Lecture 6, 28th September 2012

Analogue Electronics 3: AC Circuits – capacitors

We leave the regime of just working with DC currents and open our view to electronics behaviour over time, using alternating currents (AC).

Capacitors – an analogue component with memory:

A **capacitor** is a *break in a circuit*. In a DC circuit (static) you only know this as a switch, with limited functionality. However, in circuits with changing the voltages and currents it becomes extremely useful, because a capacitor also can *store charge* and can *release the charge* again.

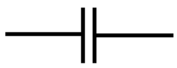
Using this ability capacitors are used as part of circuits which:

- respond to changes (differentiate),
- perform averaging (integrate),
- select frequency ranges (filter).

Before we jump into the dynamics we briefly will look again at the static properties of capacitors. You have already seen this in Physics 2A and probably elsewhere.

Static properties of the capacitor: (H&H, 1.12, p. 20)

The symbol for a capacitor is two parallel lines with a gap between them. This represents the simplest sort of capacitor: a pair of parallel plates with a small gap between them. Generally the same symbol is used for all capacitor designs.



Only if they are sensitive to polarity, like electrolyte capacities are, then a "+" is added to the side which has to be wired to the positive potential to prevent the capacitor from becoming conductive or being destroyed ("to blow up" in laboratory slang, as it goes with a bang). Or arrows may be added across the symbol for adjustable capacities.

A capacitor can store a fixed amount of charge at a particular voltage:

$$Q = C\,V \qquad \text{(units: C=Coulomb, F=Farad and V=Volt)}$$

If you place a fixed voltage, V, across a capacitor current will flow until there is a charge of +Q on one surface and –Q on the other surface.

Obviously several capacitors can be combined in the same circuit. The rules for combining the capacitances are exactly opposite to those for combining resistances:

$$\text{In series: } \frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

$$\text{In parallel: } C_{total} = C_1 + C_2 + \cdots$$

A Farad is a large unit, as it needs large surfaces and small gaps to store lots of charge. In electric circuits usually the following ranges of capacities are in use: pF, nF and at most μF.

You may need to combine several capacitors to get the values you require in the labs. Also note that the production accuracy of capacities is significantly worse than for other elements.

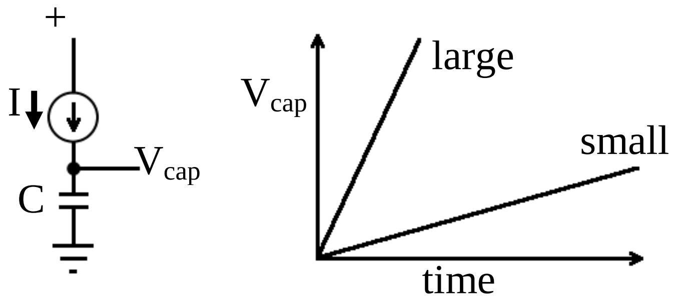
Dynamic properties of the capacitor: (H&H, 1.13, p. 23)
The interesting properties of capacitors occur in response to voltages or currents that change as a function of time. One way to summarize the behaviour is to say: **the larger the current the faster the voltage across the capacitor changes**. Or mathematically:

$$\left(\frac{d}{dt}\right)Q = \left(\frac{d}{dt}\right)CV$$

with the capacity constant in time this simplifies to:

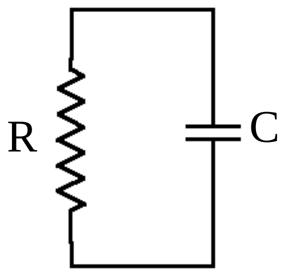
$$I = C\left(\frac{dV}{dt}\right)$$ (units: A=Ampere, F=Farad and V/s=Volta/sec)

A current source that provides a steady current to a capacitor will generate a steadily increasing voltage across the gap, like displayed below (note again the symbols for the connection to supply and ground).



The sort of voltage "ramp" that you can see in the graph is the signature of a capacitor driven by a current source. The straight lines shown here for the voltage change with time are only approximations for *small times* – what *small* means will be qualified below.

Discharging a capacitor through a resistor:
Imagine that you could collect positive and negative charges on opposite surfaces of a capacitor and then connect it to a resistor. You would have a circuit that looks like this:



In practice you would do this by having a second half of a circuit on the right with a battery that you would disconnect with one switch and then connect the resistor with a second switch.

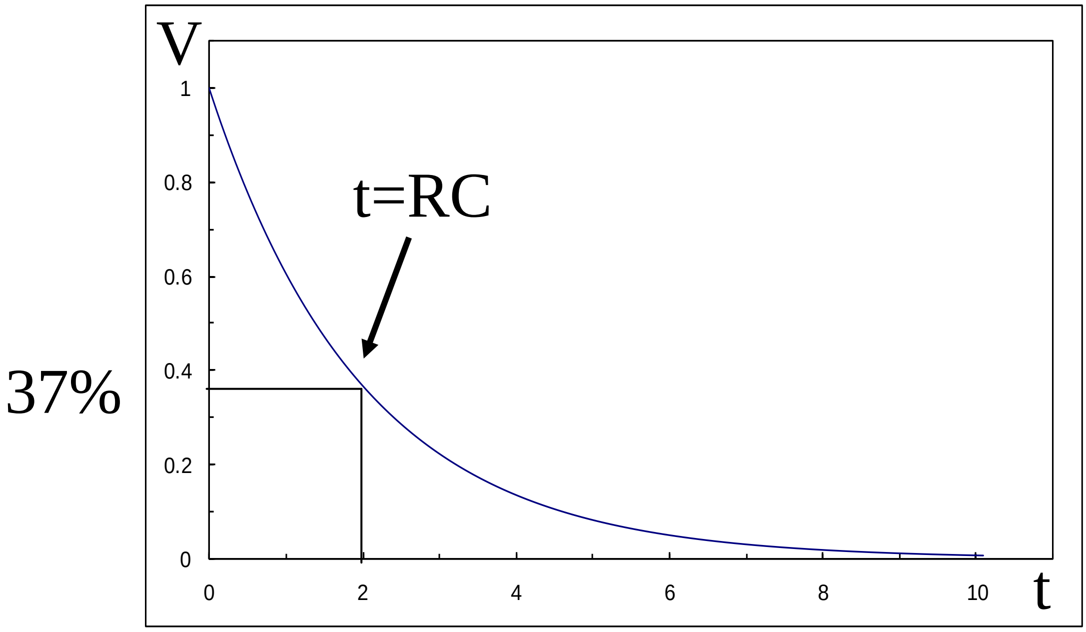
This cartoon circuit is described by:

$$C\left(\frac{dV}{dt}\right) = I = -\frac{V}{R}$$

The solution of this differential equation is:

$$V = Ae^{\left(-t/_{RC}\right)}$$

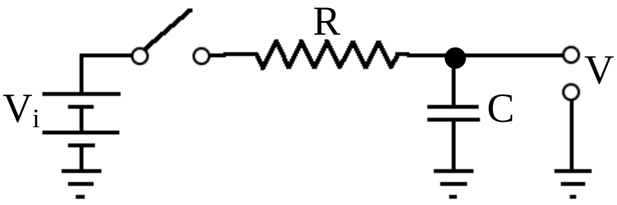
It describes an exponential decay in the voltage, V, with time, t, across the resistor. The value of the product RC is the characteristic **time constant** for the decay of the initial potential, A, across the capacity. That means that by choosing the values of "R" and "C" you can *control the time dependence* of the circuit! E.g. a time constant of t=2μs could be achieved by choosing R=50Ω and C=40nF.



As a reminder: you should know that in all exponential decays the amplitude (here the voltage across the capacity) falls by a factor of 1-1/*e* = 0.63 (or to a remaining factor of 1/*e* = 0.37) *during each time constant*. E.g. after 5 time constants the initial signal will be diminished to 0.7% of its original size.

Charging a capacitor through a resistor:

This is marginally more complicated, that is why we are looking at it second. In the circuit below we have batteries that can be connected via a switch to a resistor and a capacitor. You may notice that the arrangement of resistor and capacitor is the same as that of the two resistors in a voltage divider.



We are going to determine the voltage across the capacitor as a function of time since the switch was closed (t=0). This circuit is described by:

$$I = C\left(\frac{dV}{dt}\right) = \frac{V_i - V}{R}$$

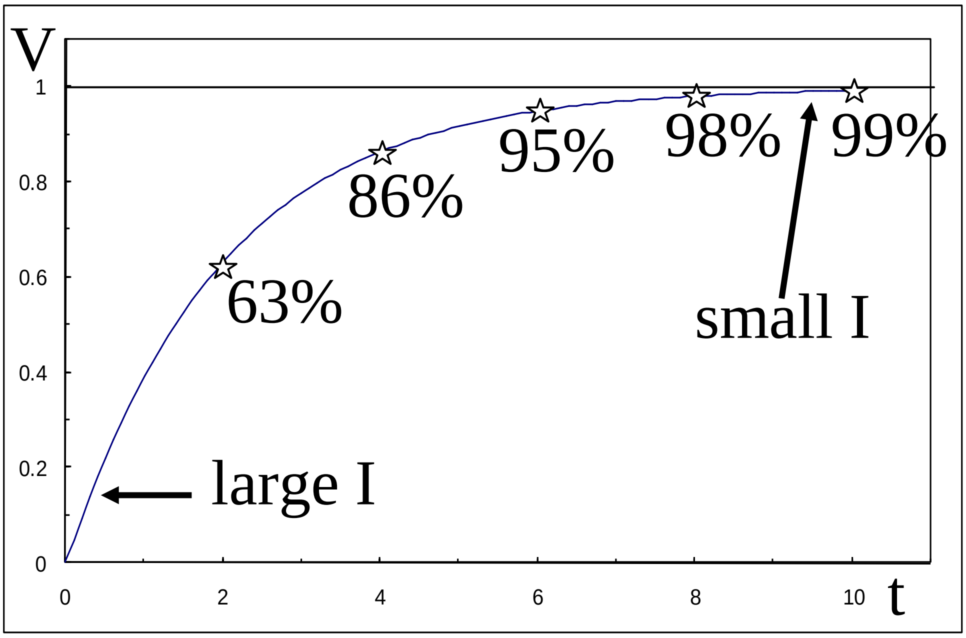
The solution of this differential equation is:

$$V = V_i \left(1 - e^{-t/_{RC}}\right)$$

Once the switch is closed the voltage V_i is spanning across both the resistor and capacitor together. What changes as a function of time is the proportion of the voltage drop which is across the two individual components.

At the moment that the switch is closed the voltage across the capacity is zero, V=0, and the entire voltage drop, V_i, appears across the resistor. As the capacitor begins to charge the balance changes until $V=V_i$, that is all of the voltage drop appears across the capacitor. The time dependence of this change is again controlled by the characteristic RC **time constant**. The change in V as a function of time is shown on the graph below. t=0 is the moment when the switch is closed.

In one t=1*RC the capacitor has charged up to 63% of its capacity. In t=5*RCs it has charged up to larger than 99%.



We have now seen the basics operation: *discharging and charging of a capacitor*. Both are controlled by the *same time constant* (RC) which can be chosen by *selecting components*.

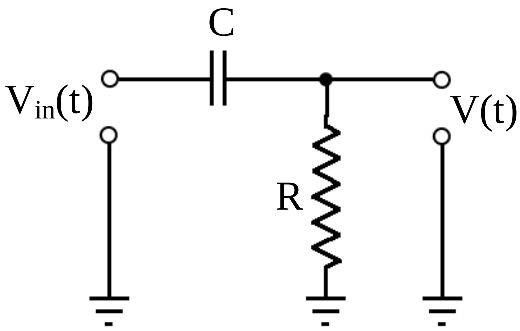
Note that the exponential curve can be approximated by a linear description for time scales small with respect to the time constant.

Next we are going to get a little more serious, and look at how capacitors can be used to perform calculus operations.

Differentiators: (H&H, 1.14, p. 25)
The defining equation for a current through a capacitor involves a derivative:

$$I = C\left(\frac{dV}{dt}\right)$$

Therefore it should be no surprise to discover that these components can then be harnessed to carry out calculus operations. Regard the following arrangement of a RC unit, in the configuration of a **differentiator**, between a time dependent input voltage signal, $V_{in}(t)$, and output voltage signal, V(t):



The voltage across C is (V_{in} – V) so:

$$I = C\left(\frac{d}{dt}\right)(V_{in} - V)$$

The resistor, R, acts as a **current sink** for the output circuit:

$$I = \frac{V}{R}$$

Together we have:

$$\frac{V}{R} = C\left(\frac{d}{dt}\right)(V_{in} - V)$$

V_{in} is no longer just a static voltage from a battery. Instead it may be any time dependent voltage which we might want to differentiate. The output voltage as a function of time is:

$$V(t) = RC\left(\frac{dV_{in}}{dt} - \frac{dV}{dt}\right)$$

That is not exactly the derivative of the input signal, but also depends on its own derivative. This is like the output signal has gained some inertia – it cannot abruptly change its trend anymore.

To ease calculation we can use an approximation which often applies: if the product **RC is very small** then the *capacitor will respond quickly* and:

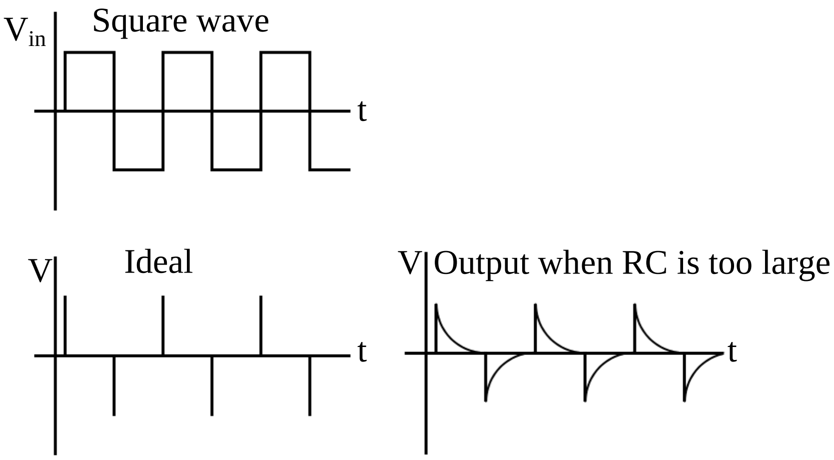
$$\frac{dV}{dt} \ll \frac{dV_{in}}{dt}$$

Thus one can approximate above equation by neglecting the second term:

$$V(t) \approx RC\frac{dV_{in}}{dt}$$

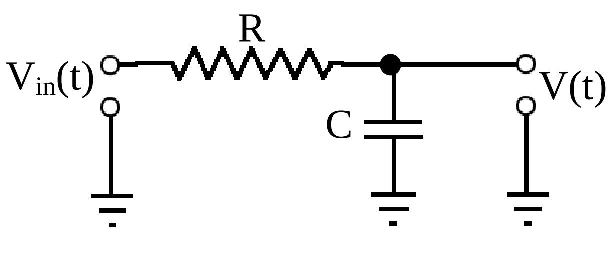
This approximation says: when the time constant of the RC element is *much shorter* than the variation of the input voltage, V_{in}, then the *current through the resistor will be proportional to the* ***change*** *of the input voltage*, with the capacity, C, being the proportionality factor.

Below are sketches of the input and output in both the *ideal case*, where RC is much smaller than the rate of change in the input, and in the case where RC is *too large* for the current to closely follow the rate of change. In the second case the charging and discharging of the capacitor smoothes the output and makes it resemble the derivative less.



In a few lectures time, you will see how this performance can be improved on via the use of operational amplifiers (op-amps), i.e. active rather than passive components.

Integrators: (H&H, 1.15, p. 26)
Integration can also be approximated in an analogous manner. The relevant circuit is pictured below. Compared to the differentiator the **integrator** circuit has the positions of the resistor and the capacitor switched:



This time it is the voltage across R which is (V_{in} – V) so:

$$I = C\left(\frac{dV}{dt}\right) = \frac{V_{in} - V}{R}$$

The capacity acts here as a **charge buffer**. What we are looking for is to find is the integral of $V_{in}(t)$. For that we need to make a similar approximation to that employed previously: if the product **RC is very large** then the *capacitor will respond slowly* and:

$$V \ll V_{in}$$

Thus one can approximate above equation by neglecting the second term:

$$C\left(\frac{dV}{dt}\right) \approx \frac{V_{in}}{R}$$

This approximation says: when the time constant of the RC element is *much larger* than the variation of the input voltage, V_{in}, then the *current through the resistor will be proportional to the **value** of the input voltage*, with the inverse capacity, 1/C, being the proportionality factor.

We can transform this into the following integral:

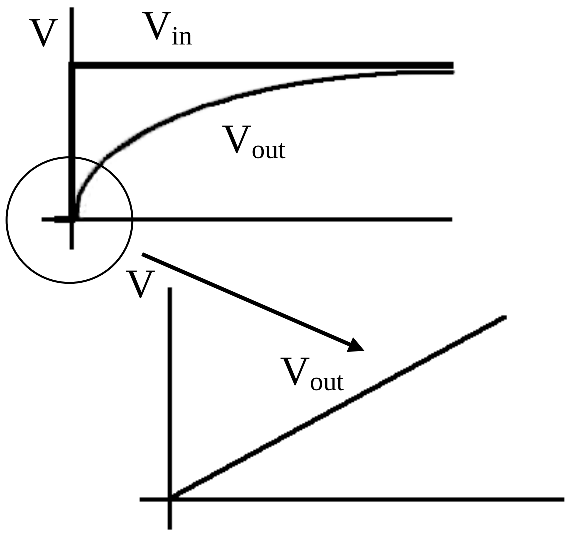
$$V(t) \approx \left(\frac{1}{RC}\right)\int V_{in}(t)\,dt + D$$

If the input was a current I(t) rather than a voltage then the output voltage would give an exact integral via:

$$I = C\frac{dV}{dt}$$

In other words: put a large resistor in series with a voltage source and you will have an approximation to a current source. Add a large capacity to ground to collect the charge and the voltage across the capacity will approximately be the integral over the input voltage, V_{in}.

Of course, the capacity will charge up using the *exponential law*, as discussed above. Thus, the integrator only will respond approximately linearly to the input signal for a short time scale compared to the RC time constant, i.e. at an early point on the charging up curve discussed before. For integration over longer time scales **saturation effects** will become visible. The nature of the approximation is pictured below.



In the top graph a square wave input, V_{in}, is overlaid on the output V. For a square wave input the integral is a ramp. The arrow to the lower graph shows the nature of the approximation. If RC is large then only the early part of the exponential charge-up is seen. This early part of the charging curve approximates a ramp and hence is the integral of the input square wave.

In summary:

- For a **differentiator**, we needed **fast response**, i.e. a **small RC** time constant.
- For an **integrator** we need a **slow response**, i.e. a **large RC** time constant.

Rule of thumb:

- A **capacity in series** with the supply with the resistor connecting as **current sink** to ground will **block DC currents**, but the output will react on changes of the input.
- A **resistor in series** with the supply with the capacity connecting as a **charge buffer** to ground will **smooth out fast changes**, but the output will react to slow changes.

Properties of an inductor: (H&H, 1.16, p. 28)
For completeness **inductors** need to be mentioned here. But they will remain a bit like the fifth wheel on the car for the rest of this lecture series, only employed when needed.

An inductor is essentially a wire arranged in the form of a coil, or an element which is acting like one. While capacitors store energy in the *electric field* which is induced between its surfaces, inductors store energy in the *magnetic field* which is induced in the centre of the curled up current.

Dynamic properties:
In an **inductance, L**, voltage is generated proportional to the rate of change of the current in a circuit:

$$V = L\frac{dI}{dt} \qquad \text{(units: V=Volt, H=Henry and A/s=Ampere/sec)}$$

Self inductance in a single circuit and **mutual inductance** between two circuits, mediated by a mutual magnetic field, are distinguished.

Essentially inductors *act the opposite way* than capacities. However, they are rarely used as components for electrical circuits in practice, and if, usually only as components to efficiently damp oscillations. Inductors tend to be more bulky, more expensive and they suffer more performance problems than capacitors.

Note however that other components, like wires, resistors and even capacities, also bear *some inductive behaviour* in real-world applications. But the inductive couplings usually are so *tiny* that they can safely be neglected and seldom are mentioned at all. Only the capacitive couplings of the elements usually are taken into account, if they become significant.

Self and mutual inductance may become a significant correction for the *high frequency behaviour* of tracks on *printed circuit boards* or in *cables* (above ~100MHz or 1GHz). Modern programs to design the layout of such tracks take inductive couplings into account (as an additional correction to the typically significantly larger capacitive couplings).

Mutual inductance becomes the main tool of trade in **transformers**, but these are not covered in this lecture.