 \quad \text{when } \gamma = 2\omega_0$$
$$a = 1 \quad b = \gamma/2\omega \quad \text{when } \gamma < 2\omega_0$$

You should check these results for yourself.

Checkpoint Task

Write an interactive JAVA program to compute and display via the `SimpleGraph` class the solution for x against t for t in the range $0 \rightarrow 5\pi/\omega_0$. Your program should:

1. Read in the *value* of ω_0 and γ from the a `Display` input panel.
2. Calculate and plot the output with the `SimpleGraph` class.
3. Re-prompt for new *values of* ω_0 and γ.
4. Replot the output *either* on the same graph, **or** in a new window.

To obtain a good smooth graph you need to calculate the above expression for at least 200 evenly spaced values of t.

Warning: The structure of this program is reasonably complex. Think very carefully how you are going to structure it before you start. Also be **very careful** when coding the expression for the amplitude of the oscillation, in particular make sure it is calculating what you expect!

End of Checkpoint

When you have completed your program, call a demonstrator and show them the code and the program working for all **three** dampling conditions. This is the end of **checkpoint 4**. Ensure that the demonstrator checks off your name.

Material Needed

In addition to the material for Checkpoint *3 you will need material from the following documents:*

1. *Loops.*
2. *Graphical Output.*

What Next?

You should now read the next two, fairly complex, sections on *Array* and *Methods* before attempting the next Checkpoint.