Lecture 4, 24th September 2012

Analogue Electronics 1: DC Circuit Basics

We have now completed all you need to know to do the digital aspects of the lab checkpoints – we now begin the analogue part. On a fundamental level this precedes digital electronics. On a practical level (in the lab) analogue is sometimes seen as more challenging than digital electronics. We first will only regard direct currents (DC) – a static flow of charge, constant in time.

Linear circuits: Resistors, electromotive force, batteries & Kirchoff's laws

The first analogue electronics lecture is revision of things you know about from school. We are going to look at the properties of resistors and batteries and then combine them in simple circuits. We will reiterate Kirchoff's laws and go through a few examples of how they are applied to analyse circuits.

Resistors: (H&H, 1.02, p.4)

In electronics we often make use of components that have useful variations in current corresponding to a change in voltage. Resistors are the simplest elements in this regard.

The potential difference, i.e. the **voltage V**, across a resistor corresponding to a particular **current, I**, flowing through the resistor, with **resistivity R**, is described by **Ohm's law**:

$$V = I\,R \qquad \text{(units: V=Volt, A=Ampere and } \Omega \text{=Ohm)}$$

This relationship describes the change in drift velocity of charges, or in a different view the amount of charge passing a cross-section of the conductor per unit time, I, in response to a change in an applied electric field,V. In the simplest type of resistors, Ohmic resistors, this relation is linear. In these the flow of electric current is impeded by the thermal motion of ions and other electrons. Consequently the resistance the moving charges experience is temperature dependent. The coupling between the flowing current and the "static" parts of the resistor means that a flowing current causes heating.

Metal wires are the simplest example for this. They heat up as the transmitted power P=VI increases. This increases the resistivity and hence the heating, until an equilibrium is reached with the heat loss via radiation or external cooling – or until the wire melts. Fortunately most resistors have properties that don't vary much for heating power P < 0.25W.

Resistive properties of materials are sometimes talked about in terms of their **conductance**, the inverse of the resistance **G**=1/R, measured in Siemens (or mhos – as the inverse of ohms).

Electromotive force (EMF) and batteries: (H&H, 1.04, p.9)

In order to use resistors to control the flow of charge in a circuit we need to provide either a potential difference (V) or a current (I). A power supply that sets the potential difference (V) is called a "voltage source". Unsurprisingly, power supply that sets the current (I) is called a "current source". A battery is a voltage source:
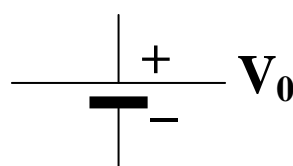
An **Electromotive Force (EMF)** is the (non electrical) source of energy that drives a current around an electric circuit.

In a **battery**, it is chemical forces that cause a separation of positive and negative charges in spite of the mutual attraction between them. As a result, a potential difference, V_0, is produced and maintained between the battery terminals. The behaviour of a battery can be understood by considering the two circuits pictured below. Note the conventional symbols for a battery and a resistor.

Switch open (left):
Chemical forces will have caused a separation of charge; that is, excess positive charge will have collected on the electrode connected to the upper terminal of the battery, and negative charge will have collected on the electrode connected to the lower terminal. Further charge separation by the chemical forces ceases when the electric field built up within the battery is just enough to stop further separation. At this point the upper battery terminal and the wire connected to it have reached some potential V_0 above that of the negative terminal and the wire and resistor connected to it. This potential difference V_0 appears across the terminals of the switch.

Switch closed (right):
The potential difference across the switch becomes zero, and current through the resistor R adjusts itself until the voltage V_0 provided by the EMF is developed across the resistance. That is, the current becomes $I = V_0/R$.
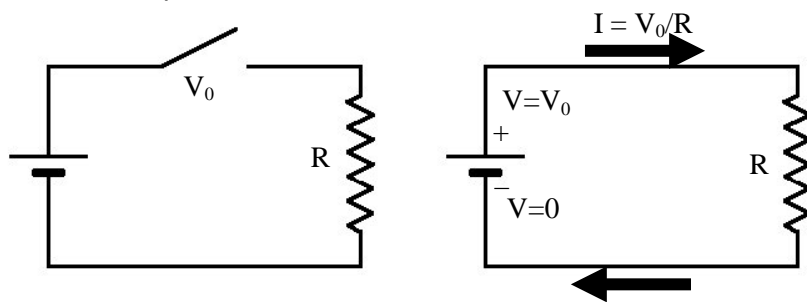
[Work done and power:](#) (H&H, 1.01, p.2 – see also p.7)
While we are in the business of revising – let's quantify the work and power associated with a current flowing through a resistor. The **work done, W**, is the charge, Q, which is moved along the potential difference, V:

$$W = V\,Q \qquad \text{(units: J=Joule, V=Volt and C=Coulomb)}$$

Usually we are more interested in the **power, P**, the work done per unit time:

$$P = \left(\frac{d}{dt}\right) W \;\; = \left(\frac{d}{dt}\right) V\,Q \; = V\,I$$

The current is the charge flowing per unit time. The resulting expression for the power is called **Joule's Law**:

$$P = V\,I \qquad \text{(units: W=Watt, V=Volt and A=Ampere)}$$

In the specific case of resistors we can eliminate either V or I from this equation, using Ohm's law:

$$P = I^2 R \; = \; \frac{V^2}{R}$$

This is the **heating power** caused by the flow of current through a resistor.

Kirchoff's Laws: (H&H, 1.01, p.3)

The process of analysing electrical circuits has been systemised in Kirchoff's laws. By applying these two laws to any circuit involving combinations of resistors and batteries (such as the one pictured below) you can determine the distribution of voltages across and the currents through the various components.
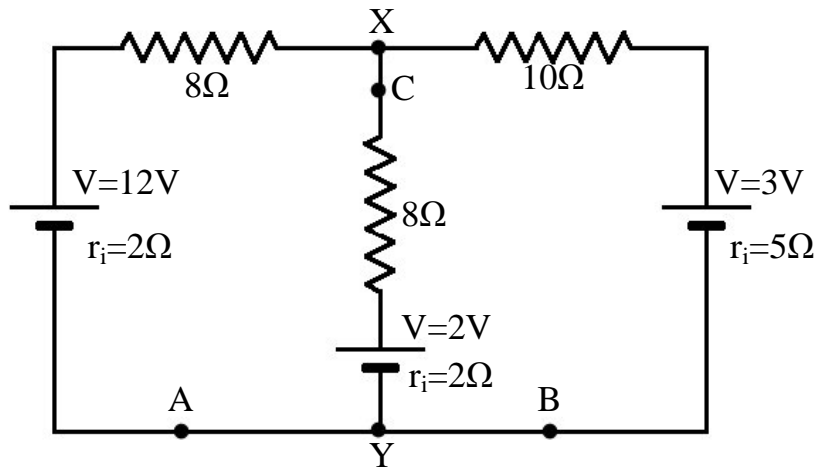
Kirchoff's current Law:

The sum of currents into a node in a circuit is equal to the sum of currents out.

This is a statement of the fundamental principle that charge is conserved. The following cartoon represents a **node** in a circuit where one wire is attached to another. This is a junction such as that labelled Y in the circuit above.
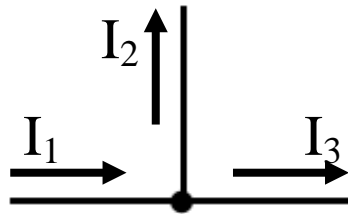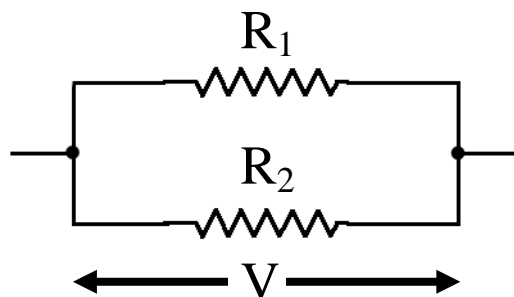

The current law in equation form in this case is: $I_1 = I_2 + I_3$

Kirchoff's coltage law:

**The sum of the voltage drops around any closed path is zero.**

This is a statement that electrical forces are conservative. Practically it means that elements wired in parallel have the same voltage across them, like the two half-loops with R_1 and R_2 below:

Resistors in series and in parallel: (H&H, 1.02, p.6)

The ability to find the value of resistances when they are combined in either serial or parallel arrangements is essential knowledge for the lab checkpoints. It is assumed that you know how to do this from here on.

A **series arrangement** is one where the resistors are combined one after another along the same conducting pathway. Here, any current has to pass through all of the resistors. This configuration is pictured in the circuit fragment for two resistors below:
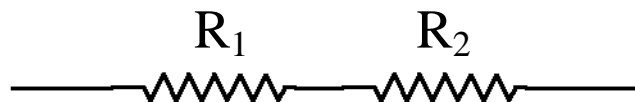
The total resistance R_T of the combination is:

$$R_T = R_1 + R_2$$

The **parallel arrangement** is to combine resistors by placing them side-by-side in different conducting pathways. The situation is pictured below for two resistors in parallel. Here, different fractions of the current pass through different the resistors – the distribution of current between the resistors is inversely proportional to their resistances.
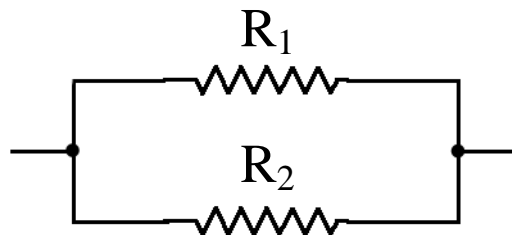
The total resistance R_T can be found using:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Rules of thumb:
To get a feel for how various resistor arrangements are going to behave you might want to consider the following approximations, valid if the resistances are very different:

- In series: If $R_1 \gg R_2$ then $R_T \approx R_1$

- In parallel: If $R_1 \gg R_2$ then $R_T \approx R_2$

To 'see' these relations think about the extreme values of resistance, 0 and ∞.

Voltage divider: (H&H, 1.03, p.8)

The **voltage divider** is ubiquitous for the remainder of this course. So, it pays to spend a little time discussing it. The illustration below is only a fragment of the circuit – like the wire junction, series resistors and parallel resistors pictured before. We need to analyse this fragment of a circuit to understand how the relationship between V_{IN} and V_{OUT} changes with the values of the resistors R_1 and R_2.
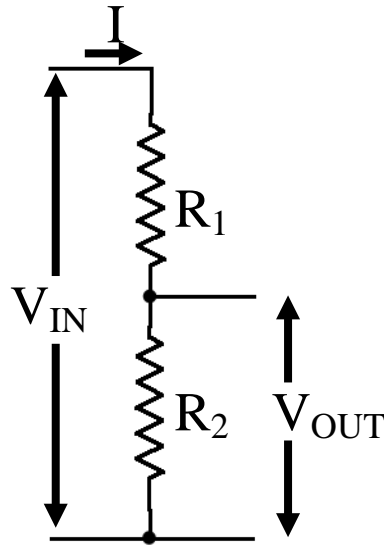
Let's first glance at the overall behaviour:

A voltage, V_{IN}, is supplied on the left, leading to a current, I, flowing from the top to bottom. Assume that no current is leaving the fragment from the node between the resistors R_1 and R_2. Thus, the currents through R_1 and R_2 are equal and the voltage drops are proportional to their resistances.

For a quantitative description apply now Ohm's law to the case of two resistors in series, then look at the voltage drop across the second and finally combine the two equations:

$$I = \frac{V_{IN}}{(R_1 + R_2)} \quad \text{and} \quad V_{OUT} = I\,R_2 \quad \Rightarrow \quad V_{OUT} = \frac{V_{IN}R_2}{(R_1 + R_2)}$$

As mentioned, voltage dividers are extremely common, and need to be considered often in the electronics lab. We are now going to give a quick thought to the *relationship* between *voltage dividers* and both *voltmeters* and *power supplies* (both of which you are using in the lab).

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How do we want the multimeter to behave, i.e. the voltmeter part of it?

You are used to attaching the red and black leads across different parts of your circuit, when you want to know what the voltage is. Say that you are trying to measure the voltage drop across R_2 in the circuit fragment below – M.M. stands here for **multimeter**. If the meter would have a low or medium internal ohmic resistance, R_M, then a significant part of the current, I, would flow through the meter instead of flowing through the resistance R_2. The resistances R_2 and R_M would be parallel, their effective resistivity would be reduced and in turn the *potential* at the point between R_1 and R_2 been *shifted to a lower value* accordingly. This way a **systematic error** is introduced in the voltage measurement, proportional to the relative size of R_1 and R_2.
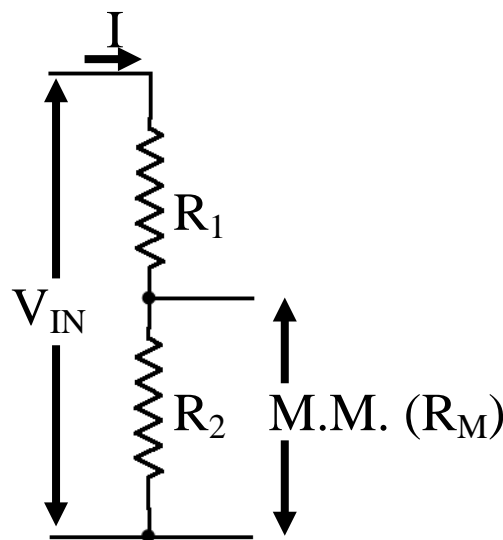

In order for the voltmeter not to significantly modify the circuit in this way it has to have **high input impedance**. Good hand-held meters usually have an input impedance of 10MΩ. This will be more quantitatively been discussed in the next lecture.
With that voltage drops around 1MΩ resistors are measured with a 5% systematic error, which one could correct for. And voltage drops around 100kΩ resistors are systematically accurate to 0.5%, i.e. this error is already smaller than the calibration accuracy of the instrument.

You may have noticed the shift in terminology from *resistance* to **impedance**. Strictly speaking the impedance of an element may only be reduced to its ohmic resistance for DC currents, i.e. at frequency 0Hz. The impedance describes its general properties to resist the free flow of charge over the *full frequency spectrum*, including *all sources of resistance* (in general: ohmic, capacitive and inductive). We will come back to that in later lectures.

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How do we want the power supply to behave?

C.C.
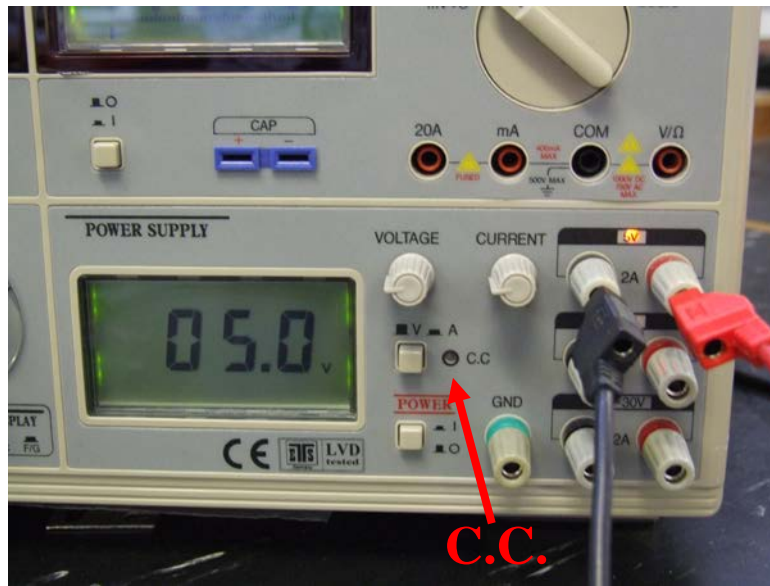
So far you have been using the **power supply** to give you 5V to drive the chips in your logic and flip flop circuits. This means that you are using it as a **voltage source** – the voltage is constant while the current changes. The switch next to the C.C. light means that you can choose the supply to operate in constant current mode instead – i.e. you can make the supply a **current source** if you want to.

Very roughly: A power supply can be thought of as a supply where the output leads come from either side of resistor R_2 – as pictured below. If you want to work at *constant voltage* then you need the voltage drop across R_2 to be constant irrespective of what the leads are plugged into. The internal resistances of a voltage supply therefore need to be *small compared to the external circuit*.
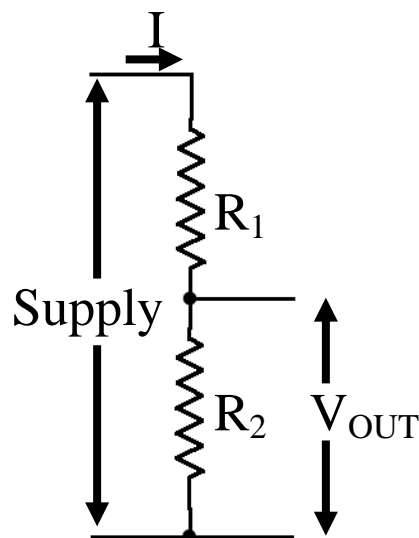
If you want to work at *constant current* then you need the current through the external circuit never to change irrespective of what that circuit is. For this operation the resistances in the voltage divider need to be *very large* – so that all the current flows through the external circuit.

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We finish this set of notes with a series of circuits for you to take a look at. These sorts of exercises may well be familiar – you use Kirchoff's laws to answer them.
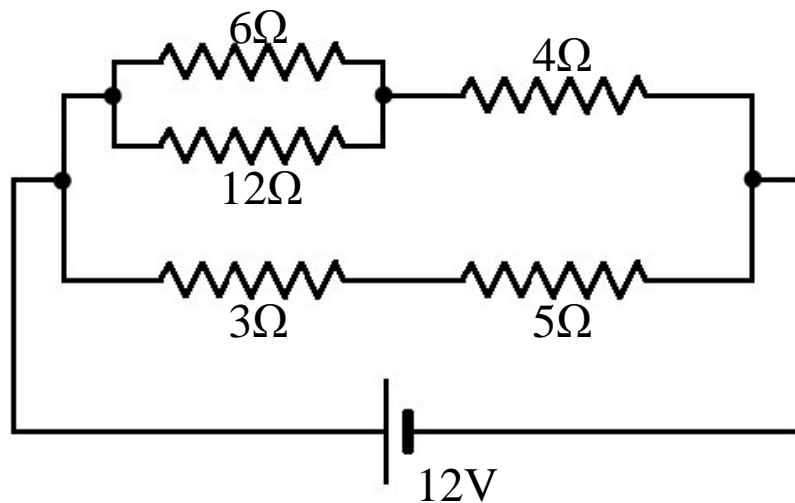
Example 1:
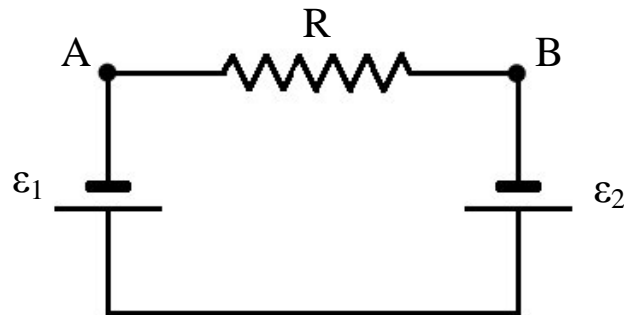
ε_1 = 12V and ε_2 =8V.
What is the direction of the current through the resistor?
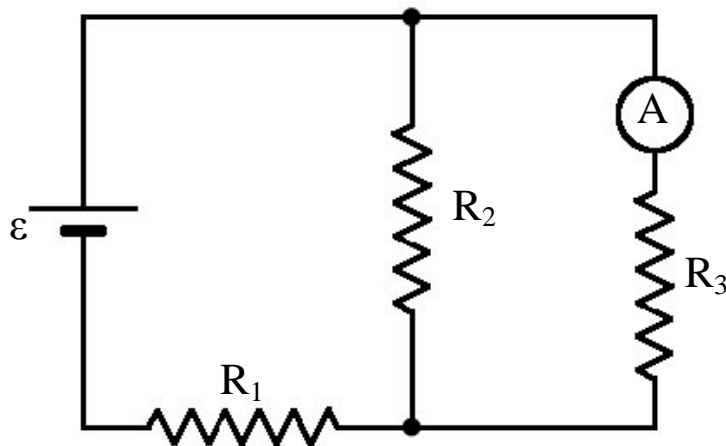Which EMF is doing positive work?
Which point, A or B, is at higher potential?

Example 2:

A circuit containing five resistors connected to a 12V battery is shown. What is the voltage drop across the 5Ω resistor?
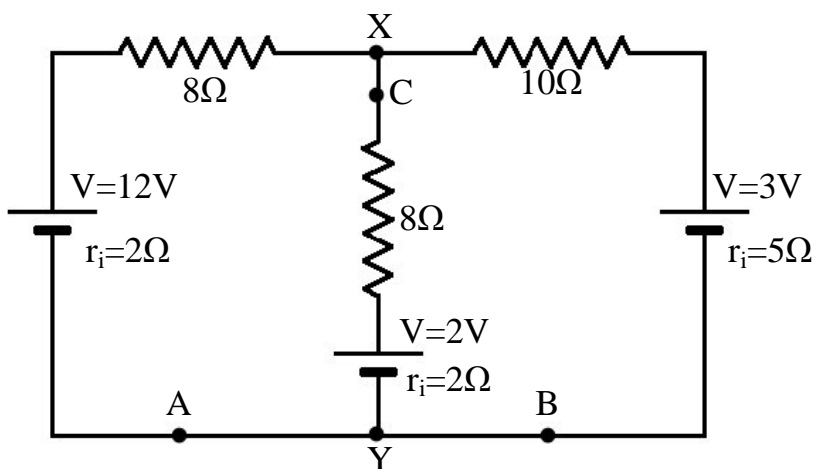
Example 3:

Imagine an ampere-meter inserted in the branch containing R_3.
   (a) What will it read, assuming $\varepsilon=5V$, $R_1=2\Omega$, $R_2=4\Omega$ and $R_3=6\Omega$?
   (b) The ampere-meter and the source of EMF are now physically interchanged. Show that the ampere-meter reading remains unchanged.

Example 4:

Find:
   • the current at the points A, B and C,
   • the potential difference between X and Y,
   • the power dissipated as heat in the circuit,
   • the power delivered by the 12V cell,

Lecture 4: key equations

For the rest of the course the following is assumed knowledge.

The behaviour of a resistor can be understood using **Ohm's law**:

$$V = I\,R \qquad \text{(units: V=Volt, A=Ampere and } \Omega\text{=Ohm)}$$

The **work done, W**, is the charge, Q, which is moved along the potential difference, V:

$$W = V\,Q \qquad \text{(units: J=Joule, V=Volt and C=Coulomb)}$$

The **power, P**, is the work done per unit time It is related to potential and current via **Joule's Law**:

$$P = V\,I \qquad \text{(units: W=Watt, V=Volt and A=Ampere)}$$

which for resistors gives:

$$\text{Power } P \; = I^2 R \; = \; \frac{V^2}{R}$$

This power appears as **heat** and is known as Joule heating.

Circuits can be analysed using **Kirchoff's laws**:

Kirchoff's current Law: The sum of currents into a node in a circuit is equal to the sum of currents out.

Kirchoff's voltage law: The sum of the voltage drops around any closed path is zero.

Resistors in **series** and **parallel**:

In series: $R_T = R_1 + R_2$

In parallel: $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Voltage divider (see circuit):

$$V_{OUT} = \frac{V_{IN} R_2}{(R_1 + R_2)}$$