Lecture 5, 27th September 2012

Analogue Electronics 2: More on DC Circuits

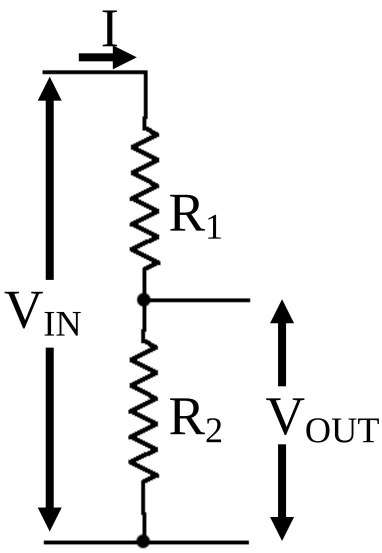
We began talking about voltage dividers and power supplies in the first analogue electronics lecture. Here we are going to complete the picture. This involves introducing a concept known as Thevenin's equivalent circuit.

Voltage: Voltage dividers, Thevenin's Equivalent Circuits, Power supplies

The key point that you need to take from this lecture is that **a circuit is modified when you attach other things to it**. This type of issue comes up frequently – for example when you start trying to attach electronics modules – used in nuclear and particle physics – to a computer so that you can manipulate the data. More prosaically – you need to *watch out when you attach* a power supply or multimeter to your circuits in the lab.

Thevenin's equivalent circuit: (H&H, 1.05, p. 11)

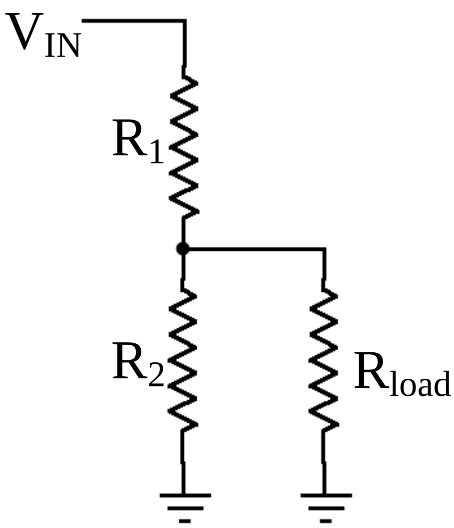
To motivate the next idea, let's take again the voltage divider as an example:



As was shown in the last lecture:

$$V_{OUT} = \frac{V_{IN}R_2}{(R_1+R_2)}$$

After you have characterized your divider to determine the value of V_{OUT} a **load** is attached across the two leads on the right hand side – this situation is pictured below.

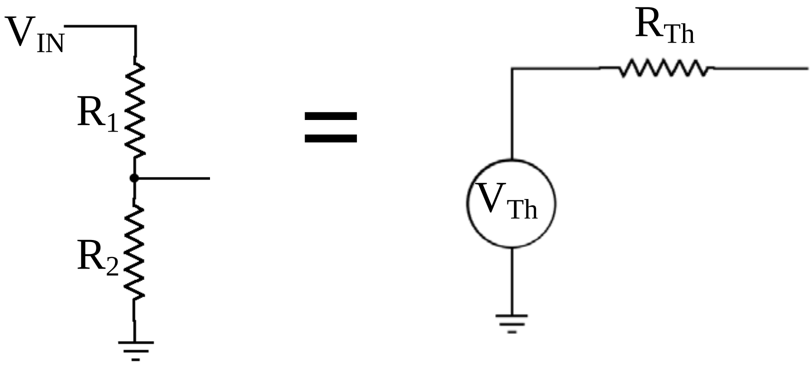


What is V_{OUT} now that the load has been attached?

If your voltage divider is the output of a device (such as a power supply, ipod etc.) then the value of R_{load} could be changed regularly. Rather than having to *reanalyse* resistances in series and parallel every time something changes we need the **response of a circuit to any load**.

By the way: As before, the series of dashes of diminishing length at the bottom of the lowest two wires is the symbol for a connection to a **common ground** (or earth). This is the point in the circuit which is a 0V (by convention the *black lead* of the power supply or the multimeter). The two points at the end of these two wires are effectively connected together.

Now the important new idea: **Thevenin's theorem:**
Any two-terminal network of resistors and voltage sources is equivalent to a single resistor in series with a single voltage source. This is shown in the picture below:



This simplifies life, since when you attach a load to the open side of the circuit it becomes easy to determine what the voltage across and current through the load.

In order to make use of this theorem we need to know how to find the values of V_{Th} and R_{Th} for any particular circuit. The required algorithms are:

$\mathbf{V_{Thevenin}}$: the voltage on the terminals when *nothing is attached* (i.e. in the case of zero load).

$\mathbf{R_{Thevenin}}$: given by $V_{Thevenin}$ / $I_{short\text{-}circuit}$, which is the current which flows from the circuit output to ground if you *short-circuit the output to ground*.

Let's try to use these algorithms for the case of a voltage divider:

$$V_{Th} = \frac{V_{IN}R_2}{(R_1+R_2)}$$

$$R_{Th} = \frac{V_{Th}}{I_{short-circuit}} = \frac{R_1R_2}{(R_1+R_2)} = R_{||}$$

Note that the effective output resistance is equal as if the resistances R_1 and R_2 would be parallel.

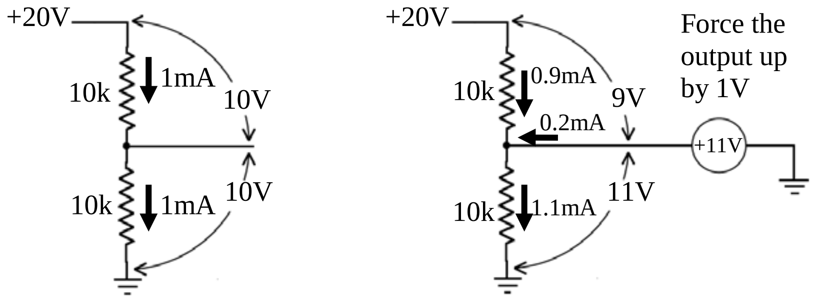
Why is this model useful? – Remember: **A circuit is modified if anything is attached.** Thevenin helps determine *by how much* and gives a *universal description* for all loads.

Input and output impedance – still in the DC limit:

We come back now to the **impedance** of circuits, but still only it in the limit of static currents and using ohmic resistances only.

Any *non-ideal voltage source* "droops" when loaded. By how much it droops depends on its **output impedance**. One can use the Thevenin equivalent circuit and express the output impedance as R_{Th}. A small R_{Th} means that most of V_{Th} will be across the load – that is the desired behaviour. But what if the resistance of the load, R_{load}, becomes small as well, i.e. of the order of R_{Th}? The voltage across the load will start to drop significantly as it always distributes across R_{Th} and R_{load}. At the same time the voltage supply will have to increase its current output to maintain the voltage V_{Th}. At some low R_{load} the supply will hit its capabilities to supply current, often signalled by a *current limit indicator*, and from here on will lower its output voltage V_{Th}.

Can we understand why for the *voltage divider* R_{Th} becomes $R_{||}$, i.e. why the output impedance of these series resistors appears to be that of parallel resistors? Let's apply the **small signal version** of **Ohm's Law**: *apply a ΔV, then find ΔI*. Assume a voltage divider as given below on the left: input voltage, resistances, voltage distribution and resulting currents are given.



Then apply a voltage supply which forces the mid-point up by 1V, as shown on the right. The voltage distributions changes accordingly. The currents through the two resistors adjust. In order to maintain the voltage shift the new supply has to provide an additional current to balance the node between the two resistors. Result: impedance = ΔV/ΔI = 1V/0.2mA = 5kΩ. Or expressed in a different way: a change at the output to the voltage divider corresponds to a *change in the current through both of the resistors* – as though they were in parallel. Hence the output impedance effectively is the resistance of these two resistors in parallel.

The **input impedance** is a similar idea. It is the resistance that the circuit, which is going to be receiving the current, appears to have. It is the effective resistance of the "load". As you have heard in the last lecture for an accurate *voltage measurement* a large input impedance is needed. This is quantitatively been discussed below.
In contrast for an accurate *current measurement* a small input impedance is needed. Else part of the current would flow past the ampere-meter and would not be accounted for.

In summary:

for	output needs:	input needs:
voltage supply/measurement	small impedance	large impedance
current supply/measurement	large impedance	small impedance

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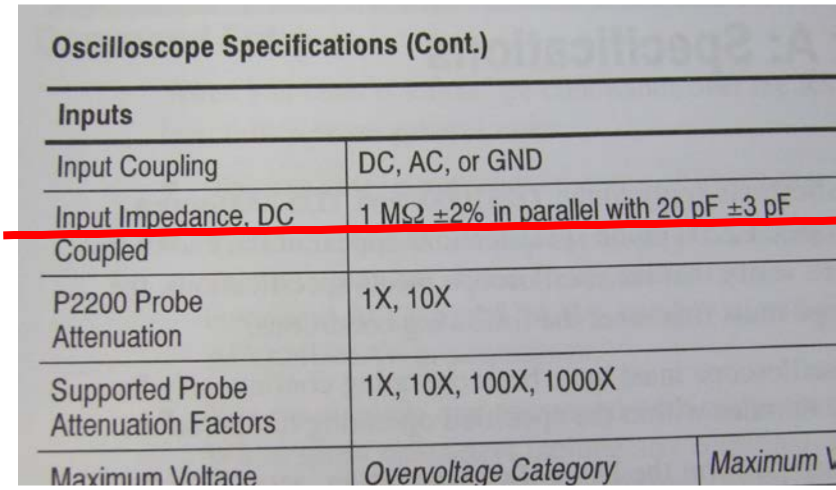
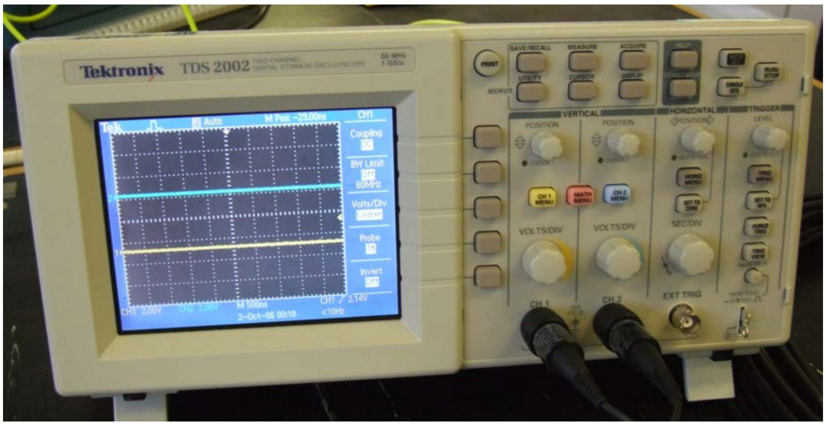
You will need to consider input and output impedances whenever you are attaching two pieces of equipment. Here are a few examples:

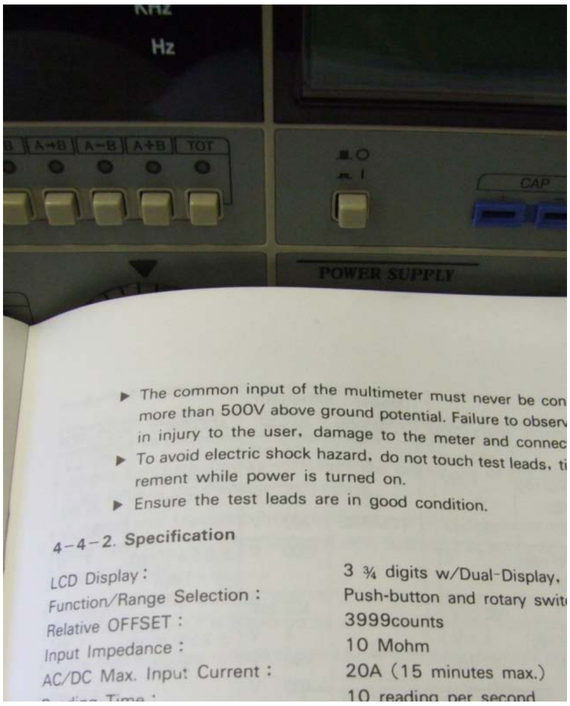
Your **oscilloscope** has an input impedance of 1 MΩ:


Oscilloscope Specifications (Cont.)

Inputs	
Input Coupling	DC, AC, or GND
Input Impedance, DC Coupled	1 MΩ ±2% in parallel with 20 pF ±3 pF
P2200 Probe Attenuation	1X, 10X
Supported Probe Attenuation Factors	1X, 10X, 100X, 1000X
Maximum Voltage	Overvoltage Category



Your **multimeter** has an input impedance of 10 MΩ:


▸ The common input of the multimeter must never be con
more than 500V above ground potential. Failure to observ
in injury to the user, damage to the meter and connec

▸ To avoid electric shock hazard, do not touch test leads, ti
rement while power is turned on.

▸ Ensure the test leads are in good condition.

4-4-2. Specification

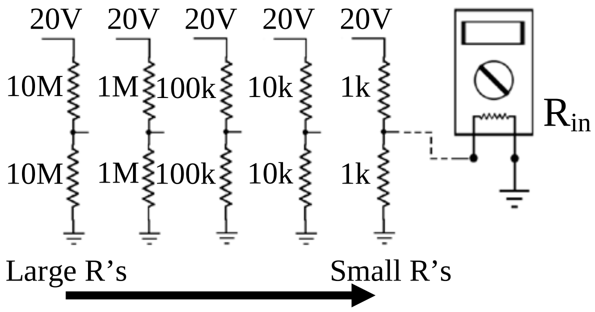
LCD Display :	3 ¾ digits w/Dual-Display,
Function/Range Selection :	Push-button and rotary swit
Relative OFFSET :	3999counts
Input Impedance :	10 Mohm
AC/DC Max. Input Current :	20A (15 minutes max.)

Your **function generator** has an output impedance of 50Ω or 600Ω:

Example: The digital voltmeter (multimeter):

The voltages measured depend on the relative size of output (R_{Th}) and input (R_{in}) impedances.

Pictured below are a series of five voltage dividers. Each has V_{in} = 20V and using a voltmeter you are trying to measure V_{out}. Since the two resistors that comprise each voltage divider in each case are both of equal value every time you should measure the same V_{out} = 10V.



However, on the left the two resistors are each 10MΩ, i.e. equal to R_{in} of the voltmeter. They decrease by one order of magnitude for each successive voltage divider until the right most circuit has a pair of 1kΩ resistors. As R_{in} is finite, part of the current will branch out and flow through the voltmeter, causing R_{in} and the lower resistor to form a parallel arrangement. Consequently the voltage between the two resistors of the divider will be shifted to a lower value, according to:

$$V_{OUT} = \frac{V_{IN}}{\left(2 + \frac{R}{R_{in}}\right)} \quad \text{or generally with unequal R} \quad V_{OUT} = \frac{V_{IN} R_2}{\left(R_1 + R_2 + \frac{R_1 R_2}{R_{in}}\right)}$$

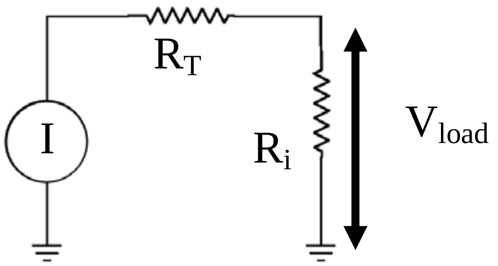
That is, the resistance measured by the voltmeter will depend on the relatively sizes of the resistors in the voltage divider and the input impedance of the voltmeter. Numerically for the above example the measurements of the true V_{out} = 10V will be:

R value	V_{out} measured	% error	systematic deviation
10MΩ	6.67V	49.9%	severe
1MΩ	9.52V	5.0%	moderate
100kΩ	9.95V	0.5%	well within calibration error

One can see that if the resistor values of the voltage divider are *two orders of magnitude smaller* than the input impedance of the voltmeter the systematic error from the load introduced by attaching the meter is significantly smaller than the **calibration accuracy** of the meter (which typically is of the order of 2%).

Control question: In the above example, if you were using the divider with two 1MΩ resistors and you would measure V_{out} = 8.00V. What would you conclude the input impedance, R_{in}, of the voltmeter to be?

Optimal values of input impedance: (H&H, p. 13)
Rather than drawing the Thevenin equivalent circuit as a fragment with a loose wire hanging off the circuit is completed with the load to be driven attached. The resistance of the load is the input impedance of that device.



The **power transferred** from the driving circuit to the load is the current squared multiplied by the input impedance of the load:

$$P = I^2 R_{in}$$

To understand this let's first look at the extreme cases: $R_{in} = 0$ and $R_{in} = \infty$.

If $R_{in} = 0$	then	$V_{load} = 0$	and	$I_{load} = I_{source}$	so	$P = V_{load} I_{load} = 0$
If $R_{in} = \infty$	then	$V_{load} = V_{source}$	and	$I_{load} = 0$	so	$P = V_{load} I_{load} = 0$

The power transferred, P, is *zero at the extremes*. Only at finite loads and source currents power is been transferred. There must be a *maximum* somewhere between these two extremes. To calculate the **maximum power transferred** we dust off our calculus skills:

$$P = I^2 R_{in} = \left(\frac{V_{Th}}{R_{Th} + R_{in}}\right)^2 R_{in}$$

$$\frac{dP}{dR_{in}} = \left(\frac{{V_{Th}}^2}{(R_{Th} + R_{in})^3}\right)(R_{Th} - R_{in})$$

We find the maximum by setting the derivative to zero and conclude:

$$\frac{dP}{dR_{in}} = 0 \qquad \text{if} \qquad R_{Th} = R_{in}$$

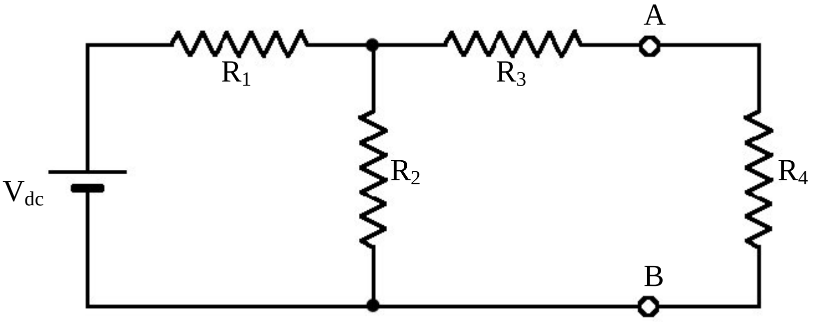
And it must be a maximum, since P = 0 for R_{in} = 0 and $R_{in} = \infty$ and P > 0 holds in-between.

Examples:

Remember Thevenin's theorem: *Any two-terminal network of resistors and voltage sources is equivalent to a single resistor in series with a single voltage source.*

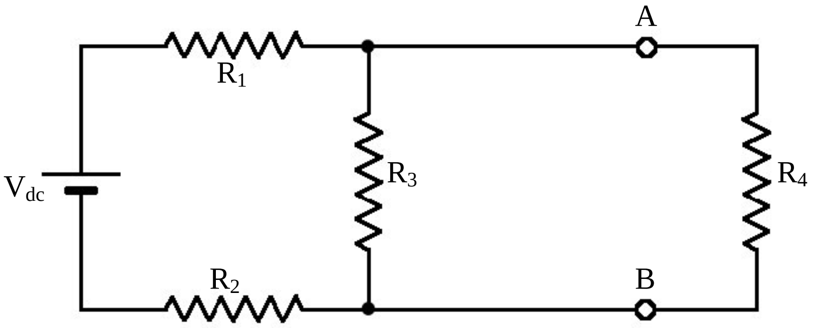
In the following three examples are given where you can go through and apply the rules to calculate Thevenin's equivalent circuit. In each case A and B are the two terminals.

Example 1:



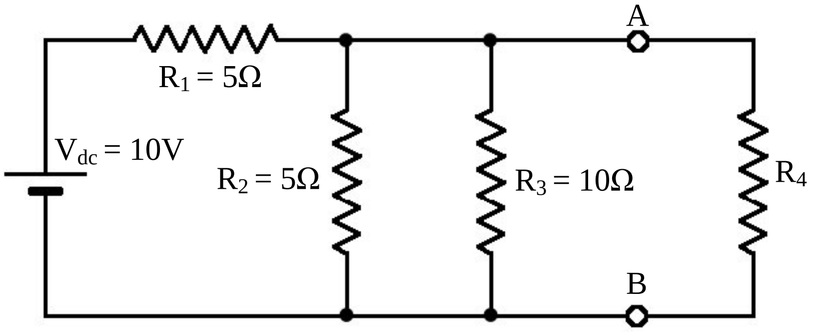
Take: V_{dc} = 12V, R_1 = 3Ω, R_2 = 6Ω, R_3 = 7Ω and R_4 = 3Ω
Use Thevenin's equivalent of the circuit to find the voltage across the output resistor R_4.

Example 2:



Take: V_{dc} = 20V, R_1 = 6Ω, R_2 = 6Ω and R_3 = 10Ω
Determine the Thevenin's equivalent circuit.

Example 3:



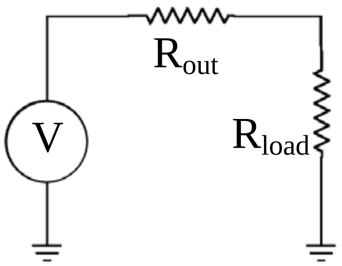
Determine the Thevenin's equivalent circuit.

Voltage and Current Sources – revisited:

In the previous lecture we began to outline the general considerations about voltage and current sources. Now that we have learnt about Thevenin's equivalent circuits and about input and output impedances we can look at these issues in more detail.

The Voltage Source:

In the illustration below R_{out} is the output impedance of the voltage source while R_{load} is the input impedance of the device which is being driven by the power supply.



We want a device that supplies a fixed voltage regardless of the load resistance. This is a tall order. Our power supply needs to keep V fixed for whatever R_{load} is attached – so it must supply a current:

$$I = \frac{V}{R_{out} + R_{load}}$$

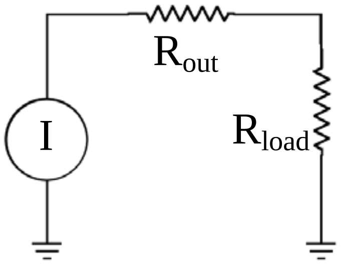
In reality a voltage source can only supply a *limited amount of current* – hence eventually V will begin to drop. In addition, in order that essentially all of V should appear across the load, R_{out} must be extremely *small compared to the likely range* of R_{load}. A "stiff" voltage source is one that doesn't bend under the load.

To achieve that in reality, active components are used to build power supplies. Here the effective R_{out} of the circuits can be as low as milliohms.

A voltage source is happy to stay open circuit, but will break down if you short-circuit it.

The Current Source:

Here we use the same schematic circuit except here we are interested in the current rather than the voltage.



We want a device that supplies a fixed current regardless of the load resistance. The power supply must keep

$$V = I\ (R_{out} + R_{load})$$

for whatever R_{load} is attached. In reality a current source can only provide a *limited voltage* – hence I begins to drop when R_{load} is high enough. In addition, in order that the current through the load should remain constant, R_{out} must be extremely *large compared to the likely range* of R_{load}. The maximum voltage that a current source can supply is called the **output voltage compliance**.

Again, in reality, the high value of R_{out} in the MΩ range is achieved using active components.

Evidently a current source is happy to be short circuited. By contrast it will be very unhappy to find itself trying to drive its current through a huge resistance. At open circuit it naturally falls flat.

Summary:

- A circuit is modified when other elements are attached – a measurement unit is such an element.
- Thevenin's theorem: Any two-terminal network of resistors and voltage sources is equivalent to a single resistor in series with a single voltage source.
- The relationship between devices that get attached can be discussed in terms of input and output impedances.