Topic 5: Noise in Images

Aim: Covers the origins of noise in image system. This includes fixed pattern noise from CCD systems, imaging striping and methods for removal. The Guassian noise model is detailed showing the approximations needed to obtain zero meaned additive signal independant Guassian noise. The baics propertis of additive noise and signal to noise ratio are introduced.

Contents:

Introduction

Consider all processes effecting the image **NOT** related to the object as being *Noise*

Range of origins:

1. Discrete nature of radiation
2. Detector sensitivity
3. Electrical noise
4. Film grain
5. Data transmission errors
6. Air turbulence
7. Image Quantisation

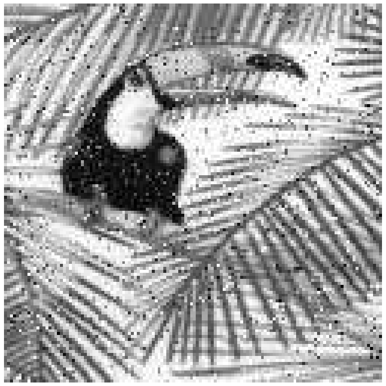
In this section we will consider some of the simple noise models, including data transmissin errors, and intrinsic noise resulting from the disctere nature of radiation.

Data Drop-Out Noise

In many data transmission systems, random bits corrupted or lost on a data channel.

Typically appears as *snow* on images.

1 in 100 bits



1 in 20 bits

Corruption very common in satellite images and video systems where bits are set wrong,

Type of corruption is not correlated with the image data, and can be significantly reduced by;

- Threshold Average Filter
- Median Filter

With filtering error rate of about 1.5% can be removed without significant degradation of the image.

These filters will be discussed in detail in Lectures 9-10.

Fixed Pattern Noise

Two-Dimensional CCD systems, variable sensitivity of detectors. Typical problem on CCD sensors due to variability in manufacture.

Noise can be correct on a point by point basis be calibration of each sensor.

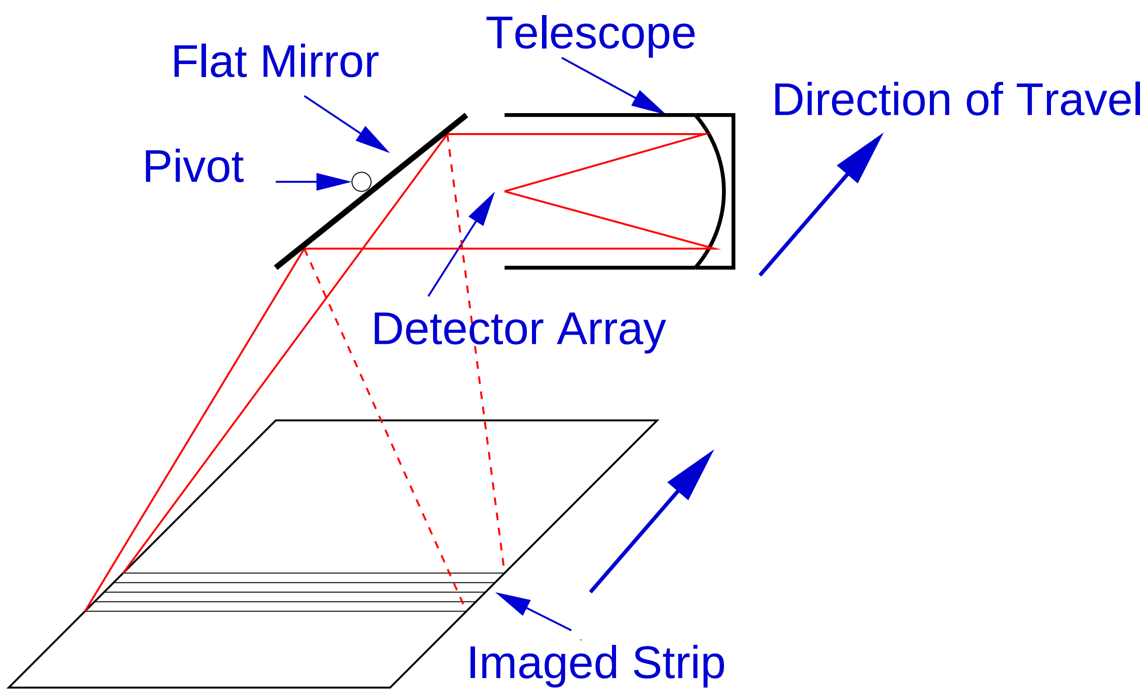
Typical fixed pattern noise from a CCD camera, (image stretch to about 5 grey levels).

Take measures at a range of light intensities, build-up sensitivity profile for each pixel point.

Typically only needed for critical applications, such as astronomy, or when sensor is very poor (Infra-red detector arrays).

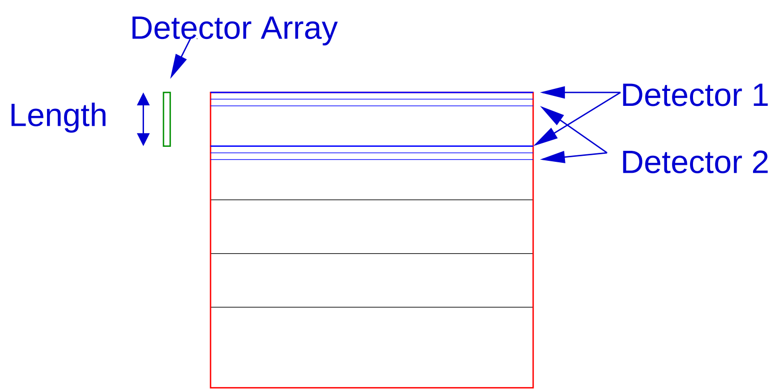
Satellite Image Striping

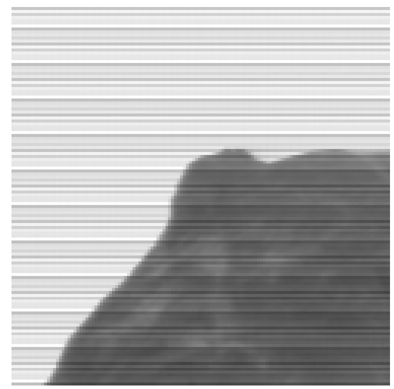
Many scanning systems systems use a one-dimensional



Satellite Image Striping I

Giving an image geometry of





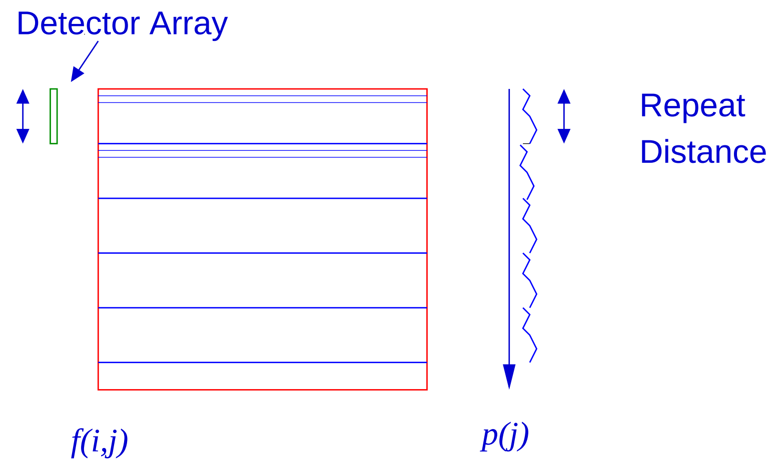
1-D scanning system show striping due to varying detector sensitivity.

Satellite Image Striping II

Form a projection,

$$p(j) = \sum_{i=0}^{N-1} f(i,j)$$

sensor of K pixels long, periodic structure of period K associated with detector sensitivity, so can be calculated and corrected for.

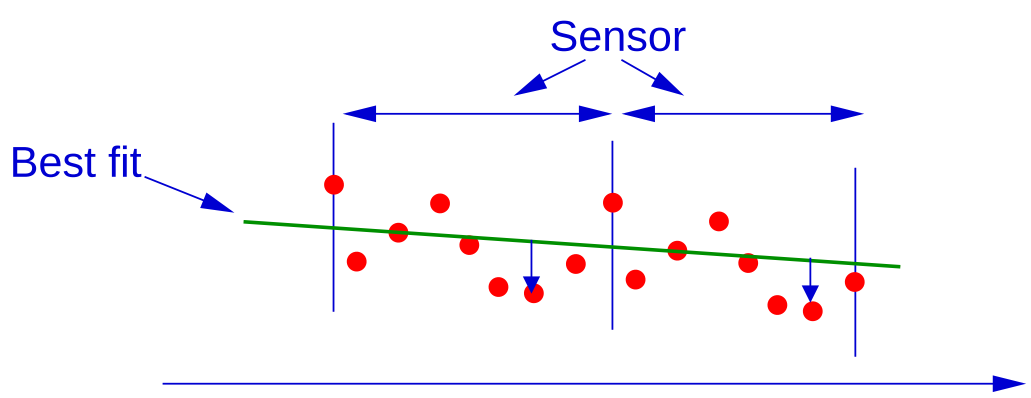


Satellite Image Striping II

If this is repeated on many images taken with the same sensor, able to obtain a good estimate of sensor variability.

- Additive error (charge leakage from CCD)
- Multiplicative error (gain variability in amplifier)
- General non-linear errors, need look-up-table for each detector.

Assume a that projection is smooth, deviation from best fit will give sensor partameters



Also possible: Fourier transform filtering, information causing striping at particular spatial frequency, so can be removed by suitable Fourier filters.

Statistical Method

If an image is scaled by a multipliative factor a and additive term

$$g(i,j) = a\,f(i,j) + b$$

we have that

$$\langle g \rangle = a \langle f \rangle + b$$

and

$$\sigma_g^2 = a^2 \langle f^2 \rangle - a^2 \langle f \rangle^2 = a^2 \sigma_f^2$$

If we have a detector with K elements, we effectively detect K images

$$g_k = a_k\, f + b_k$$

with each detected image being of size $N/K \times N$, being a reduce resolution version of $f(i,j)$.

The image statistics on mean are variance do not depend on the resolution, so and

$$\langle g_k \rangle = a_k \langle f \rangle + b_k \quad \text{and} \quad \sigma_{g_k}^2 = a_k^2 \sigma_f^2$$

so allowing us to solve for a_k and b_k.

One you have the a_k and b_k you can easily reform the real image $f(i,j)$ from the subimages.

Basis of the DIA Programming task

Detector or Shot Noise

Noise process *intrinsic* to the process of measurement, not related to an imaging system.

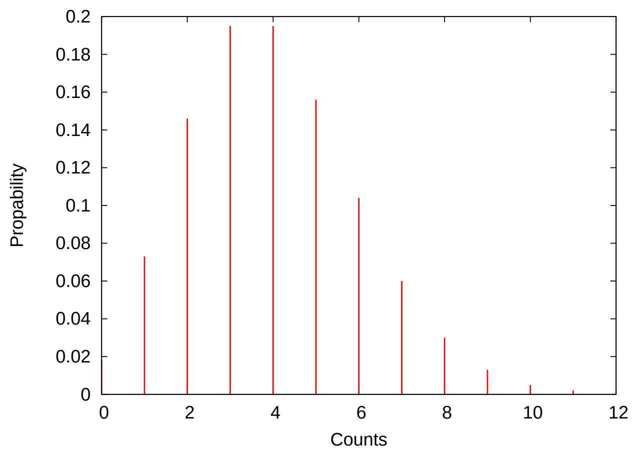
All imaging systems actually count *particles*, (electrons or photons), which are governed by statistical and physical laws.

For a *source* of average brightness $\langle \mu \rangle$ *expected* observed value is

$$\langle f \rangle = \Delta t \langle \mu \rangle$$

However a single observation is random variable from the PDF

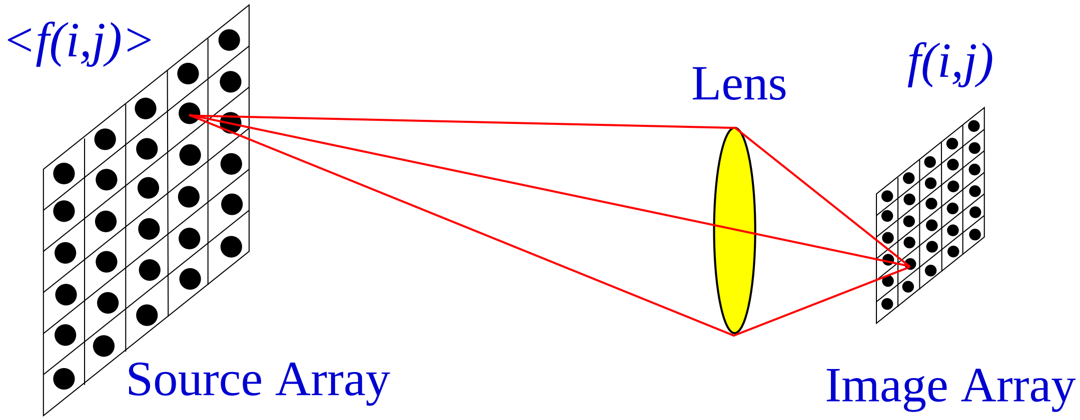
$$p(f) = \frac{\langle f \rangle^{f} \exp(-\langle f \rangle)}{f!}$$



PDF of a poisson distribution with $\langle f \rangle = 4$

Two-Dimensional Image

For the 2-D case we have, we assume that we have a two-dimensional array of sources.



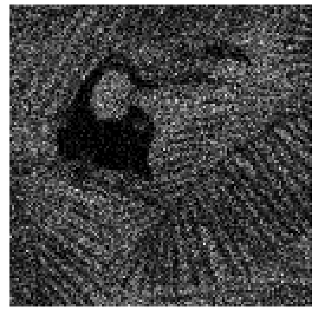
Source brightness of: $\langle f \rangle(i,j)$ then PDF for each pixel,

$$p(f(i,j)) = \frac{\langle f \rangle(i,j)^{f(i,j)} \exp(-\langle f \rangle(i,j))}{f(i,j)!}$$

Simulated Example

Digital simulation with average number of photons per pixel specified.

1 Photons/Pixel



4 Photons/Pixel

16 Photons/Pixel

64 Photons/Pixel

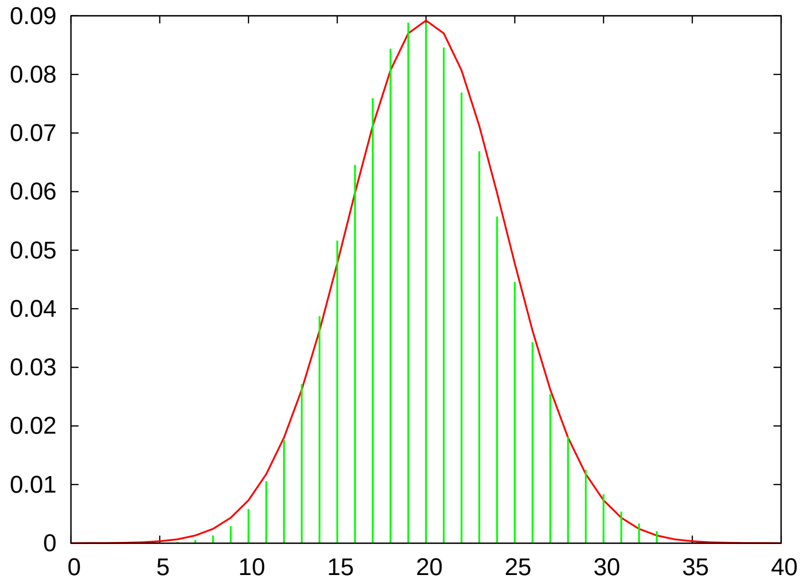
Low numbers of photons, noise dominates, but as number increase the image become usable.

Gaussian Approximation

The Poisson distribution is mathematically difficult, (discrete distribution).

For large *expected* values, this may be approximated by a Gaussian of **mean** u and **halfwidth** $\sqrt{2u}$

$$p(n) = \frac{u^n \exp(-u)}{n!} \to \frac{1}{(2\pi u)^{1/2}} \exp\left(\frac{-(n-u)^2}{2u}\right)$$



This has an error of $\approx 1\%$ for $u > 20$.

Gaussian Approximation I

So for mean of $\langle f \rangle$, approximate $p(f)$ by

$$p(f) = \frac{1}{(2\pi\langle f \rangle)^{1/2}} \exp\left(\frac{-(f - \langle f \rangle)^2}{2\langle f \rangle}\right)$$

So if we Regard the measured value f as

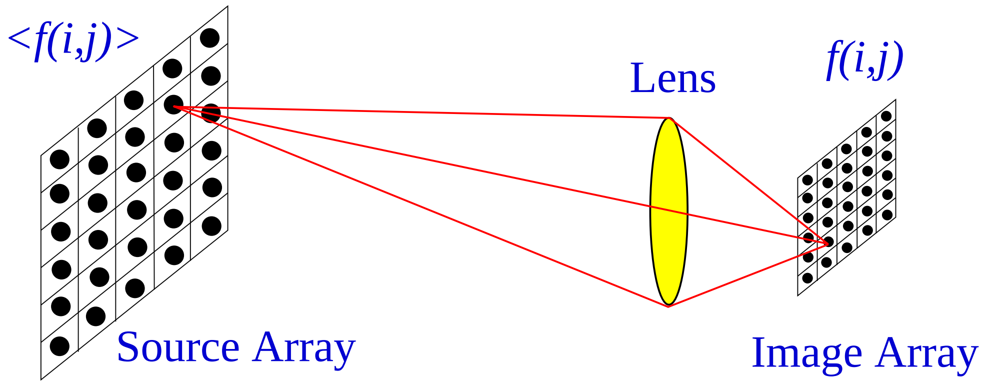
$$f = \langle f \rangle + n$$

where n is the noise so the PDF of the noise is,

$$p(n) = \frac{1}{(2\pi\langle f \rangle)^{1/2}} \exp\left(\frac{-n^2}{2\langle f \rangle}\right)$$

Two-Dimensional Image

For the 2-D case we have, we assume that we have a two-dimensional array of sources.



Source brightness of: $\langle f\rangle(i,j)$ then we measure,

$$f(i,j) = \langle f\rangle(i,j) + n(i,j)$$

where the PDF of the noise is given by.

$$p(n(i,j)) = \frac{1}{(2\pi\langle f\rangle(i,j))^{1/2}} \exp\left(\frac{-n(i,j)^2}{2\langle f\rangle(i,j)}\right)$$

This is an additive noise model, where the PDF of the noise **depends** on the signal. Known as

Signal Dependant Additive Noise.

This model assumes the imaging system is space invariant and linear.

Low Contrast Approximation

If we assume the image is *Low Contrast* so

$$\langle f \rangle(i,j) \approx \text{const} = \mu$$

we have that

$$p\left(n(i,j)\right) = \frac{1}{(2\pi\mu)^{1/2}} \exp\left(\frac{-n(i,j)^2}{2\mu}\right)$$

which is **independent** of the structure of the image.

Additive noise, with PDF being **Zero Mean Gaussian** with variance μ, the mean of the expected image value,

$$\mu = \langle (\langle f \rangle(i,j)) \rangle$$

This is the typical assumption for image noise models with are

Signal Independant Additive Gaussian

This assumes imaging system is:

- Linear and Space Invariant
- High brightness (many photons/electrons)
- Low contrast

for this noise model to be valid.

Validity of Additive Noise

In taking *additive* signal independent model, we assume *High brightness* & *Low contrast*.

Assumption valid in many imaging systems,

1. Video images (Many thousands on photons/electrons)
2. Infra-red (Usually very low contrast)
3. Electron microscope (Usually very low contrast)
4. CT and MRI Medical imaging (low contrast)

Not valid in,

1. γ-camera (small count number)
2. Astronomical images (small count number **and/or** very high contrast).
3. Image intensifier systems (small count number)
4. High magnification electron microscope (small count number)

If the additive signal independant noise model is not valid, processing is much more difficult.

Most processing used additive signal independant noise model, even if it is not *really* valid.

Properties of Additive Noise

Take the additive noise model as:

$$f(i,j) = s(i,j) + n(i,j)$$

where $s(i,j)$ is signal and $n(i,j)$ is noise.

The PDF of $n(i,j)$ is zero mean Gaussian so:

$$\langle n(i,j)\rangle = 0 \quad \text{and} \quad \left\langle |n(i,j)|^2 \right\rangle = \sigma_n^2$$

so that

$$\langle s(i,j)\rangle = \langle f(i,j)\rangle = \mu$$

Noise is independent of signal $s(i,j)$, so un-correlated, mathematically that:

$$\langle s(i,j) n(i,j)\rangle = 0$$

so the variance of $f(i,j)$ is:

$$\sigma_f^2 = \left\langle |f(i,j) - \langle f(i,j)\rangle|^2 \right\rangle$$

By substitution, and above properties, this can be expanded to give

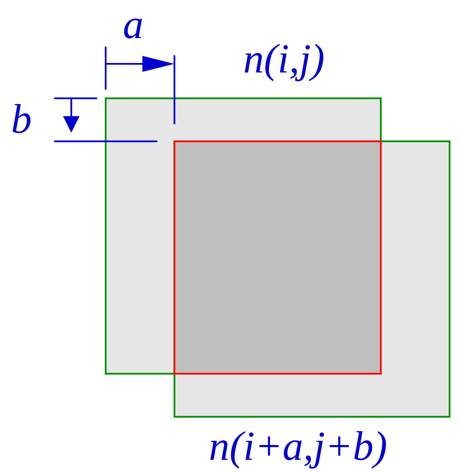
$$\sigma_f^2 = \sigma_s^2 + \sigma_n^2$$

where σ_s^2 is the Variance of the signal.

So the addition of noise alters the Variance but not the Mean.

Properties of Additive Noise I

We have assumed that each image point is independant, so the noise is not correlated with-itself,



mathematially this means:

$$n(i,j) \otimes n(i,j) = \delta_{i,j} \langle |n(i,j)|^2 \rangle = \delta_{i,j} \sigma_n^2$$

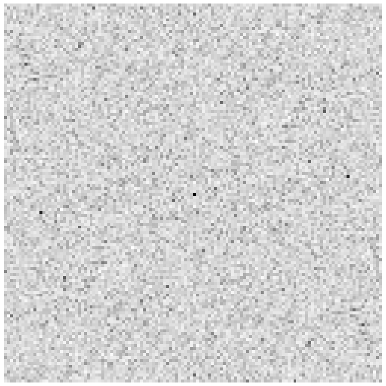
so the Auto-correlation is a δ-Function.

From the (auto)-correlation theorm, the Fourier Transform of the *Auto-correlation* is the *Power Spectrum*, so:

$$|N(k,l)|^2 = \text{Constant}$$

Properties of Additive Noise I

Known as *White Noise* with equal power at all spatial frequencies.



Power Spectrum of single realisation not actually constant, but equal power over each region of Fourier space.

Parseval's Theorm
We have that the power in real space and Fourier space is the same, so that

$$\langle |N(k,l)|^2 \rangle = \langle |n(i,j)|^2 \rangle = \sigma_n^2$$

Processing of Noisy Images

In Fourier space we have that

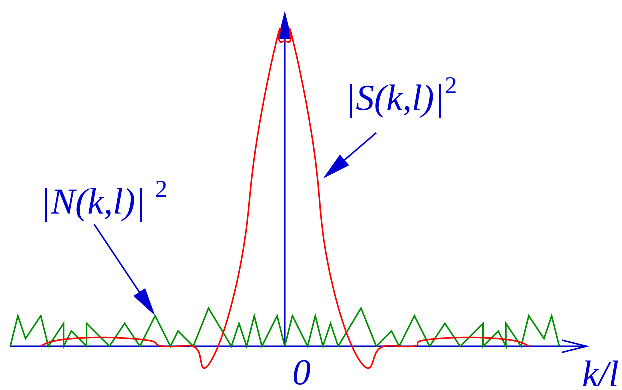
$$F(k,l) = S(k,l) + N(k,l)$$

we know that

$$\begin{aligned} S(k,l) &= \text{Sharply peaked about low spatial frequencies} \\ N(k,l) &= \text{Constant at all spatial frequencies} \end{aligned}$$

so that when

$$\begin{aligned} |S(k,l)| &\gg |N(k,l)| \quad \text{Little effect} \\ |S(k,l)| &\approx |N(k,l)| \quad \text{Signal corrupted} \end{aligned}$$



Greatest effect at high spatial frequencies where $S(k,l)$ is small. So high spatial frequencies corrupted by the noise.

Processing of Noisy Images I

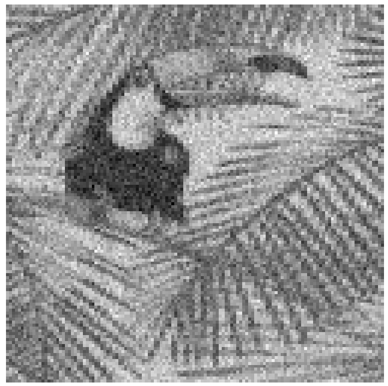
Reduce effect of noise by Low-Pass filtering

- Fourier low pass
- Real Space averaging
- Median filter

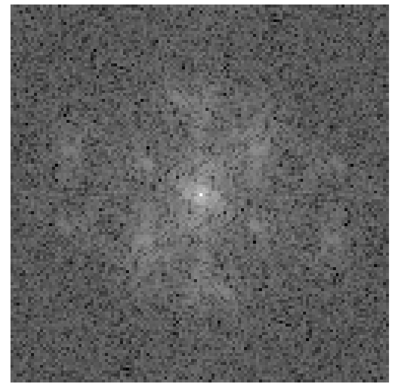
all of which remove or reduce high spatial frequenies, thus the effect of noise.

Deatils in next two lectures.

Digital Example

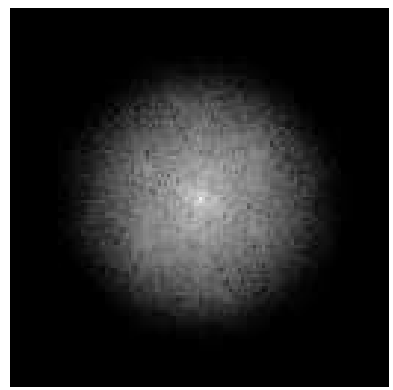


Noisy Image



Fourier Transform

Low-pass filtered



Fourier Transform

Signal to Noise Ratio

The use of SNR (Signal to Noise Ratio) is a confusing topic, since

- There is a range (about 10) definitions.
- Can a single number really classify how good an image is.
- In practice image quality is very strongly image dependant.

For signal independent additive noise,

$$f(i,j) = s(i,j) + n(i,j)$$

Signal & Noise variances are:

$$\begin{aligned} \sigma_s^2 &= \langle |s(i,j) - \langle s(i,j) \rangle|^2 \rangle \\ \sigma_n^2 &= \langle |n(i,j)|^2 \rangle \end{aligned}$$

Define SNR by

$$\mathrm{SNR} = \frac{\sigma_s}{\sigma_n}$$

Signal to Noise Ratio I

Noting that the signal and the noise are uncorrelated, we have

$$\sigma_f^2 = \sigma_s^2 + \sigma_n^2$$

so we can write the SNR as:

$$\mathrm{SNR} = \sqrt{\frac{\sigma_f^2}{\sigma_n^2} - 1}$$

To calculate SNR, need 2 of σ_s, σ_f, or σ_n.

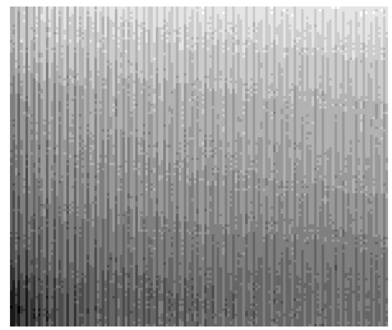
Calculation of SNR for Single Image

From single image can **only** find σ_f.

If region of image with **NO** signal can estimate σ_n from that region.

This method works for an image that contains a large region of "water" or "sky" where these is not signal.

Whole Image



Piece of Sky

Sky region shows typical CCD array fixed pattern noise.

Calculated vales are $\sigma_f^2 = 5287$ and $\sigma_n^2 = 1.85$, so that

$$\mathrm{SNR} \approx 53.4$$

Calcualte SNR for Multiple Images

Assume that we have **two** realisation of same scene, so:

$$\begin{aligned} f(i,j) &= s(i,j) + n(i,j) \\ g(i,j) &= s(i,j) + m(i,j) \end{aligned}$$

which is equivalent to two image of the same scene take at different times.

The noise in each realsiation have the same PDF, so that

$$\begin{aligned} \langle n(i,j) \rangle = \langle m(i,j) \rangle = 0 \\ \langle |n(i,j)|^2 \rangle = \langle |m(i,j)|^2 \rangle = \sigma_n^2 \end{aligned}$$

The two noise realisations were measured at *different* times, so they are uncorrelated, so:

$$\langle n(i,j)\, m(i,j) \rangle = 0$$

In both cases the noise is uncorrelated with the signal, so that

$$\langle s(i,j)\, n(i,j) \rangle = \langle s(i,j)\, m(i,j) \rangle = 0$$

Calcualte SNR for Multiple Images I

If we then define the *Normalised* Correlation between $f(i,j)$ and $g(i,j)$ as:

$$r = \frac{\langle (fg - \langle f \rangle \langle g \rangle) \rangle}{[\langle | f - \langle f \rangle |^2 \rangle \langle |g - \langle g \rangle |^2 \rangle]^{1/2}}$$

Then be expanding, collecting terms, and cancelling all zero terms, *it-can-be-shown* that

$$r = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}$$

to we can form the SNR from

$$\mathrm{SNR} = \sqrt{\frac{r}{1-r}}$$

So allowing direct calculation of the SNR independant of the type of the signal.

Calcualte SNR for Multiple Images II

If there is more than two realiastion available a better estimate for SNR can be found by forming the normalised correlation between pairs of images and averaging.

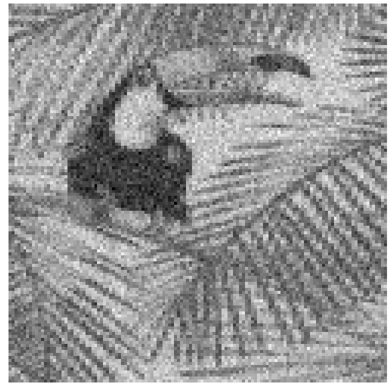
In practice this measure of SNR looks about right for most images. For examples:

SNR	$>$	20	Little visible noise
SNR	$\approx$	10	Some noise visibile
SNR	$\approx$	4	Noise clearly visible
SNR	$\approx$	2	Image severly degraded
SNR	$\approx$	1	Is there an image?

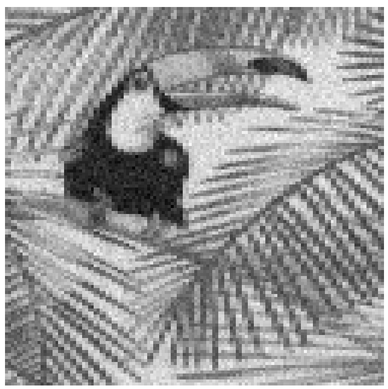
Digital Simulation

Digitally simulated “noisy” images:

$\mathrm{SNR} = 1$



$\mathrm{SNR} = 2$



$\mathrm{SNR} = 4$

$\mathrm{SNR} = 8$

Images formed by addition of Gaussian random noise.

Summary

In this section we have considered:

- Types of noise.
- Data drop-out noise
- Fixed pattern noise for two-dimensional CCDs
- Fixed pattern noise of one-dimensional sensors and how to correct.
- Detector or shot noise and Gaussian approximation.
- Properties of additive noise and underlying assumptions.
- Signal to Noise Ratio and methods of measuring.