Topic 9:

Holographic Interferometry

Aim: Covers the basics of frozen fringe, live fringe and time averaged holography is simple geometries.

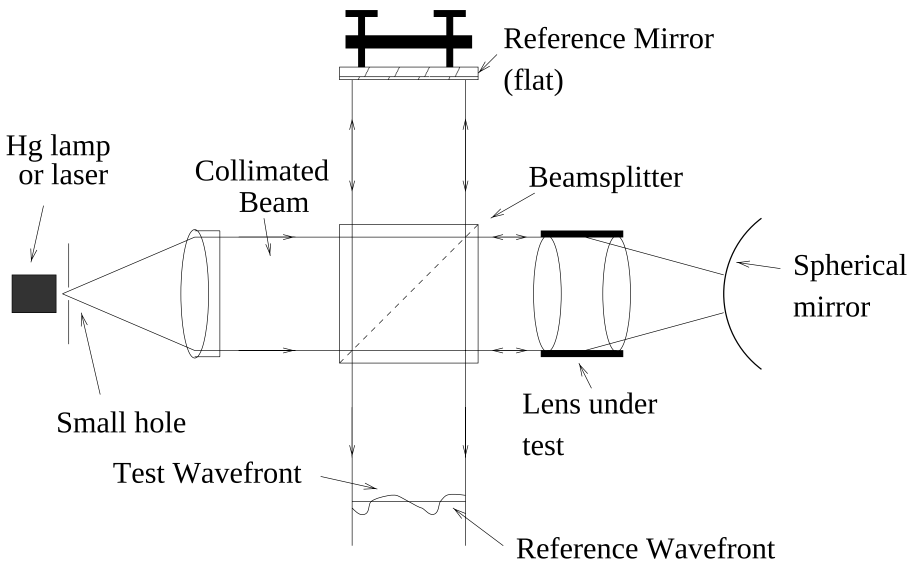
Contents:

- Concepts of Interferometry.
- Types of Holographic Interferometry.
- Two Wave Interferometry.
- Rigid Object Restriction.
- Time Averaged Holography

Interferometry

All two beam interferometers rely on Constructive and Destructive interference to give fringes.

Fringes give **Contours** of Optical Path Difference.



Bright Fringe when

$$\mathrm{OPD} = \pm n\lambda$$

Holography: Replace the mirrors with **two** objects waves to obtain interference between them.

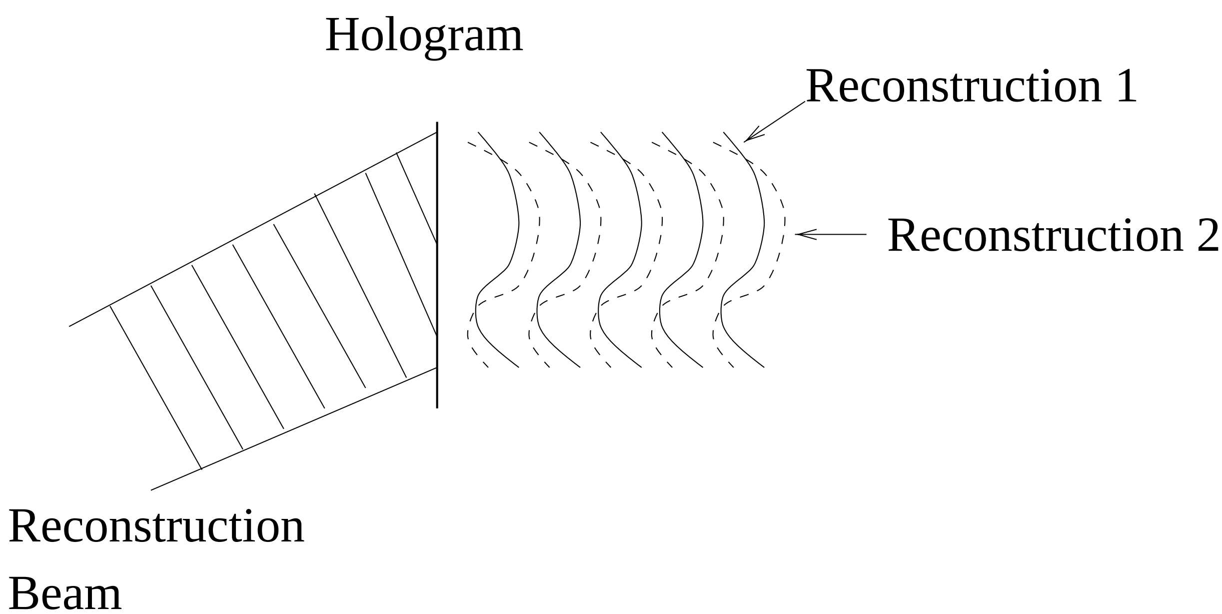
Three Types of holographic interferometry, being

1. Frozen Fringe
2. Life Fringe
3. Time Averaged

We will now look at these three types.

1) Frozen Fringe:
Expose **Two** holograms on the same plate with a movement between.

We get **two** coherent reconstructions, with interference between them.

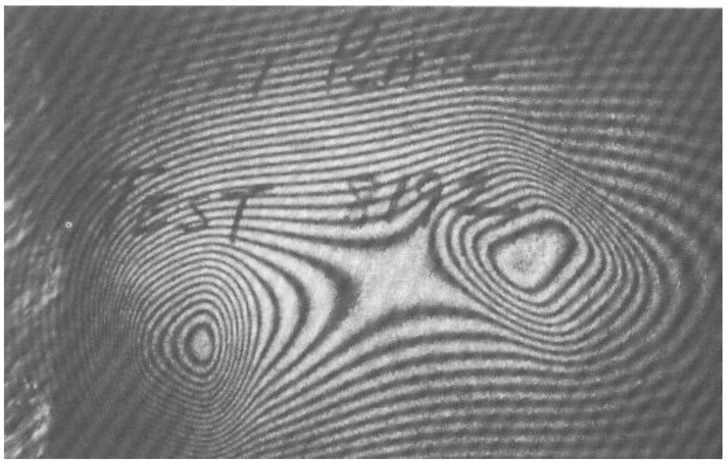


Two reconstructions will interfere and give fringes of constant OPD between the two reconstructions. (Measure difference from fringes).

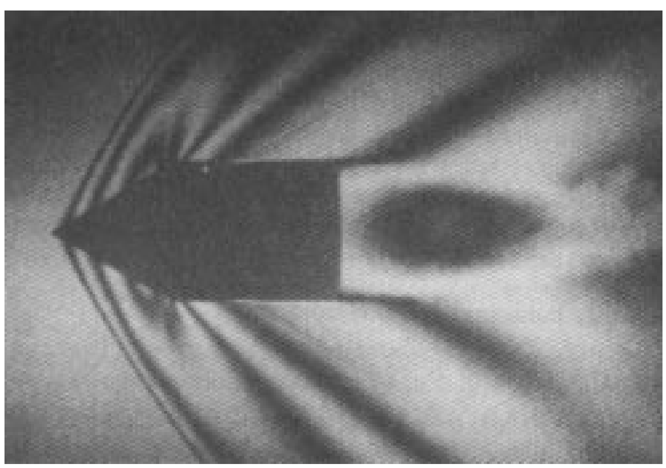
Fringes *Frozen* into the hologram. Used to take "snap-shot" of system. Can be used with pulsed lasers to analyse moving or distorting objects.

Examples

Double exposure hologram of a flat metal plate which was stressed between exposures from Hilton & Mayville, Opt. Eng **24** 757-768 (1985)

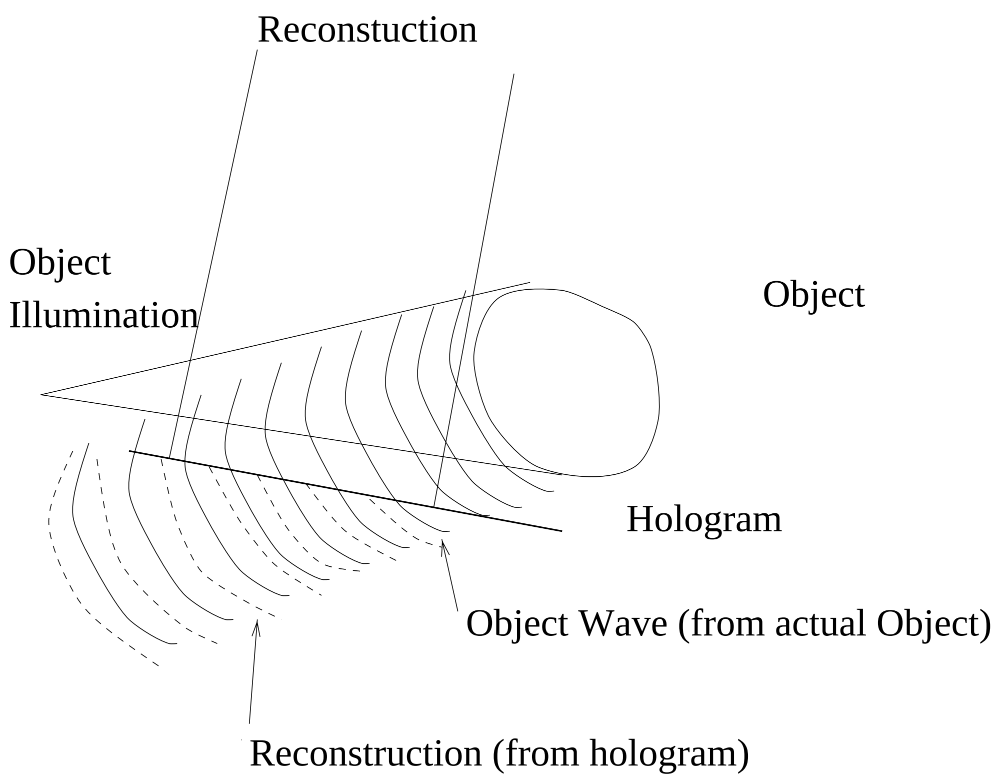


Double exposure hologram of bullet in flight taken with pulser Q-switched ruby laser showing shape of pressure wave about the bullet.



2) Live Fringe:
Make a single exposure hologram, and replace in original location.

Original reference beams become reconstruction beam.



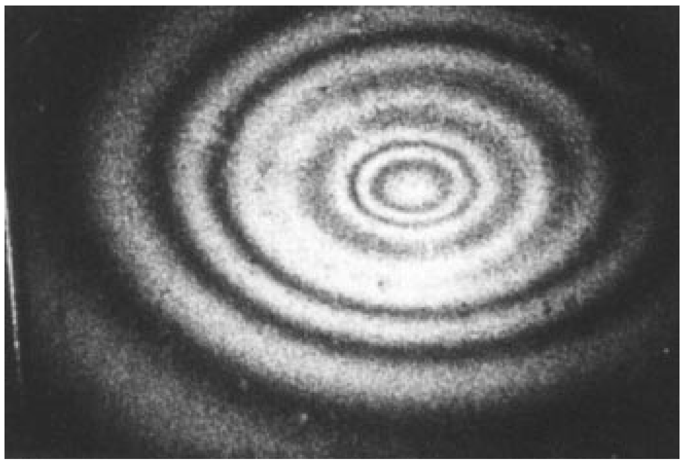
Interference between object and static reconstruction. Distort object, *Live Fringes*.

Very difficult to set-up. Replace plate to better than $\lambda/2$, (develop the plate in place).

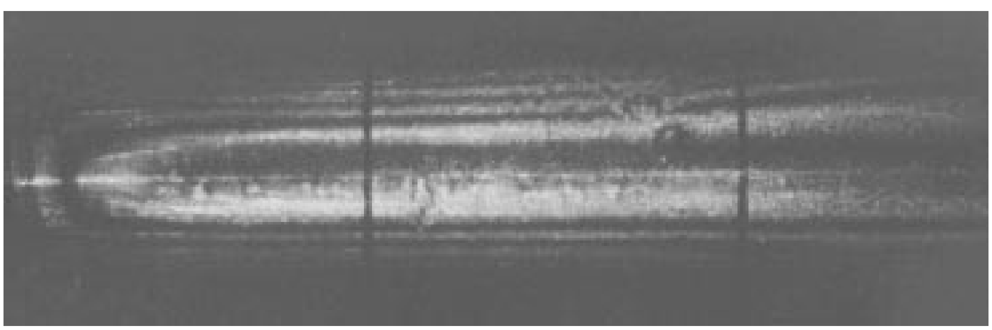
Ideal with the thermo-plastic camera.

Examples

Live fringe image for forced mass transfer experiment with air jet on swollen polymer film. Fringes caused by shrinkage of polyer due to removal of swelling agent.



Live fringe image of convective mass transfer experiment of model AGR fuel-rod using swollen polymer technique.



Both image taken in Chemical Engineering (U of E) using Newport thermo-plastic holographic camera. Nebrensky, (1996)

3) Time Averaged
Periodic motion of object (vibration).

We take **long exposure** (much longer than period of vibration).

Bright regions at nodes of vibration and fringes giving amplitude of vibration.

User for analysis of vibrating objects, from loudspeakers to parts of jet engines. (The most frequently used holographic analysis technique).

Examples: See end of lecture.

Summary

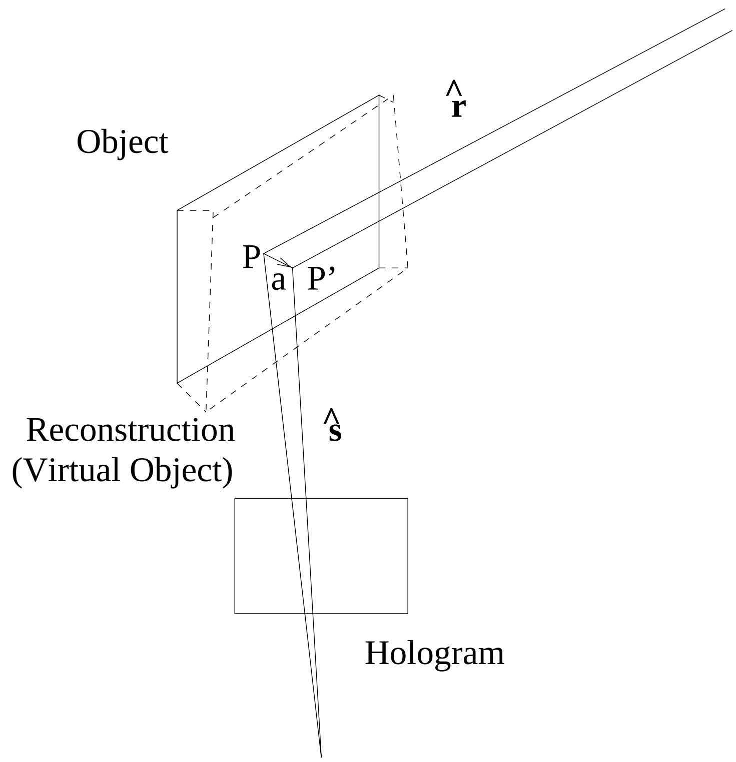
a) Two wave systems (frozen or live) fringes used to analysis movement, (also stress and distortion).

b) Time Averaged to get amplitude of vibration.

Both techniques **very sensitive**, able to measure movements and vibrations of order of λ. Often too sensitive for practical systems.

Two Wave Systems

Two images of an object (either frozen or live fringes).



Point P on object moves to point P' on reconstruction (virtual object).

$\hat{\mathbf{r}}$	$\rightarrow$	Illumination direction
$\hat{\mathbf{s}}$	$\rightarrow$	Viewing direction
$\mathbf{a}$	$\rightarrow$	Displacement of $P \rightarrow P'$

Optical Path difference in the two rays is

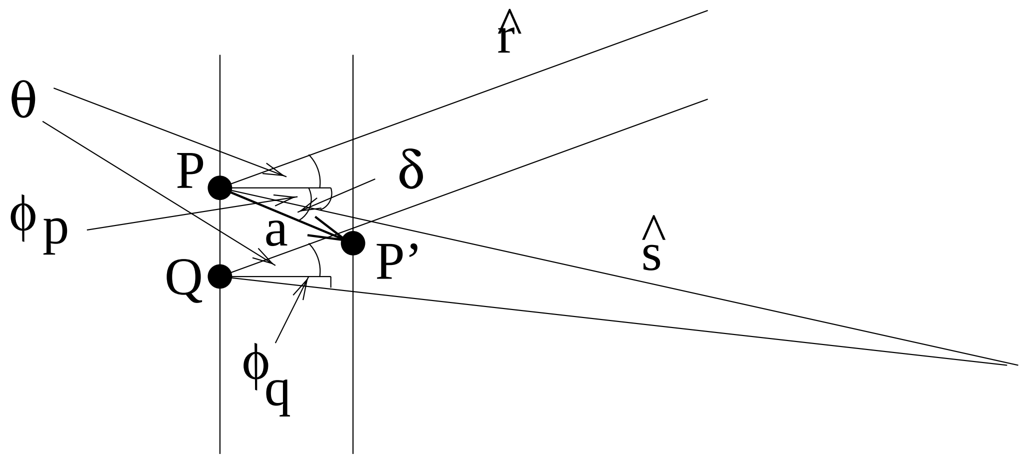
$$\Delta = \mathbf{a}.\hat{\mathbf{r}} + \mathbf{a}.\hat{\mathbf{s}} = \mathbf{a}.(\hat{\mathbf{r}} + \hat{\mathbf{s}})$$

Rays will interfere, so general condition for a “Bright Fringe” is,

$$\mathbf{a}.(\hat{\mathbf{r}} + \hat{\mathbf{s}}) = \pm n\lambda$$

Difficult to deal with in general.

Rigid Object



Where θ is illumination direction, ϕ_P and ϕ_Q are viewing directions and δ is angle of movement.

At the point P,

$$\mathbf{a}.(\hat{\mathbf{r}} + \hat{\mathbf{s}}) = a\cos(\theta + \delta) + a\cos(\phi - \delta)$$

So we get a bright fringe when,

$$a\cos(\theta + \delta) + a\cos(\phi - \delta) = \pm n\lambda$$

Assume there **is** a bright P, and the **next** one at Q, then

$$\begin{aligned} a\cos(\theta + \delta) + a\cos(\phi_P - \delta) &= n\lambda \\ a\cos(\theta + \delta) + a\cos(\phi_Q - \delta) &= (n+1)\lambda \end{aligned}$$

we have, by subtraction that,

$$\cos(\phi_P - \delta) - \cos(\phi_Q - \delta) = \lambda/a$$

Let us write.

$$\phi = \phi_P \quad \text{and} \quad \phi_Q = \phi + \Delta\phi \quad \text{and} \quad \alpha = \phi - \delta$$

Then by substitution we have that

$$\cos(\alpha - \Delta\phi) - \cos(\alpha) = \lambda/a.$$

Noting that

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

We get that

$$\cos(\alpha)\cos(\Delta\phi) + \sin(\alpha)\sin(\Delta\phi) - \cos(\alpha) = \lambda/a$$

Now of $\Delta\phi$ is small, then we can take the approximations that

$$\cos(\Delta\phi) \approx 1 \quad \text{and} \quad \sin(\Delta\phi) \approx \Delta\phi$$

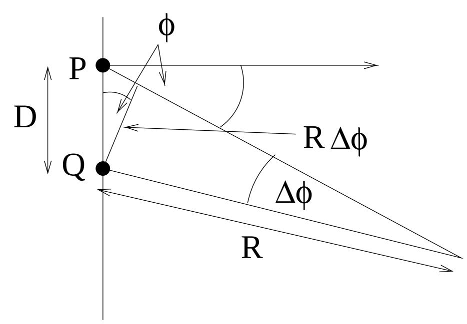
so giving that

$$\Delta\phi \sin(\alpha) = \lambda/a$$

Or that the angular separation of the fringes is:

$$|\Delta\phi| = \frac{\lambda}{a\sin(\phi - \delta)}$$

View Object from a distance R,



Separation of fringes on the object is

$$D = \frac{R\Delta\phi}{\cos(\phi)} = \frac{R\lambda}{a\cos(\phi)\sin(\phi-\delta)}$$

Which does not give a unique solution for a and δ.

Simple Geometries

1) Out of Plane Displacement:
We have movement perpendicular to the surface, so

$$\delta = 0$$

so that fringe separation

$$D = \frac{R\lambda}{a\cos(\phi)\sin(\phi)}$$

2) In plane Displacement:
We have movement parallel to the surface, so

$$\delta = \frac{\pi}{2}$$

so that fringe separation

$$D = \frac{R\lambda}{a\cos^2(\phi)}$$

Note: in both cases we have that

$$D \propto \frac{1}{a}$$

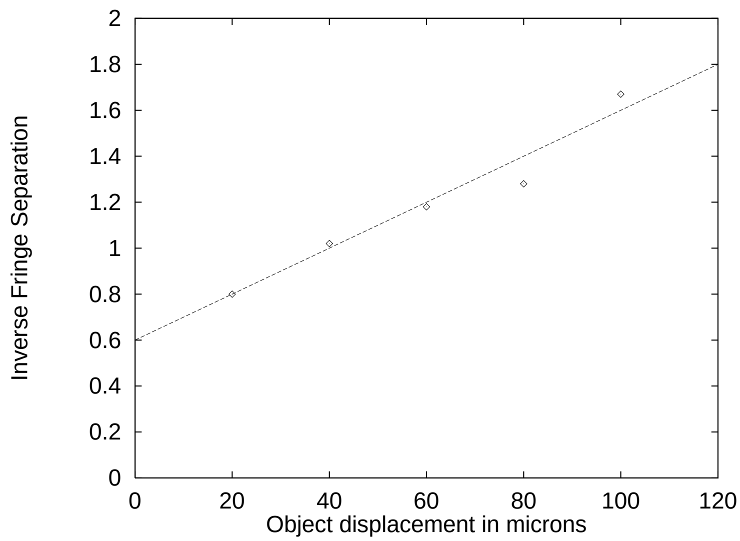
Using this technique it is possible to measure displacements of the order of λ.

Typical Results

With “in-plane” displacement of a small piece of alumiminium.

$$\delta = 90^\circ \quad R = 200\,\mathrm{mm} \quad \phi \approx 45^\circ$$

we get a plot of $1/D$ against Displacement a gives,



Results show correct linear relation, but graph displaced from origin.

Problem: In practice finges are not localised in the object plane due to a combination of effects not considered in the above analysis. Results in systematic error see above.

Time Averaged Holography

We have a vibrating object and take long exposure.

Assume the object wave from a stationary object is

$$O(x,y)\exp(\imath\Phi(x,y))$$

Now if the object vibrates, then if the amplitude if the vibration is small then

$$O(x,y) \approx \text{constant}$$

so the object wave of the vibrating object is,

$$O(x,y)\exp(\imath\Phi(x,y,t))$$

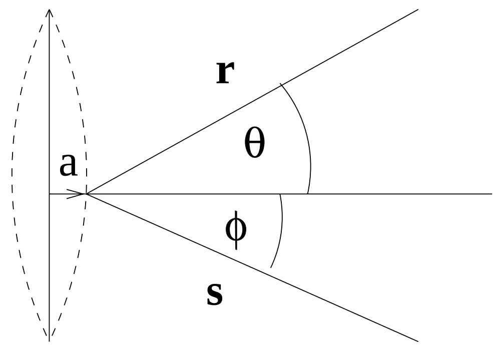
If a hologram of this object is formed with exposure time τ then the average object wave recorded is

$$\int_0^\tau O(x,y)\exp(\imath\Phi(x,y,t))\,\mathrm{d}t$$

which will be highly dependent on the form of $\Phi(x,y,t)$.

We will consider the simplest case only.

Plane Vibrating Sheet



where the amplitude of the vibration is a.

The OPD caused by the vibration is

$$a\,(\cos(\theta)+\cos(\phi))$$

where a can be written as

$$a(x,t) = a_0(x)\cos(\omega t)$$

where $a_0(x)$ is the amplitude and ω is the angular frequency.

The phase term of the object is now

$$\Phi(x,y,t) = \overbrace{\Phi_0(x,y)}^{\text{Stationary}} + \kappa a(x,y,t)\,(\cos(\theta)+\cos(\phi))$$

which can be written as

$$\Phi_0(x,y) + A_0\cos(\omega t)$$

where we have written

$$A_0 = \kappa a_0(x,y)\,(\cos(\theta)+\cos(\phi))$$

The object wave is now

$$\underbrace{O(x,y)\exp(\imath\Phi_0(x,y))}_{\text{Stationary}}\underbrace{\int_0^{\tau}\exp\left(\imath A_0\cos(\omega t)\right)\,\mathrm{d}t}_{\text{Vibration}}$$

Assume period of vibration **much** less that exposure time,

$$\tau \gg \frac{2\pi}{\omega} = \tau_0$$

In addition assume that there are exactly N period of the vibration in the exposure, so that

$$\tau = N\tau_0 \quad N \text{ is large}$$

The object wave now becomes,

$$O(x,y)\exp(\imath\Phi_0(x,y))N\int_0^{2\pi/\omega}\exp\left(\imath A_0\cos(\omega t)\right)\,\mathrm{d}t$$

which is be substitute $\beta = \omega t$ we have,

$$O(x,y)\exp(\imath\Phi_0(x,y))\frac{N\tau_0}{2\pi}\int_0^{2\pi}\exp\left(\imath A_0\cos(\beta)\right)\,\mathrm{d}\beta$$

This integral is the familiar $\mathrm{J}_0()$ so the resultant recorder object wave is

$$\underbrace{O(x,y)\exp(\imath\Phi_0(x,y))}_{\text{Stationary}}\underbrace{N\tau_0\mathrm{J}_0\left(A_0(x,y)\right)}_{\text{Vibration}}$$

The intensity of the reconstruction is therefore modulated by

$$\left|\mathrm{J}_0\left(A_0(x,y)\right)\right|^2$$

which for the above system can be explicitly written as,

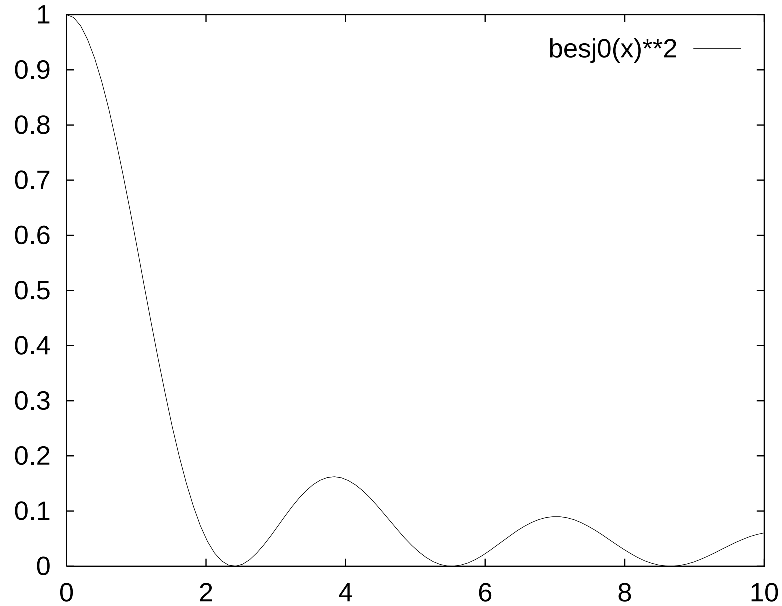
$$\left|\mathrm{J}_0\left(\kappa a_0(x,y)(\cos\theta + \cos\phi)\right)\right|^2$$

where θ and ϕ are the illumination and viewing angles, and $a_0(x,y)$ is the amplitude of the vibration.

This analysis is valid for all flat vibrating plates.

Shape of Fringe Pattern

Shape of $|\mathrm{J}_0(x)|^2$



Take the special case of $\theta = \phi = 0$, then the fringe contrast becomes

$$\left| \mathrm{J}_0 \left(\frac{4\pi}{\lambda} a_0(x,y) \right) \right|^2$$

which has a max at $a_0 = 0$, (node of vibration), and zeros at

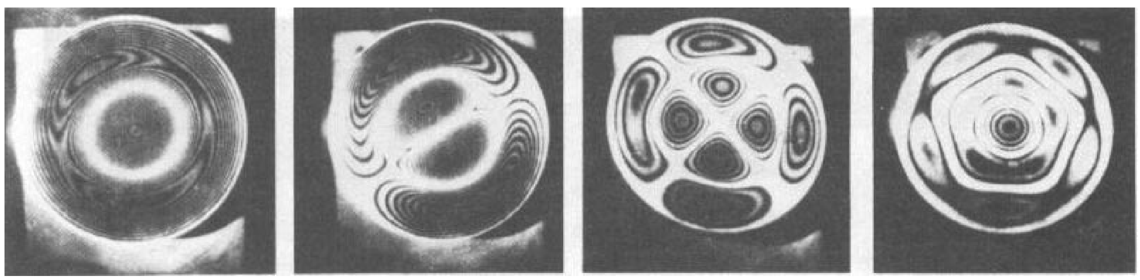
$$\begin{aligned} a_0 &= 0.19\lambda \\ a_0 &= 0.43\lambda \\ a_0 &= 0.68\lambda \end{aligned}$$

so we get a contour map of Vibration Amplitude. (Very sensitive technique).

Note: When we use this technique on complex vibrating objects with "large" amplitude, we typically only see the "nodes" of the vibration and the secondary fringes are lost.

Examples Time Averaged Results

Bottom of 35 mm film container moved by electric solenoid at different frequencies. Very early result from Powell & Stetson, JOSA, **55** 1593-1598 (1965)



Vibrating guitar at (a) 185 Hz and (b) 285 Hz, from Molin & Stetson, Institute of Optical Research, Stockholm, (1971)