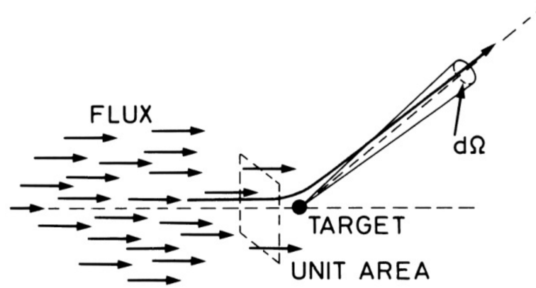
Interaction of Elementary Particles with Matter

- Cross section & mean free path
- Interaction of charged particles
- Interaction of neutral particles

Cross Section

- Total and Differential cross section on single target:

$$\sigma(E) = \int d\Omega \frac{d\sigma}{d\Omega}, \quad \frac{d\sigma}{d\Omega}(E,\Omega) = \frac{1}{F}\frac{dN_s}{d\Omega}$$



  - N_s = scattered particles / time
  - F = flux = N_{inc} / (area · time)
  - $[d(\sigma)]$ = 1 barn = 10^{-28} m^2
- Mass thickness of target = $\rho \cdot t$ (aka. 'surface density')
  - ρ = density [g/cm^3]
  - t = thickness [cm]
  - $[d(\rho \cdot t)]$ = g/cm^2
- Differential cross section per unit mass of extended, but thin target:

$$\frac{d\sigma}{d\Omega}(E,\Omega) = \frac{1}{FA\rho\delta x}\frac{dN_s}{d\Omega}$$

  - A = target area
  - δx = target thickness
- Probability of interaction for single particle per unit mass of extended, but thin target:

$$N_{tot} = FA\sigma\rho\delta x \quad \Rightarrow \quad w\delta x = \sigma\rho\delta x$$

  - N_{tot} = total number of scattered particles / time into all angles

Mean Free Path

- Probabilities of:
  - having an interaction between x and dx: w dx
  - not having an interaction between x and dx: P(x+dx) = P(x)(1- w dx)
  - not having an interaction after distance x: P(x) 'survival prob.'
- Exponential decay of survival probability:

$$P(x) = e^{(-wx)}$$

- Mean free path λ (mean distance without interaction):

$$\lambda = \frac{\int xP(x)dx}{\int P(x)dx} = \frac{1}{w} = \frac{1}{\rho\sigma}$$

- Survival probability:

$$P(x) = e^{(-\rho\sigma\cdot x)} = e^{\left(-\frac{x}{\lambda}\right)}$$

- Interaction probability:

$$P_{\text{int}}(x) = 1 - e^{(-\rho\sigma\cdot x)} = 1 - e^{\left(-\frac{x}{\lambda}\right)}$$

- Interaction probability in dx after survival of travel through x:

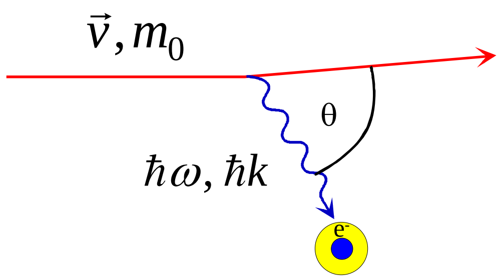
$$P(x+dx) = e^{(-\rho\sigma\cdot x)}\rho\sigma\cdot dx = e^{\left(-\frac{x}{\lambda}\right)}\frac{dx}{\lambda}$$

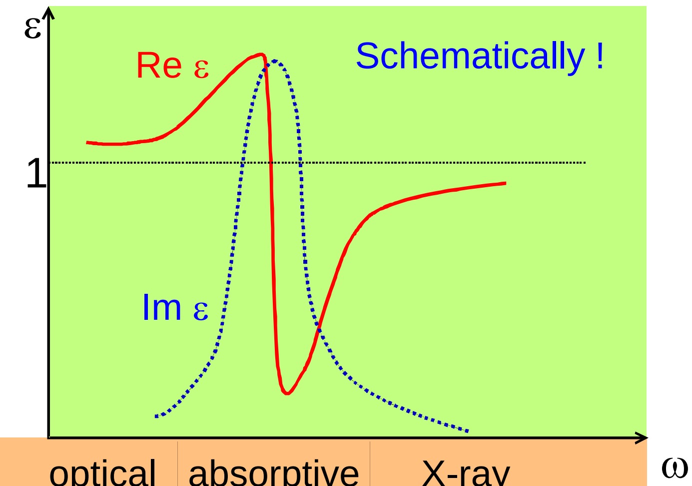
Interactions in Matter

- Charged Particles:
- Passage of charged particles through matter:
  - Energy loss: inelastic collisions with atomic electrons — very frequent
    - soft: exitation (scintillation)
    - hard: ionisation (knock-on electrons (‘δ-rays’))
  - Deflection: elastic scattering from nuclei — frequent
  - Bremsstrahlung:
  - Photon Emission: (Cherenkov & Transition radiation)
  - Nuclear reactions: (neutrons, alphas)
  - Weak interactions: (neutrinos)
- Divided into two classes:
  - heavy particles: μ, π, K, p, d, α, …
  - particles of electron mass: e^-, e^+
- Neutral Particles:
- Photons
  - Photo electric effect
  - Compton Scattering
  - Pair production
- Neutrons
  - nuclear reactions

Interactions of Charged Particles

- Collisions with nuclei not important ($m_e \ll m_N$).
- Collisions with atomic electrons of absorber material





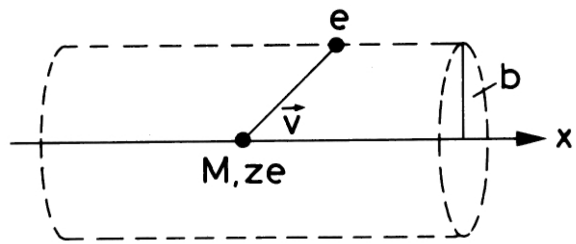
regime:	optical	absorptive	X-ray
effect:	Cherenkov radiation	ionisation	transition radiation

- Dispersion relation:

$$\omega = 2\pi\nu = 2\pi\frac{1}{\lambda}\frac{c}{n} = k\frac{c}{n} \qquad \omega^2 - \frac{k^2c^2}{\varepsilon} = 0 \qquad \varepsilon = n^2 \qquad \varepsilon = \varepsilon_1 + i\varepsilon_2$$

- Optical region: Cherenkov $\varepsilon_1 > 1 \quad \varepsilon_2 \ll 1 \qquad \varepsilon$ real $\quad \cos\theta = \frac{1}{n\beta} = \frac{1}{\sqrt{\varepsilon}\beta}$
- Absorptive region: excitation, ionisation $\varepsilon_2 \gg 0$
- X-ray region: Transition radiation $\varepsilon_1 < 1 \quad \varepsilon_2 \ll 1$

Classical Energy Loss (Bohr)



- Consider:
  - heavy particle ($M \gg m_e$) with charge ze and velocity v
  - quasi-free atomic electron at impact parameter b
  - restriction: interaction short, i.e. electron static
- Calculate momentum transfer: current seen by electron

$$I = \int F dt = \int eE_\perp dt = \int eE_\perp \frac{dx}{v} \qquad \int E_\perp 2\pi b dx \overset{Gauss}{=} 4\pi ze \;\Rightarrow\; \int E_\perp dx = \frac{2ze}{b}$$

$$= \frac{2ze^2}{bv}$$

- Calculate momentum transfer: energy gained by electron

$$\Delta E(b) = \frac{I^2}{2m_e} = \frac{2z^2e^4}{m_e v^2 b^2}$$

- Energy lost to all electrons in volume $dV = 2\pi b\, db\, dx$ with electron density N_e

$$-dE(b) = \Delta E(b) N_e dV = \frac{4\pi z^2 e^4}{m_e v^2} N_e \frac{db}{b} dx \quad \Rightarrow \quad -\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{b_{\max}}{b_{\min}}$$

Classical Energy Loss (Bohr)

- Integration limits: naïve approach fails
  - b=0: infinite energy loss
  - b=∞: contradicts short interaction time
- b_{min}: maximum energy transfer in head-on collision

$$\frac{2z^2e^4}{m_e v^2 b_{\min}^2} = 2\gamma^2 m_e v^2 \quad \Rightarrow \quad b_{\min} = \frac{ze^2}{\gamma\, m_e v^2}$$

- b_{max}: electron bound with orbital frequency $\nu \rightarrow$ interaction time short wrt. $\tau=1/\nu$
  - relativistic interaction time: t~b/γv
  - upper limit for averaged electron frequencies <ν>:

$$\frac{b}{\gamma\, v} \leq \tau = \frac{1}{\langle \nu \rangle} \quad \Rightarrow \quad b_{\max} = \frac{\gamma\, v}{\langle \nu \rangle}$$

- Bohr's classical energy loss formula:

$$-\frac{dE}{dx} = \frac{4\pi\, z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m_e v^3}{ze^2 \langle \nu \rangle}$$

- Reasonable description for heavy particles like the α-particle or heavier nuclei
- Fails proton or lighter particles due to quantum effects

Bethe-Bloch Formula

- Correct quantum-mechanical calculation leads to the energy loss dE/dx [MeV/cm] due to ionisation caused by heavy ($M \geq m_\mu$) charged particles:

$$-\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e \gamma^2 c^2 \beta^2 T^{max}}{I^2} - \beta^2 - \frac{\delta}{2} - \frac{C}{Z} \right]$$

$$2\pi N_A r_e^2 m_e c^2 = 0.1535 \frac{MeV\, cm^2}{g}$$

- with maximum energy transfer in single collision T^{max}:

$$T^{max} = \frac{2 m_e \gamma^2 c^2 \beta^2}{1 + 2\frac{m_e}{M}\sqrt{1+\gamma^2\beta^2} + \frac{m_e^2}{M^2}} \simeq 2 m_e \gamma^2 c^2 \beta^2$$

- and experimental mean excitation potential I:

$$Z < 13: \quad \frac{I}{Z} = 12 + \frac{7}{Z} eV$$

$$Z \geq 13: \quad \frac{I}{Z} = 9.76 + 58.8 Z^{-1.19} eV \quad \approx \quad Z\, 10 eV$$

- and its corrections $\delta(\log_{10}(\beta\gamma))$ (density effect) and $C(I,\beta\gamma)$ (shell corrections)

– N_A = Avogadro's number
– r_e = classical electron radius
– m_e = electron mass [g]
– c = velocity of light

– ρ = density of absorber [g/cm³]
– Z = atomic number of absorber
– A = atomic weight of absorber

– z = charge of incident particle [e⁻]
– M = mass of incident particle
– β = v/c of incident particle
– γ = $1 / \sqrt{1-\beta^2}$ of incident particle

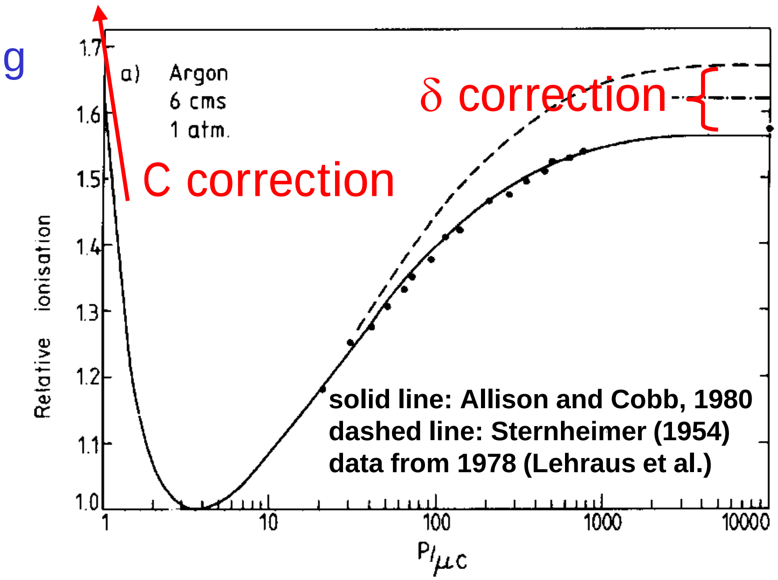
– often used convention: x =: ρx [g/cm²]
→ 'mass stopping power' dE/dx: [MeV/g cm²]

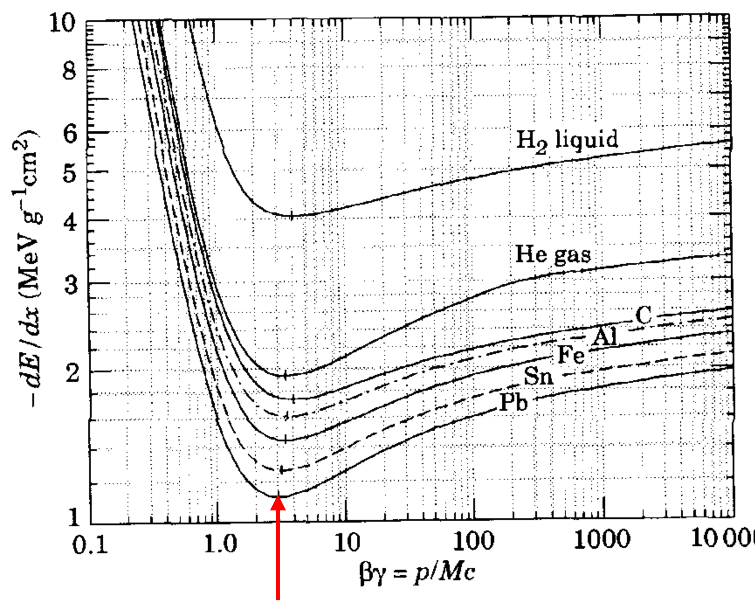
Bethe-Bloch Formula

- Not valid for e^-, e^+: $m_{proj}=m_{target}$ → Bremsstrahlung
- small β: (kinematic factor)
  - dE/dx falls like $\propto 1/\beta^2$ (more precise $\propto \beta^{-5/3}$)
  - shell correction small
- βγ ≈ 4: (minimum ionising, MIP)
  - dE/dx constant $$\frac{dE}{dx} \approx 1.2\ MeV\ g^{-1} cm^2$$
- βγ >> 1: (relativistic rise)
  - dE/dx rises like $\propto \ln \gamma^2$
  - density larger for lighter materials (gases)
- dE/dx rather independent of Z (except H_2)
- Mixed materials: (Bragg's rule)
  - add mass stopping powers by weight contributions

$$\frac{1}{\rho}\frac{dE}{dx} = \frac{w_1}{\rho_1}\left(\frac{dE}{dx}\right)_1 + \frac{w_2}{\rho_2}\left(\frac{dE}{dx}\right)_2 + \cdots$$

  - fairly good approximation





Bethe-Bloch Formula

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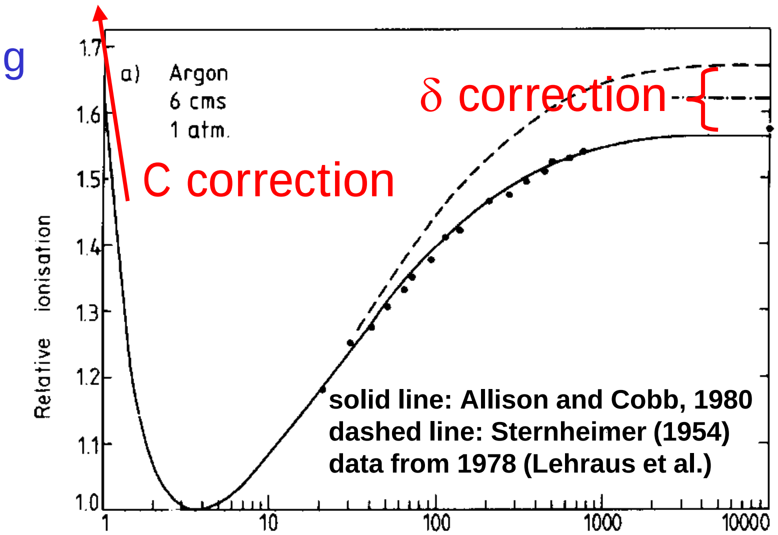
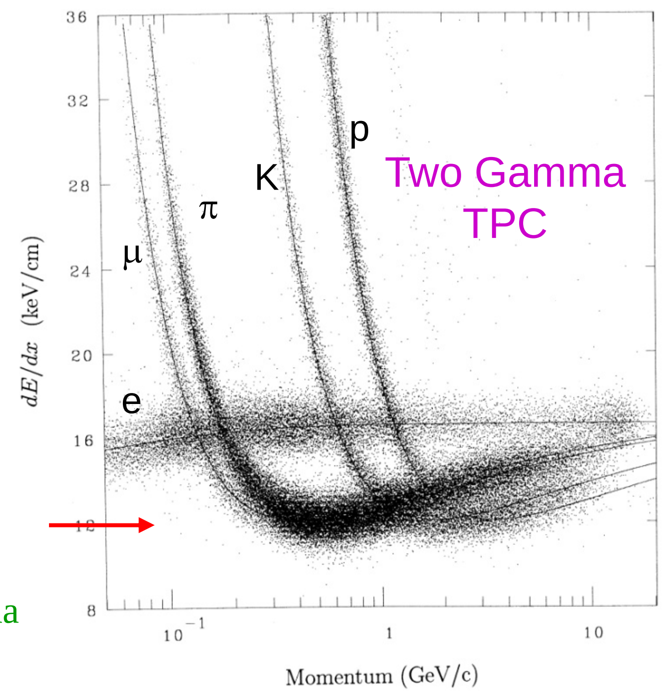




dE/dx Scaling & Range

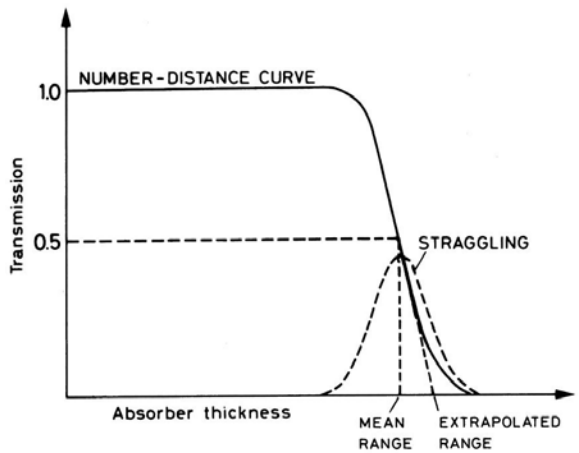
- Scaling Law:
  - T = kinetic energy
  - scaling from particle 1(M_1,T_1) to particle 2(M_2,T_2)

$$-\frac{dE}{dx} = z^2 f(\beta) = z^2 f'\left(\frac{T}{M}\right) \qquad T = (\gamma - 1)Mc^2$$

$$-\frac{dE_2}{dx}(T_2) = -\frac{z_2^2}{z_1^2}\frac{dE_1}{dx}\left(T_2\frac{M_1}{M_2}\right) \qquad T = T_2\frac{M_1}{M_2}$$

- Range:
  - T_{min} = minimum energy for dE/dx valid
  - energy loss statistical → range straggling → mean range R_{mean} & extrapolated range R

$$R(T_0) = R_0(T_{min}) + \int_{T_{min}}^{T_0}\left(\frac{dE}{dx}\right)^{-1} dE$$



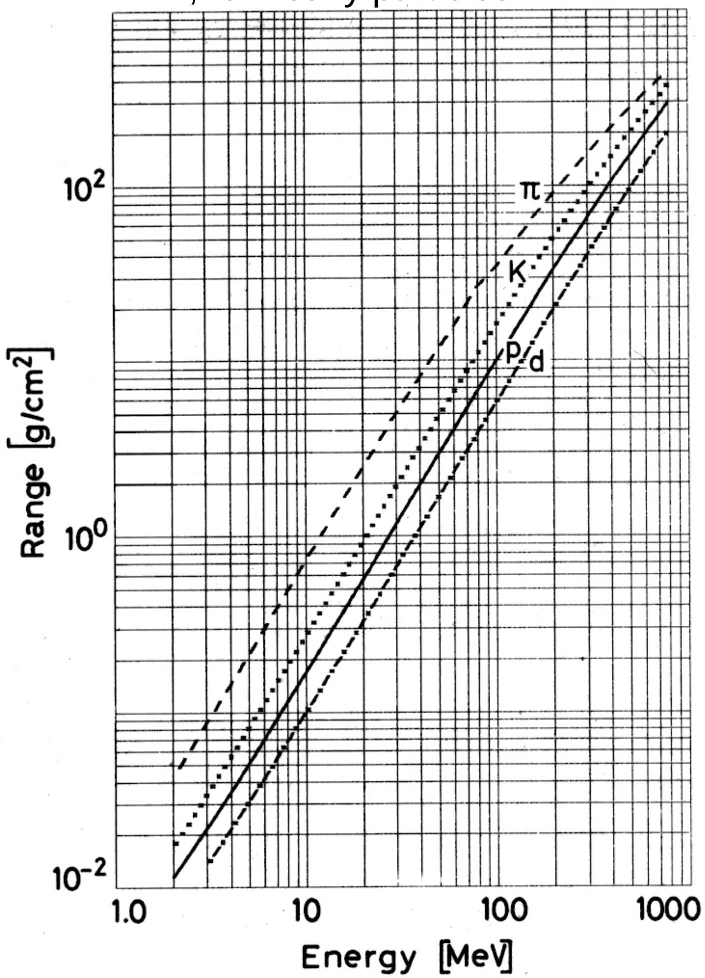
$$R_2(T_2) = \frac{M_2}{M_1}\frac{z_1^2}{z_2^2}R_1\left(T_2\frac{M_1}{M_2}\right)$$

$$\frac{R_1}{R_2} = \frac{\rho_2}{\rho_1}\frac{\sqrt{A_1}}{\sqrt{A_2}}$$

  - scaling: different particles in same medium
  - scaling: same particles in different medium

Range

- Dependence on particle mass:
  - in Al, for heavy particles



- Dependence on absorber:

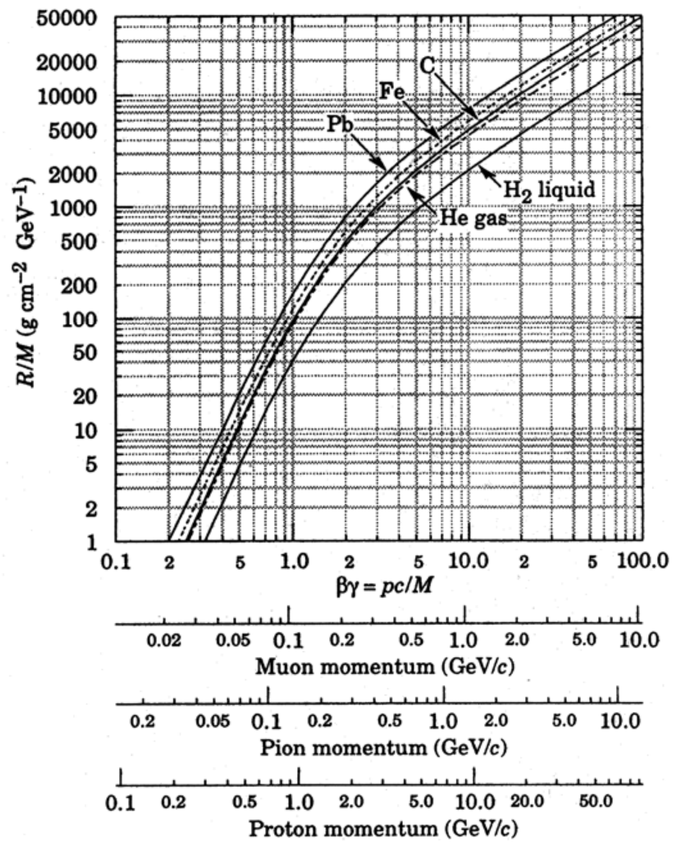


Fig. 6.5. Theoretical curves for range, R/M in g/(cm^2 GeV) as a function of $\beta\gamma$. (From Ref. 1.1.) For $\beta\gamma<1$ the β^4 behavior is evident, while the γ behavior for $\beta\gamma>10$ is also observed.

Electrons & Positrons

- Energy loss due to collision and radiation:

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{col} + \left(\frac{dE}{dx}\right)_{rad}$$

- Critical energy E_C: $\left(\frac{dE}{dx}\right)_{col}(E_C) = \left(\frac{dE}{dx}\right)_{rad}(E_C)$ $\quad E_c^{\text{solid+liq}} = \frac{610\text{ MeV}}{Z+1.24} \quad E_c^{\text{gas}} = \frac{710\text{ MeV}}{Z+1.24}$

- Energy loss due to collision → Bethe-Bloch has to be modified:
  - $m_{proj}=m_{target}$ → large angle scattering
  - e^- scatter off e^- → indistinguishable

$$T^{\max} = \frac{T_e}{2}$$

$$-\left(\frac{dE}{dx}\right)_{col} = 4\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\tfrac{1}{2}\ln\frac{(\tau+2)T_e^2}{2I^2} - F(\tau) - \frac{\delta}{2} - \frac{C}{Z}\right] \qquad \tau = \frac{T_e}{m_e c^2}$$

- Electrons:

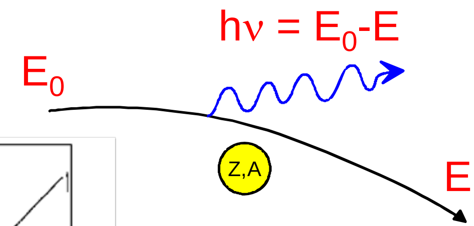
$$F(\tau) = 1-\beta^2 + \frac{\frac{\tau^2}{8} - (2r_e+1)\ln 2}{(\tau+1)^2}$$

- Positrons:

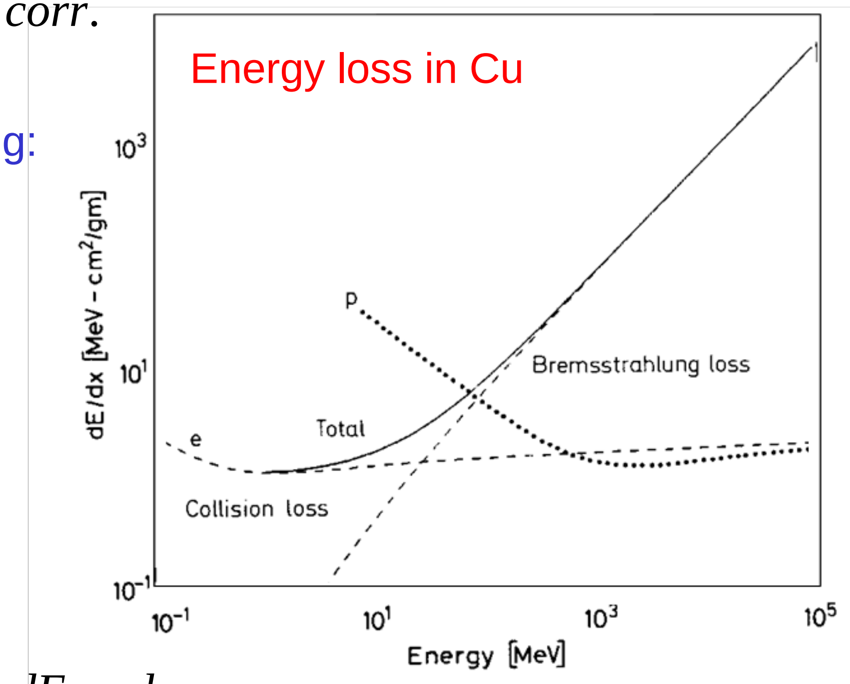
$$F(\tau) = 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right)$$

Bremsstrahlung

- Photon emission in nucleon field:



$$-\frac{dE}{dx} = 4\alpha N_A Z^2 r_e^2 E_0 \ln\frac{183}{Z^{1/3}} + corr.$$



- electron-electron Bremsstrahlung:
  - $Z^2 \rightarrow Z(Z+1)$

- Important only for e^+ and e^-:

$$\frac{\left(-\left.\frac{dE}{dx}\right|_{Brems}(e^{\pm})\right)}{\left(-\left.\frac{dE}{dx}\right|_{Brems}(\mu^{\pm})\right)} = 40000$$

- Radiation length X_0:
  - path length were $\langle E\rangle = E_0/e$

$$-\frac{dE}{E_0} = \frac{dx}{X_0} \qquad \langle E\rangle = E_0 e^{-X/X_0}$$

$$\frac{1}{X_0} \cong 4Z(Z+1)\frac{\rho N_A}{A} r_e^2 \alpha \ln\frac{183}{Z^{1/3}} + corr. \quad \left[\frac{cm^2}{g}\right]$$

Interactions of Photons

- Photon detection:
  - photons must create charged particles or transfer energy to charged particles
- As photon beam traversing matter:
  - energy is not degraded but intensity is attenuated mainly by:
    - Photoelectric effect
    - Compton scattering
    - Pair production

$$I_\gamma(x) = I_0 e^{-\mu x}$$

$$\mu = \mu_{photo} + \mu_{Compton} + \mu_{pair} + \ldots$$

$$\frac{\mu_i}{\rho} = \frac{N_A}{A}\sigma_i \quad \left[\frac{cm^2}{g}\right]$$

- I_0 = incident beam intensity
- x = absorber thickness
- μ = absorption coefficient
- μ/ρ = mass absorption coefficient
- σ = cross section

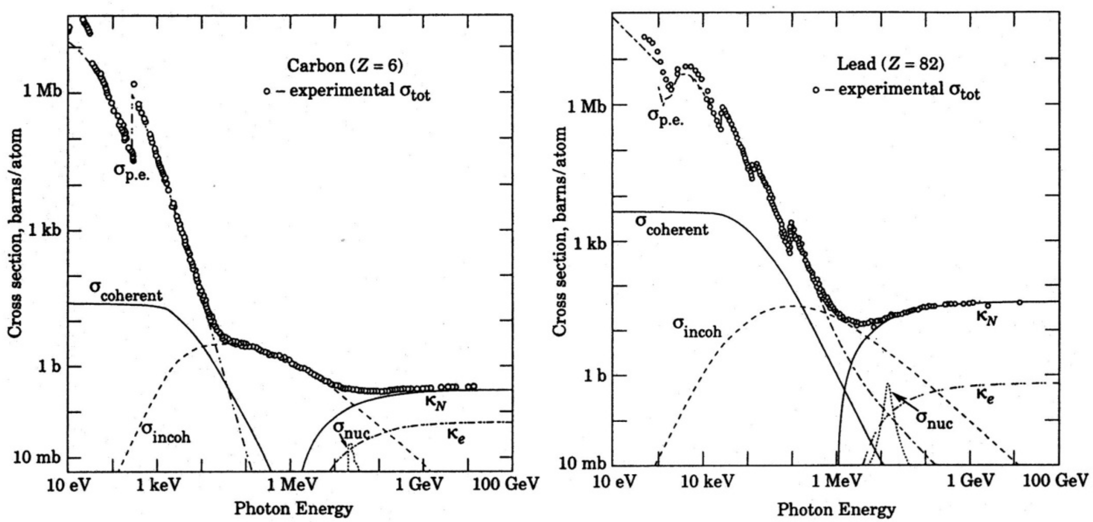
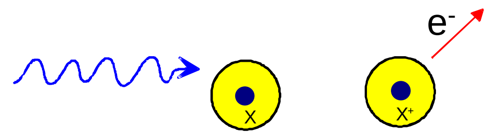


Fig. 1.10. Photon cross sections on carbon and lead as a function of photon energy. (From Ref 1.1.)

Photoelectric Effect

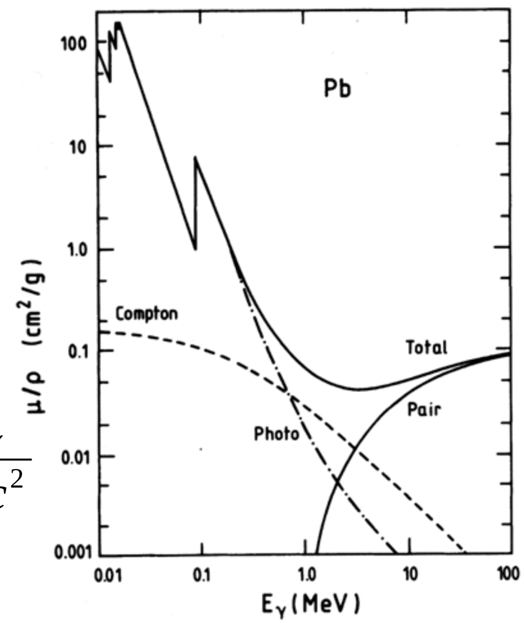
- Dominates at low photon energies:
  - E_γ < 10eV … 1 MeV



$$\gamma + \text{atom} \rightarrow \text{atom}^{+} + e^{-}$$

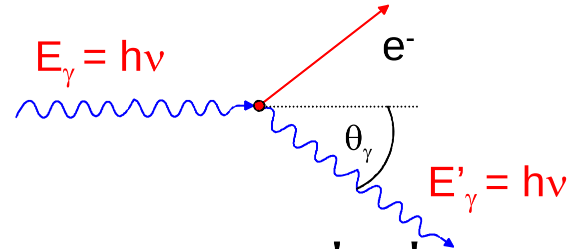
- Electron energy:
  - $E_e = h\nu$ - B.E. (B.E. = binding energy)
- Nucleus absorbs recoil momentum:
- Increase sharply at:
  - $E_\gamma \approx$ B.E. of e in atomic K- and L-shell
- Cross section:
  - $E_K < E_\gamma < m_e c^2$ $$\sigma^{K}_{photo} = \frac{32\pi}{3}\sqrt{2}\alpha^4 Z^5 \frac{1}{\varepsilon^{7/2}} r_e^2 \qquad \varepsilon = \frac{E_\gamma}{m_e c^2}$$
  - $E_\gamma >> m_e c^2$ $$\sigma^{m_e}_{photo} = 4\pi r_e^2 \alpha^4 Z^5 \frac{1}{\varepsilon}$$

Fig. 1.3. Mass absorption coefficient μ/ρ for photons in lead.



$$\sigma_{photo} \propto Z^5$$

Compton Scattering



$$\gamma + e \rightarrow \gamma' + e'$$

- Assume electron as quasi-free
- Energy of scattered photon:

$$E'_\gamma = E_\gamma \frac{1}{1+\varepsilon(1-\cos\theta_\gamma)} \qquad \varepsilon = \frac{E_\gamma}{m_e c^2}$$

- Cross section:
  - non-relativistic limit:
    no energy transfer for
    Thomson & Rayleigh scattering

    $E_\gamma < m_e c^2$

$$\sigma_c^e = \sigma_{\mathrm{Th}} = \frac{8\pi}{3} r_{\mathrm{e}}^2 = 665\ \mathrm{mb}$$

  - relativistic energies: Klein-Nishina formula:

    $E_\gamma >> m_e c^2$

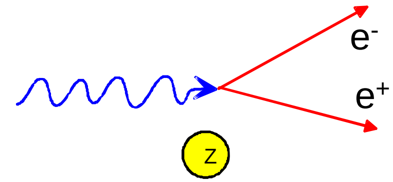
$$\sigma_c^e \cong \frac{3}{8}\sigma_{\mathrm{Th}} \frac{1}{\varepsilon}\left(\frac{1}{2} + \ln 2\varepsilon\right)$$

- Atomic Compton cross section:

$$\sigma_c^{atomic} = Z \cdot \sigma_c^e$$

Pair Production

- For momentum conservation:
  - Coulomb field of nucleus (electron) needed to absorb recoil
  - minimum energy: $E_\gamma \geq 2 m_e c^2$



$$\gamma + nucleus \rightarrow e^+ e^- + nucleus$$

- Related to Bremsstrahlung by substitution
- Cross-section:
  - low energy limit: $2 \ll \varepsilon \ll 137/Z^{1/3}$

$$\sigma_{pair} \approx 4\alpha\, r_e^2 Z^2 \left(\frac{7}{9} \ln 2\varepsilon \right)$$

  - high energy limit: $\varepsilon \gg 137/Z^{1/3}$

$$\sigma_{pair} \approx 4\alpha\, r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{\frac{1}{3}}} \right) \approx \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0} \approx \frac{A}{N_A} \frac{1}{\lambda_{pair}}$$

Independent of energy

- Mean free path: λ_{pair}
  - Radiation length X_0

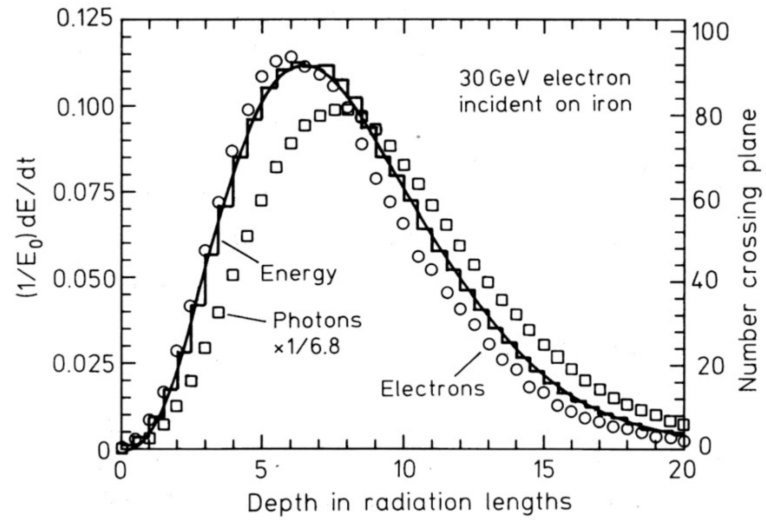
$$\lambda_{pair} = \frac{9}{7} X_0 \qquad \left[\frac{g}{cm^2} \right]$$

Electron-Photon Shower

- High energy electron or photon starts shower:
  - photon emission and pair production alternate
  - statistical process
  - in average: equal split of energy between particles
  - until energy drops below critical energy E_C where loss due to collision halt the cascade
- After t radiation lengths:
  - number of particles (e^-, e^+, γ): $N \cong 2^t$
  - average energy: $E(t) \cong \frac{E_0}{2^t}$
- Maximum penetration depth: $E(t_{max}) \cong \frac{E_0}{2^{t_{max}}} = E_C$
  - assume abrupt stop at E_C:

$$t_{max} = \frac{\ln \frac{E_0}{E_C}}{\ln 2}$$

  - maximum number of particles: $N_{max} \cong \frac{E_0}{E_C}$



Neutrons

- Neutron detection:
  - only strong interaction: needs ~10^{-15}m distance to nuclei $\rightarrow$ much rarer process
  - $\rightarrow$ neutrons are very penetrating
  - large scattering angles
- Processes: $\sigma_{tot} = \Sigma\sigma_i$

– high energy hadron shower:	np $\rightarrow$ X , nn $\rightarrow$ X	E_n >100MeV
– elastic scattering off nuclei:	A(n,n)A	dominant for E_n O(MeV)
– inelastic scattering:	A(n,n')A* +γ , A(n,2n')B+γ	E_n >1MeV
– radiative neutron capture:	n+(Z,A)$\rightarrow$ γ+(Z,A+1)	low v_n and resonances
– nuclear reactions:	(n,p), (n, d), (n, α), …	E_n O(eV)…O(keV)
– fission:	(n, f)	E_n thermal

- Mean free path (like for photons):

$$\lambda = \frac{N_A \rho}{A}\sigma_{tot} \qquad \left[\frac{g}{cm^2}\right]$$

- Attenuation:

$$N(x) = N_0 e^{-\frac{x}{\lambda}}$$

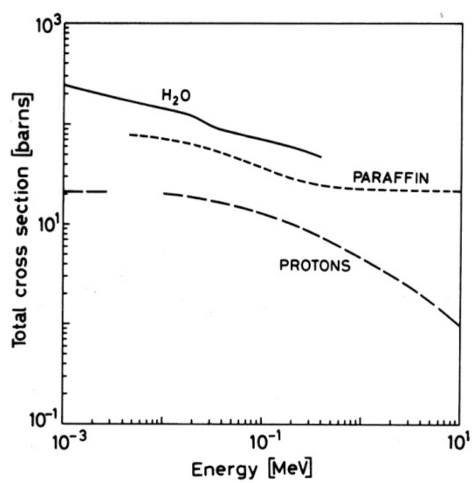


Table of Material Properties

Table 5. *Radiation length X_0, critical energy E_c and hadronic absorption length λ_{had} for some materials*

Material	X_0 (g/cm^2)	E_c (MeV)	λ_{had} (g/cm^2)
H_2	63	340	52.4
Al	24	47	106.4
Ar	18.9	35	119.7
Kr	11.3	21.5	147
Xe	8.5	14.5	168
Fe	13.8	24	131.9
Pb	6.3	6.9	193.7
Lead glass SF 5	9.6	~11.8	
Plexiglas	40.5	80	83.6
H_2O	36	93	84.9
NaI(Tl)	9.5	12.5	152.0
$Bi_4Ge_3O_{12}$	8.0	10.5	164