Tutorial Solutions

6 Practical Systems

All of these questions relavent to the course and should be tried. Some of the solutions are more details than is required for the course.

6.1 The Pinhole Camera

The famous *pinhole camera* consists box containing a piece of film and a pinhole of radius a. Assume this is used to image a distant object illuminated by incoherent green light. If the pinhole is 10 cm from the film estimate the optimal size of the pinhole using the following *two* methods and compare your answers.

Method 1:

1. Obtain an estimate for the OTF of the "camera" assuming that the pinhole is *large* enough so the PSF is given by geometric optics (no diffraction). Use this to calculate the spatial frequency cutoff being the first zero of the OTF.

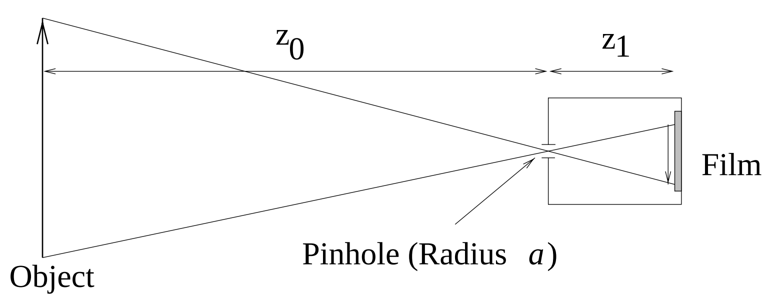
2. Obtain an estimate of the OTF of the "camera" assuming the pinhole acts as a lens of radius a.

3. From these two estimates obtain an "optimum" size for the pinhole that maximises both estimates.

Method 2:
Assume that the pinhole acts as a lens of focal length 10 cm and apply the Strehl limit to obtain the maximum allowable diameter.

Solution

the pinehole camera is as shown below,



Assume that $z_0 \to \infty$, so that $z_1 = f$ the effective focal length of the pinhole.
Method 1:
If the PSF is given by geometric optics, then the pinehole is big enough for there to be no diffraction. The PSF is just a projection of the pinhole, so for a pinhole of radius a we get

$$\begin{aligned} h(r) &= 1 \quad \text{for } r^2 = x^2 + y^2 < a^2 \\ &= 0 \quad \text{else} \end{aligned}$$

The OTF is the Fourier Transform of the PSF, so

$$H(w) = \frac{J_1(2\pi a w)}{2\pi a w}$$

which has the first zero (so cutoff frequency) at

$$2\pi a w_0 = 1.22\pi \Rightarrow w_0 = \frac{0.61}{a}$$

Note: This does not depend on wavelength since we have assumed NO diffraction.

If we assume the pinhole acts as a lens of focal length f and radius a, then the OTF is the scaled Autocorrelation of the Pupil function, which from previous lecture gives the spatial frequency cutoff to be

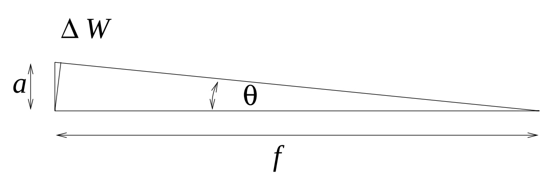
$$w_0 = \frac{2a}{\lambda f}$$

One estimate gives $w_0 \propto a$ and one $w_0 \propto \frac{1}{a}$, so the *maximum* of each estimate will occur when both estimaes give the same answer, so when:

$$\frac{0.61}{a} = \frac{2a}{\lambda f} \Rightarrow a^2 \approx 0.3\lambda f$$

for green light ($\lambda = 55m\,\text{nm}$) and $f = 10\,\text{cm}$, we get $a \approx 0.13\,\text{mm}$, so pinhole of $0.26\,\text{mm}$ diameter. This gives a spatial frequency cutoff of $4.7\,\text{mm}^{-1}$ which suggest a "reasonable" image quality.

Method 2:
If we consider the pinhole to be a lens of focal length f and radius a,



If the pinhole is illuminated by a distant object the phase fronts after the lens will be plane *not* parabolic. The wavefront aberration at the edge of the pinhole will then be given by

$$\Delta W = a\theta \approx \frac{a^2}{f}$$

To we within the Strehl limit (see previous), this wavefront aberration must be less than $\lambda/4$, so we get the maximum pinhole size of

$$a^2 \approx 0.25\lambda f$$

which gives as estimate for $a \approx 0.12\,\text{mm}$, so a pinhole of $0.24\,\text{mm}$ in diameter.

These estimates are almost identical, suggesting that both are valid.

6.2 How Good a Camera do you need?

Most cheap cameras are used to produce small ($10 \times 15\,\text{cm}$) prints that are viewed at normal reading distance. You will see a "sharp" photograph if the resolution on the print is comparable to the resolution limit of the human eye. Using this criteria, suggest a simple design for a cheap camera that uses 35 mm film (the image size on 35 mm film is $24 \times 36\,\text{mm}$.)

Solution

The angular resolution of the eye with pupil diameter $d \approx 2\,\mathrm{mm}$ and $\lambda = 550\,\mathrm{nm}$

$$\delta\theta = \frac{1.22\lambda}{d} = 3.35 \times 10^{-4}\,\mathrm{Rad}$$

so if the print is viewed 40 cm (sensible viewing distance), then the maxiumum resolvable spatial frequency on the print is,

$$u_0 \approx 7.5\,\mathrm{mm}^{-1}$$

If a $24 \times 36\,\mathrm{mm}$ negative is enlarged to a $100 \times 150\,\mathrm{mm}$ requires a magnification of 4.17, so the resolution on the film has to be a least $32\,\mathrm{mm}^{-1}$.

Aside: The resolution of cheap colour film is typically about $60\,\mathrm{mm}^{-1}$*, which good quality black-and-white film is up to* $150\,\mathrm{mm}^{-1}$.

We want a good contast at $32\,\mathrm{mm}^{-1}$, so we want the first zero of the OTF to be at least double this, at approximately $60\,\mathrm{mm}^{-1}$.

The typical lens of a simple compact 35 mm camera is a 38 mm focal length $F_{No} = 3.5$. The diffraction limit for such a lens is

$$v_0 = \frac{1}{F_{No}\lambda} = 519\,\mathrm{mm}^{-1}$$

which is a factor of $\times 10$ better than is required, so we are **not** looking for a diffraction limited system.

For a $36 \times 24\,\mathrm{mm}$ image plane the diagonal maximum off-axis distance is $\sqrt{18^2 + 12^2} = 21.6\,\mathrm{mm}$, so for a 38 mm focal length lens the maximum half-field angle $\eta = 29.6°$.

Aside: In practice the "good" compact camera (>£100), will, typically have 4 element glass Tessar type lenses, while the cheaper ones will have either all plastic three element, see Kingslake, "A History of the Photographic Lens" Academic Press, page 78, or a mixed glass and plastic three element lens with the front element being glass. When actually in use the the lens well be "stopped-down" to a lower F_{No}*, typically to* $5.6/8$*. At these apertures the glass Tessar lens will be giving essentially diffarction limited performance. The cheap plastic lens will typically give good performance on-axis (middle of the image), but rather poor performance at the edges.*

6.3 "Big" Telescopes

A Newtonian telescope with $F_{No} = 4$ primary mirror of diameter 10 cm is used for photographic imaging with the photographic plate placed at the primary focus (at the focus of the primary mirror). Calculate the distance Δz that the plate can be moved along the optical axis with the image still within the Strehl limit. Assuming that the telescope body is a steel "tube" with a coefficient of expansion of $\alpha = 11 \times 10^{-6}\,\mathrm{K}^{-1}$, calculate the maximum change in temperature that this telescope tolerate so still give a sharp image. What does this mean for the design of the telescope.

Repeat the above calculation for the Hubble Space Telescope which has a 72" (1.8 m) $F_{No} = 2.5$ primary mirror, and the Mount Palomar telescope that has a 100" (254 cm) $F_{No} = 4$ primary mirror. What does this means for the design of the Space Telescope and the Mount Palomar telescope.

Solution

The Strehl limit is when the wavefront aberration is less that $\lambda/4$ over the aperture. If we defocus an optical system by a distance Δz then the defocus,

$$D = \frac{1}{z_0} + \frac{1}{z_1 - \Delta z} - \frac{1}{f} \approx \frac{\Delta z}{z_1^2}$$

If the aperture is of radius a then the *maximum* of the wavefront aberration is at the edge of the aperture, so

$$\Delta W = \frac{Da^2}{2} = \frac{\Delta z a^2}{2z_1^2}$$

At the Strehl limit we have that $\Delta W = \lambda/4$, so that

$$\Delta z = \frac{\lambda}{2}\frac{z_1^2}{a^2}$$

but for a distant object, $z_1 = f$ (the focal length of the lens), so that

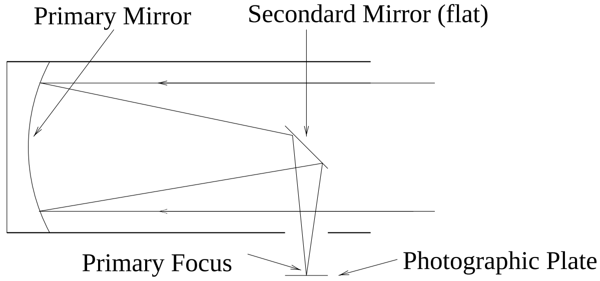
$$\Delta z = \frac{\lambda}{2}\left(\frac{f}{a}\right)^2 = 2\mathrm{F_{No}}^2\lambda$$

So for the $\mathrm{F_{No}} = 4$ telescope, taking $\lambda = 550\,\mathrm{nm}$ then

$$\Delta z = 17.6\,\mu\mathrm{m}$$

which is about twice the thickness of the photographic emulsion.

The focal length of the mirror is 40 cm the Newtonian telescope is approximately a "tube" of length 40 cm as shown below,



If this raised by temperature ΔT then the length increase is

$$\Delta l = \alpha l \Delta T$$

so with $l = f$, then to be within the Strehl limit,

$$\Delta T < \frac{\Delta z}{\alpha f} = \frac{\lambda}{2\alpha}\frac{f}{a^2}$$

so with $\alpha = 11 \times 10^{-6}\,\mathrm{K}^{-1}$, $f = 40\,\mathrm{cm}$, $a = 5\,\mathrm{cm}$ and $\lambda = 550\,\mathrm{nm}$, then

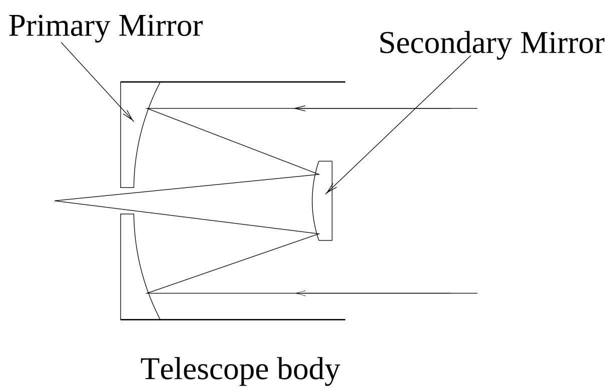
$$\Delta T < 4\,\mathrm{K}$$

This is a fairly tight, but not difficult constraint since a typical *long* exposure image may take 1 hour, then provided the temperture of the telescope remains $\pm 4\,\mathrm{K}$ during the exposure then the image will be sharp. This does however mean the telescope must be left to reach ambient temperture before the photograph is taken and it *must* be focused. It is not possible to have a fixed focus telescope. This adds considerablly to the cost of such an instrument since it requires a fine focus system and a viewing system to set the focus (this is usually done by replacing the photographic plate with a ground glass screen and the focus set by an observer by viewing the image on the screen with a microscope).

Hubble Space telescope has $F_{\mathrm{No}} = 2.5$, (to make compact system), for green light gives,

$$\Delta z = 6.8\,\mu\mathrm{m}$$

The actual design is a Casasegrian design which shortens the tube by doubling the beam path.



The image sensor is also a CCD array camera, but the principle is the same.

The "effective" length of the telescope tube is still approximately f, the focal length of the primary mirror, being approximately 4.5 m. The radius being $0.9\,\mathrm{m}$. So again assuming steel (actually stainless steel and aluminum), we get

$$\Delta T < 0.14\,\mathrm{K}$$

This is a significant problem since changes in the position of the sun during the telescope's orbit will produce fairly rapid changes in temperature. The optical system thus needs an auto-focus system to move the image plane during even relatively short exposures.

The Mount Palomar can be used in a range of optical configurations. It is now normally used as a Cassegrian telescope, but was used extensively as a large simple Newtonian telescope with a photographic plate placed at the primary focus. (This telescope is so big that that a human observer can sit in a cradle at the primary focus).

Simply putting in the numbers gives,

$$\Delta z = 17.6\,\mu\mathrm{m}$$

and with a focal length of $f = 10\,\mathrm{m}$*, and radius of* $a = 1.27\,\mathrm{m}$ *we get*

$$\Delta T < 0.16\,\mathrm{K}$$

which suggests a major problem requiring auto-focus and very careful temperature control.

However "big" telescopes are **not** *limited in resolution by their size but by the effects of the atmosphere (details of this are beyond this course). The actual resolution is about 1 arc-second,* *which is a equivalent to that for a 10 cm telescope. For long exposure images a big telescope act like a large array of 10 cm telescopes all imaging the same point of the sky. This collects more light so allows imaging of fainter objects. Thus the "effective"* $\mathrm{F_{No}} \approx 100$, *which gives*

$$\Delta z \approx 11\,\mathrm{mm}$$

which is huge, so assuming a steel frame, we get

$$\Delta T < 100\,\mathrm{K}$$

which means there is no problem at all. In practice the actual telescope dome is kept temperature stabalised and matched to the outside temperature to prevent convention currents effecting the imaging properties. Also there are other bits of the telescope, typically the spectrometer systems, that are very temperature sensitive.

6.4 Spectacle Lenses Again

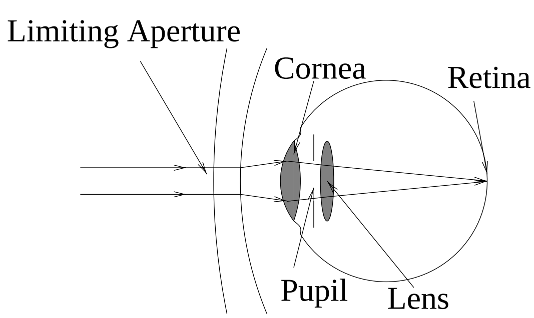
Consider a spectacle lens of power 5 Diopter and diameter 60 mm. Calculate the focal length and $\mathrm{F_{No}}$ of this lens. Given that such a lens is a singlet made of plastic discuss the effect of aberrations and why a person with the correct spectacles has every bit as good vision as people who have perfect eyesight.

Solution

A spectacle lens of 5 Diopters and diameter of 60 mm has, apparently, focal length and $\mathrm{F_{No}}$ of

$$f = \frac{1}{5} \times 1000 = 200\,\mathrm{mm} \quad \text{so} \quad \mathrm{F_{No}} = \frac{200}{60} = 3.3$$

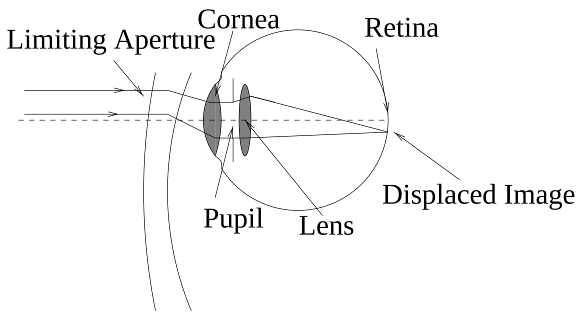
This suggests a sever problem with aberrations. However when placed infront of the human eye,



the limiting aperture is given by the pupil of the eye, which is about 2 mm in diameter. We have to consider the whole optical system of spectacle lens *plus* the eye. The human eye is fluid filled sphere of approximately 30 mm diameter. When looking at a distant object the spectacle lens *and* the two lenses in the eye focus rays from this object to a point on the retina. The effective focal length of the whole optical system is thus $f \approx 30\,\mathrm{mm}$. The effective $\mathrm{F_{No}} \approx 15$.

As seen from solution 3.6 that for optical systems with $\mathrm{F_{No}} > 8$ Spherical Aberration of a singlet is less than the Strehl limit, then at $\mathrm{F_{No}} \approx 15$ then all aberrations from singlet lenses will be insignificant. So if the focus defect in the eye is correct by a singlet lens the resolution will be essentially identical to that of a person who does not require spectacles. This fits in with the previous solutions that show that the human eye is "almost" diffraction limited.

As the eye moved it will not always look through the centre of the spectacle lens, for example,



Under these conditions the distant object is still sharply focused but “appears” displaced in the field of view. This effect is canceled out by processing in the visual system.