

Quarkonium production in the LHC era: From puzzles to understanding

PPE seminar, Edinburgh, October 28th 2014

Valentin Knünz
(HEPHY Vienna)



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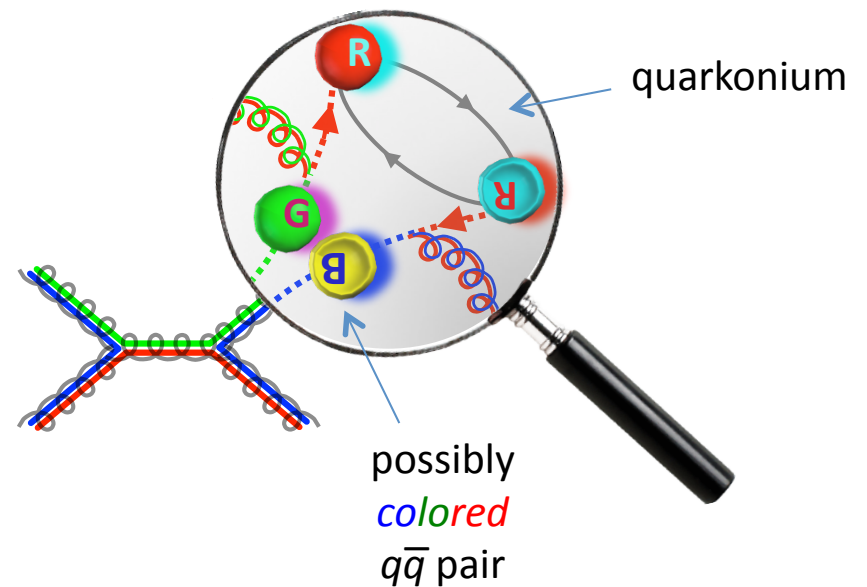
*Many thanks to C. Lourenço, P. Faccioli, H. Wöhri, J. Seixas and
I. Krätschmer for various inputs used in this presentation*

1) Motivation & introduction: the pre-LHC puzzles

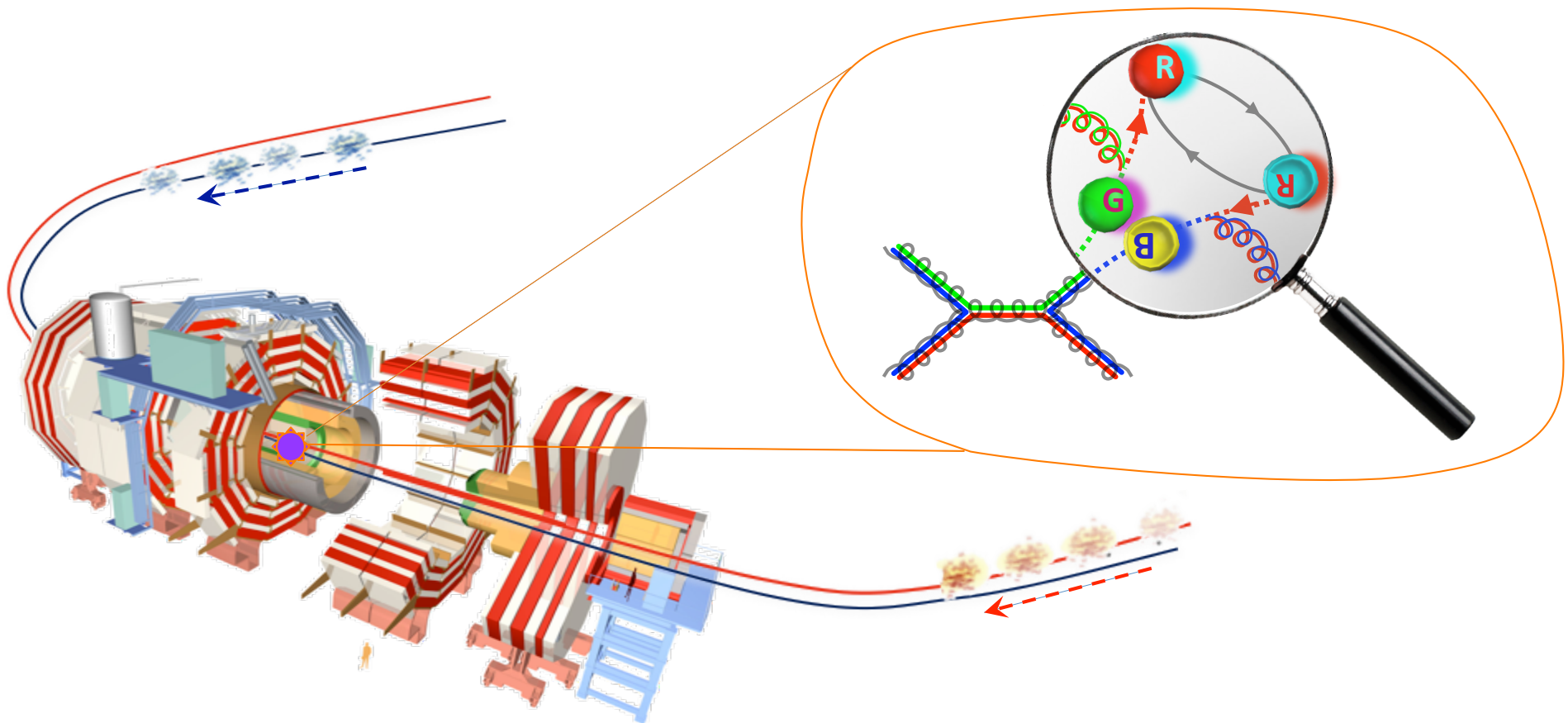
- Why do we study quarkonium production?
- Quarkonium spectrum & feed-down decays
- Quarkonium production models: Non-Relativistic QCD
- Puzzles in the pre-LHC era

2) Quarkonium production measurements at CMS

3) Interpretation of the results: a polarized perspective

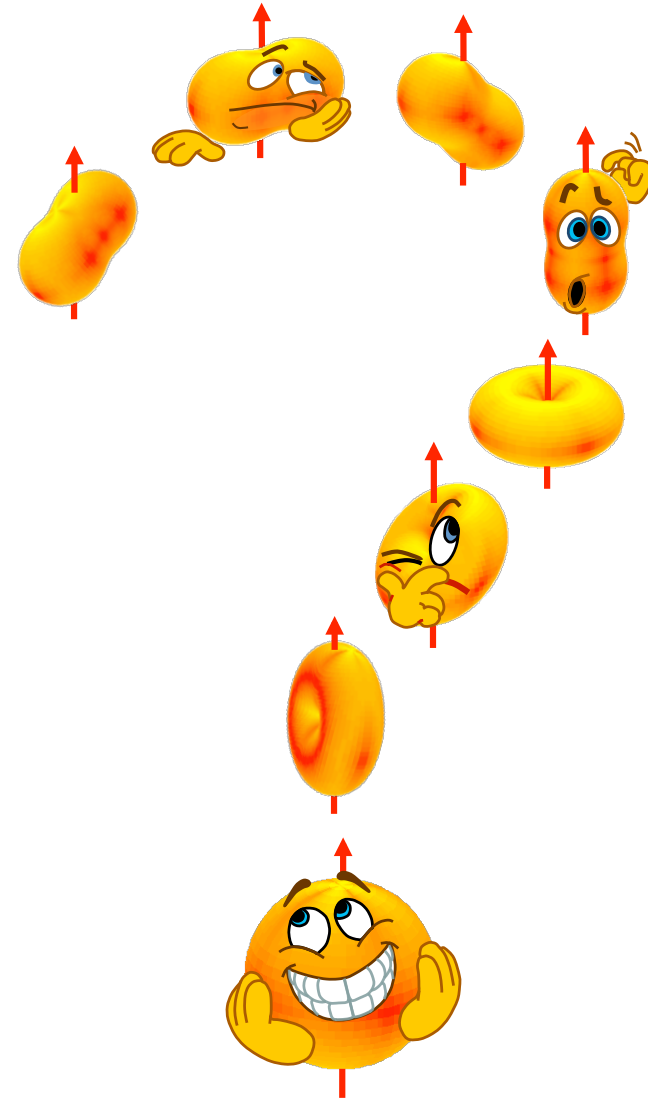


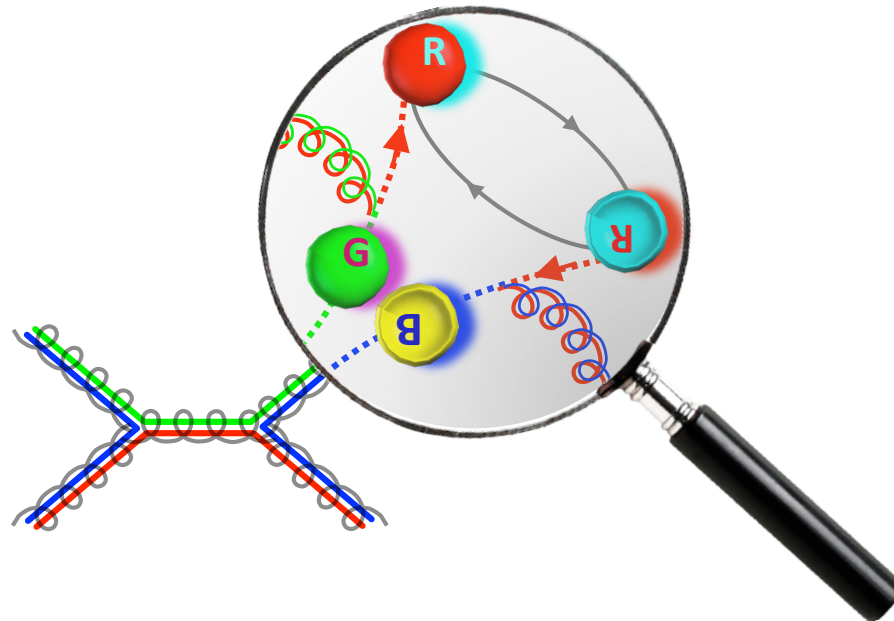
- 1) Motivation & introduction: the pre-LHC puzzles
- 2) Quarkonium production measurements at CMS**
 - Improved data analysis methodologies
 - Summary of relevant CMS data analyses
 - Overview of LHC quarkonium production results
- 3) Interpretation of the results: a polarized perspective



- 1) Motivation & introduction: the pre-LHC puzzles
- 2) Quarkonium production measurements at CMS
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- Review of existing NRQCD analyses
- A data-driven perspective
- Towards the solution of the 'quarkonium polarization puzzle'





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- 2) Quarkonium production measurements at CMS
- 3) Interpretation of the results: a polarized perspective

The big picture in a nutshell

Only 0.1% of the mass in the universe exists as truly elementary particles (Higgs mechanism);
almost all the visible matter is made of *hadrons*

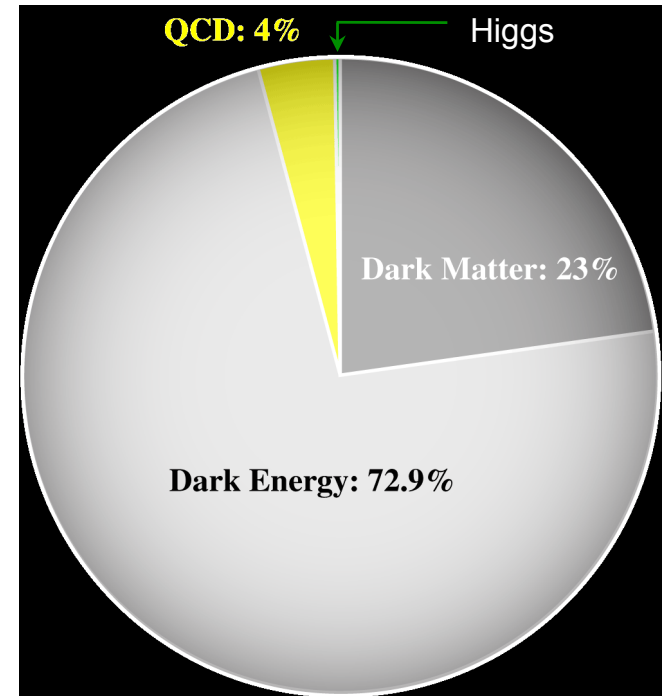
The “dark sector” is a mystery,
but hadron formation is not well understood either

“QCD is full of surprises and challenges”
(Joe Lykken, summary talk, LHCP 2013)

Quarkonium production is an ideal probe to study
hadron formation, part of the non-perturbative QCD
sector → how do quarks combine into a bound state?

Quarkonia are bound states of a heavy quark and
it's antiquark ($c\bar{c}$, $b\bar{b}$) and exist in “families” of several
states (colorless, neutral mesons)
→ QCD analogues of the hydrogen atom

Quark production and quarkonium formation are well
separated processes at distinct timescales



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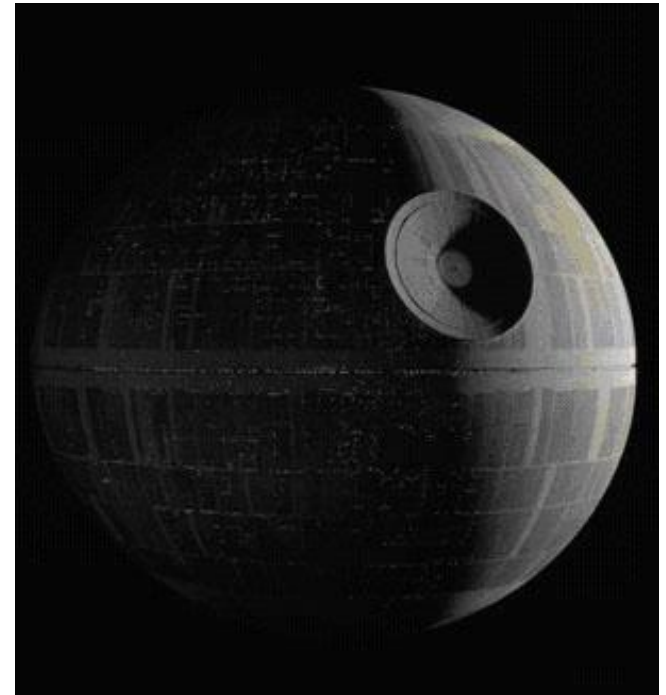
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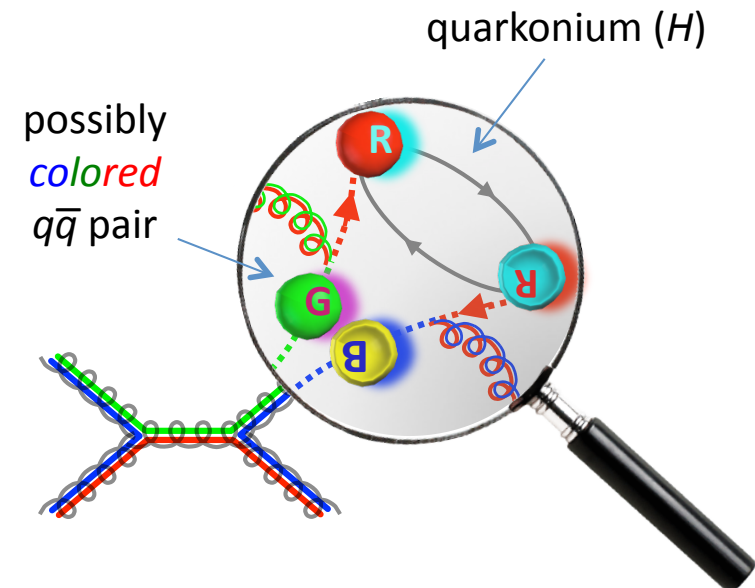
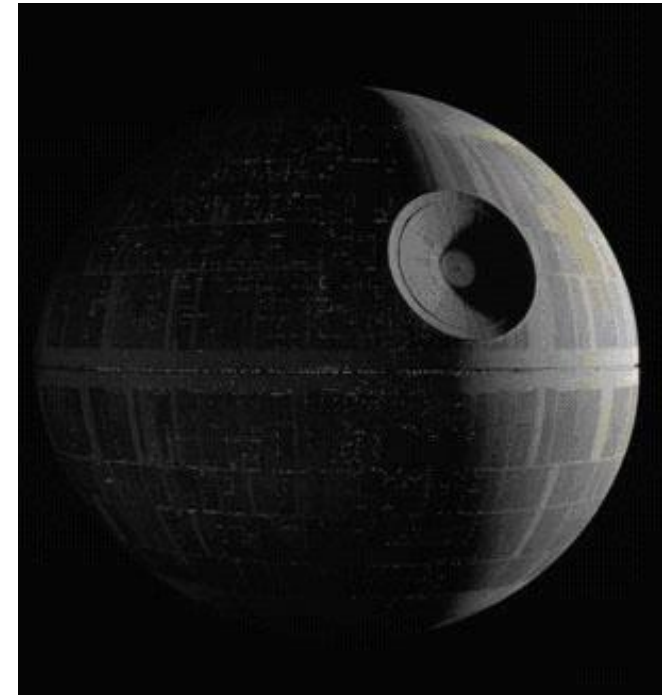
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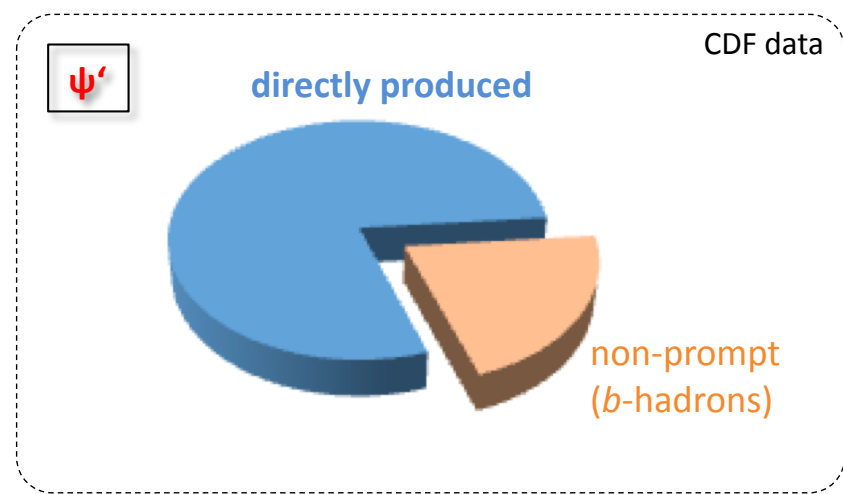
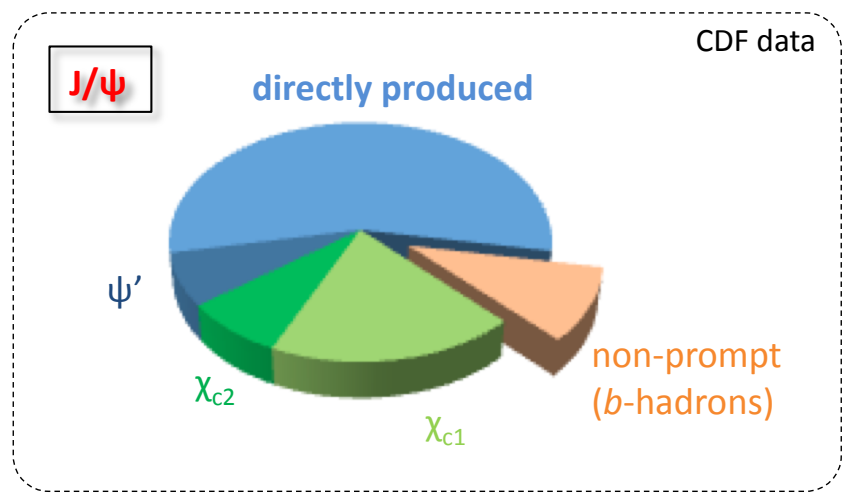
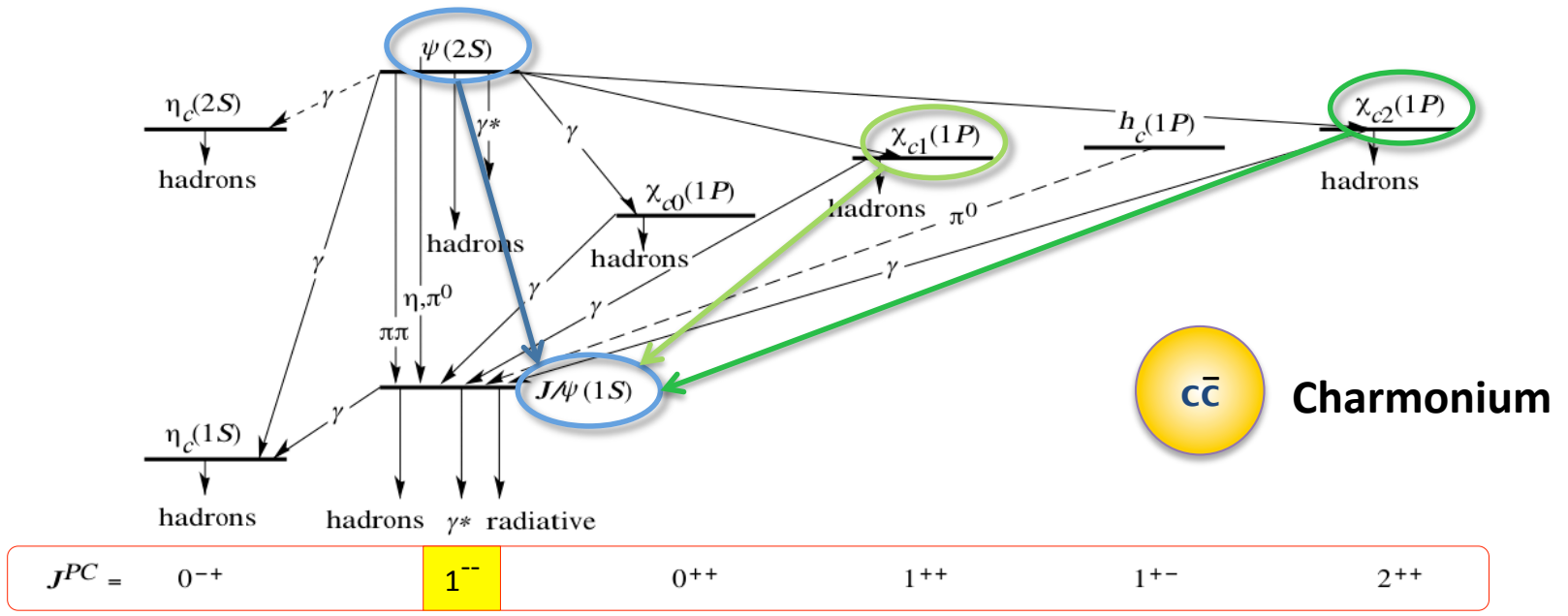
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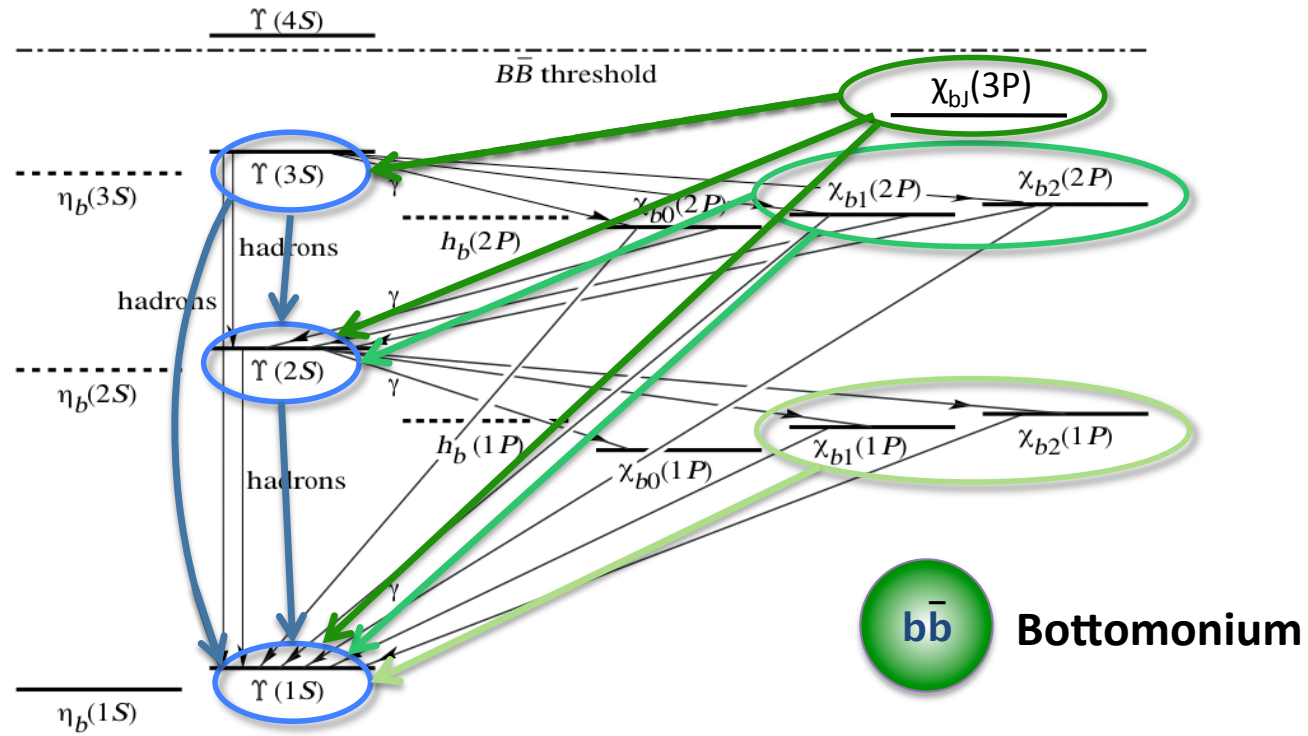


Quarkonium spectra & feed-down considerations

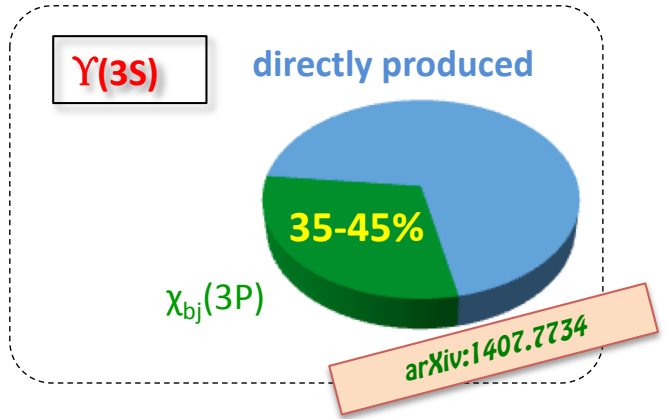
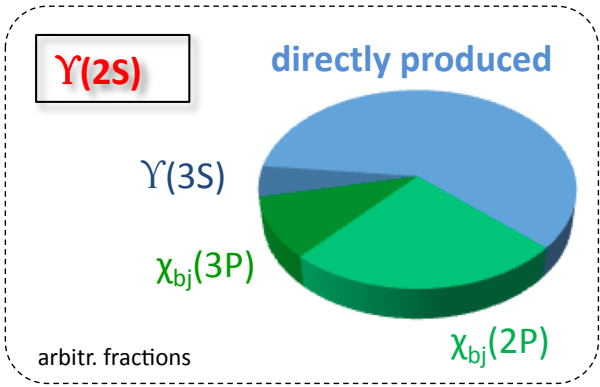
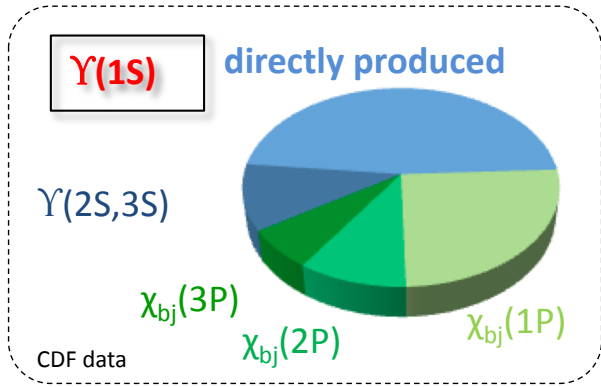


Prompt contribution = Direct production + charmonium feed-down

Quarkonium spectra & feed-down considerations



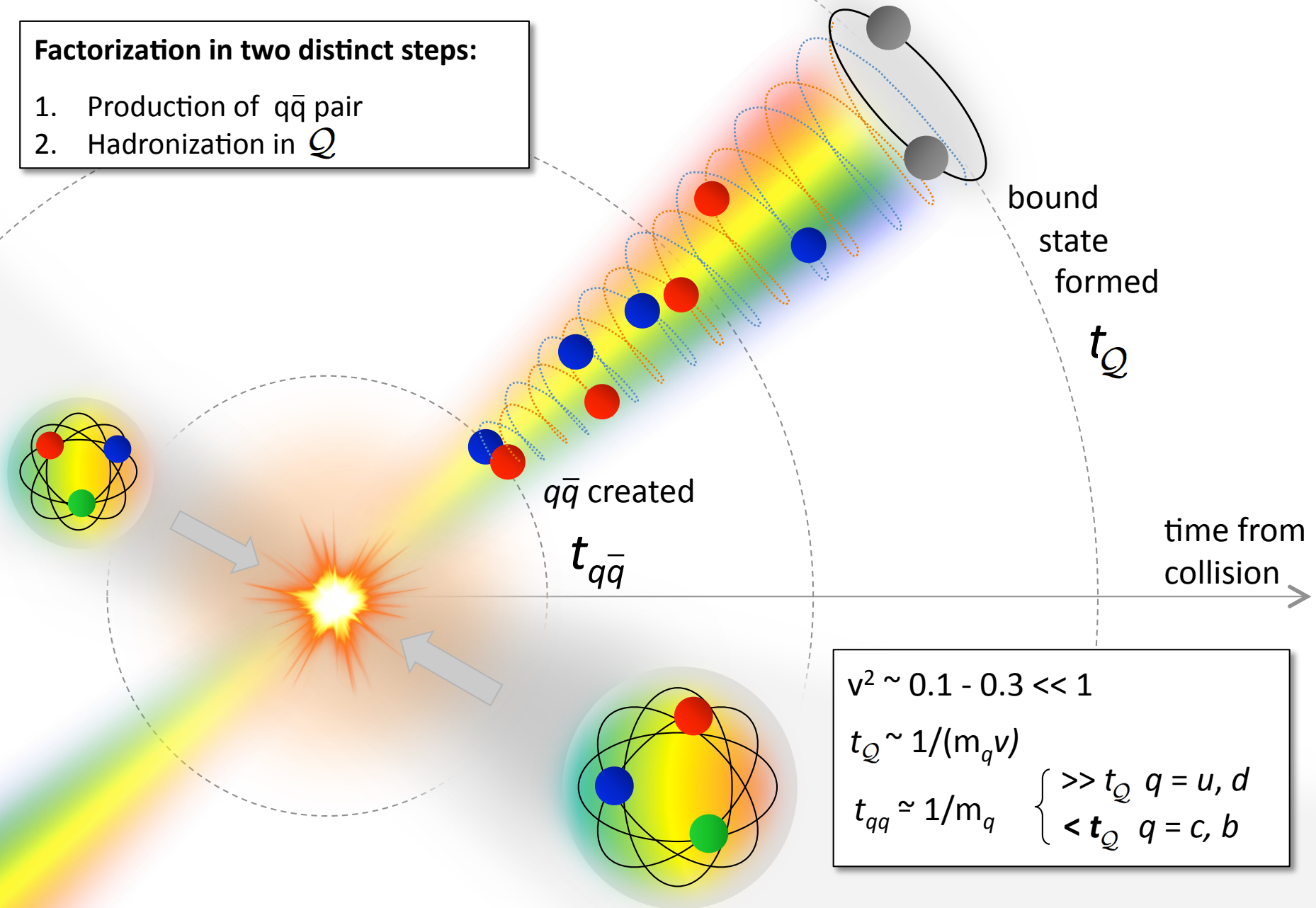
$J^{PC} =$	0^{-+}	1^{--}	1^{+-}	0^{++}	1^{++}	2^{++}
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We need heavy quarks to “see” hadron formation

Factorization in two distinct steps:

1. Production of $q\bar{q}$ pair
2. Hadronization in Q



$$v^2 \sim 0.1 - 0.3 \ll 1$$

$$t_Q \sim 1/(m_q v)$$

$$t_{q\bar{q}} \approx 1/m_q \begin{cases} \gg t_Q & q = u, d \\ < t_Q & q = c, b \end{cases}$$

What's the problem?

Quarkonium spectrum very well understood

Quarkonium decays very well understood

The problem is that quarkonium production has been plagued with *experimental puzzles*, preventing reliable progress in our physics understanding

Headlines from "The Fermilab times"

the ψ' anomaly

CDF Run 1 and Run 2 results
dramatically disagree
with each other

CDF and D0
mutually exclude each other

the quarkonium polarization puzzle

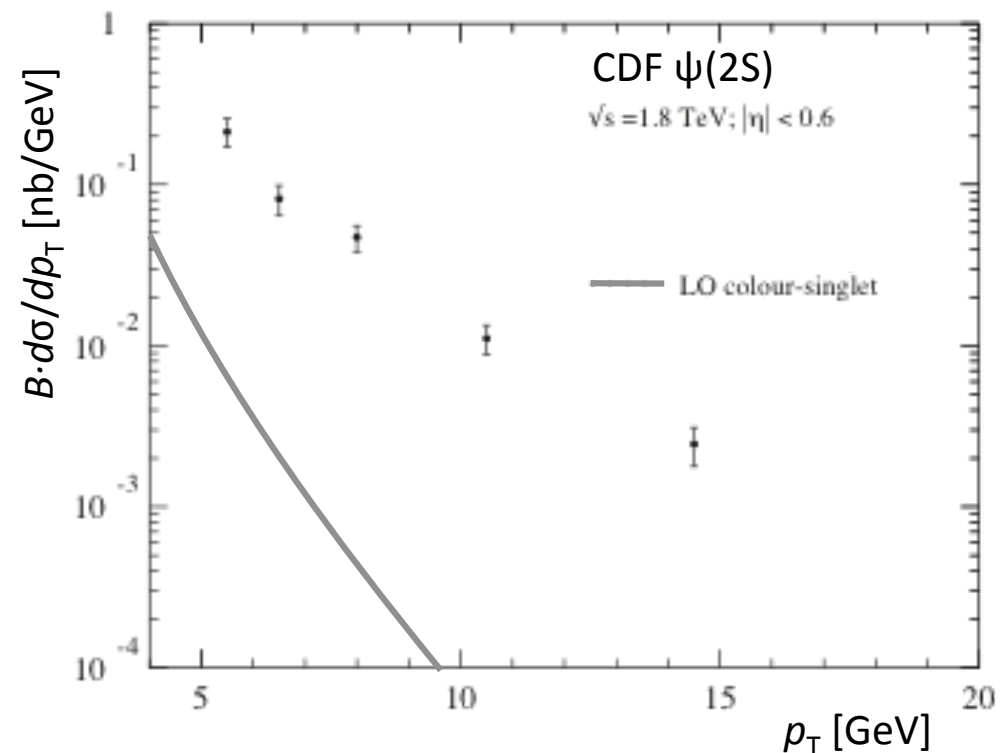
The “quarkonium polarization puzzle” in four easy steps

In the early 90’s, CDF measured a $\psi(2S)$ cross section 50 times larger than expected in the color singlet model (CSM): “the ψ' anomaly”



In 1995, Bodwin, Braaten and Lepage developed the NRQCD (non-relativistic QCD) approach, which solved the ψ' anomaly by adding a series of color octet terms, with free normalizations; given the extra freedom, the data could be reproduced

The validity of NRQCD was then probed by fixing those free parameters and comparing the resulting predictions to independent measurements; this is where the polarization enters... The outcome is well known: NRQCD predicts transverse polarization at high p_T , not observed in data



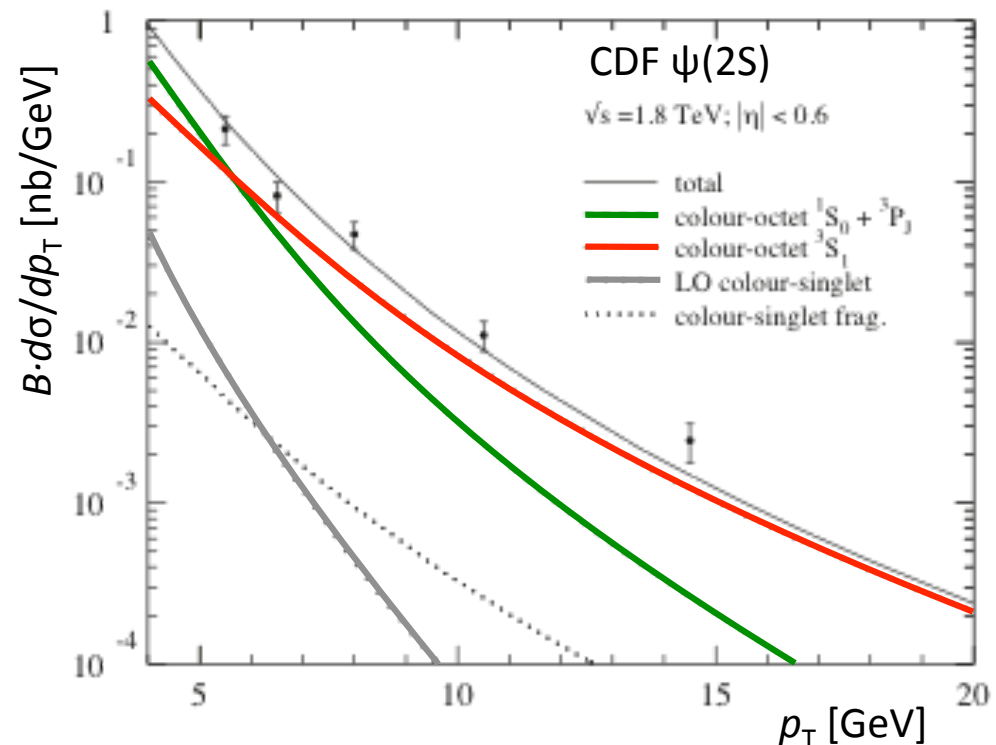
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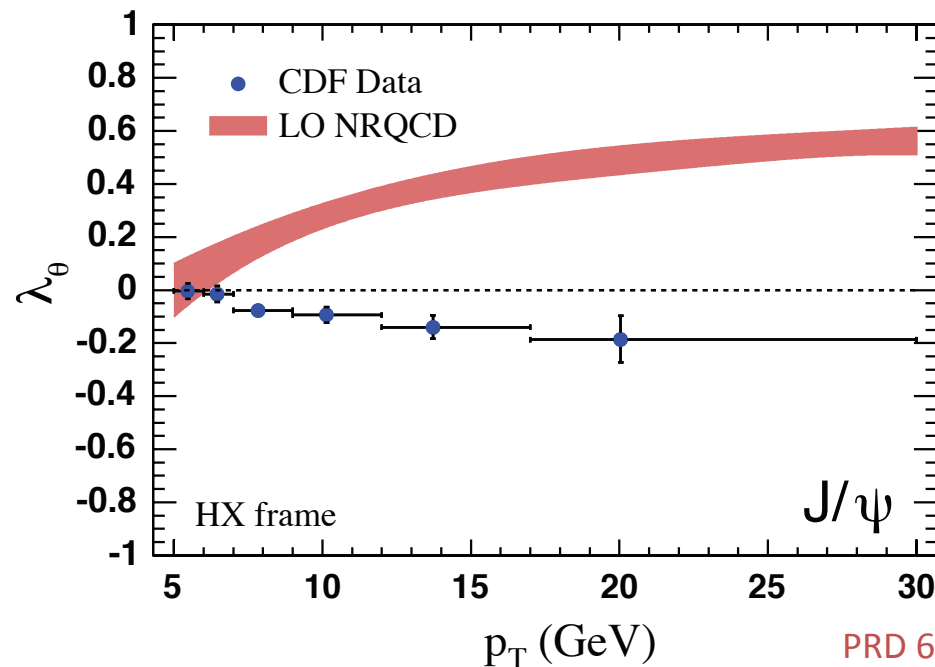
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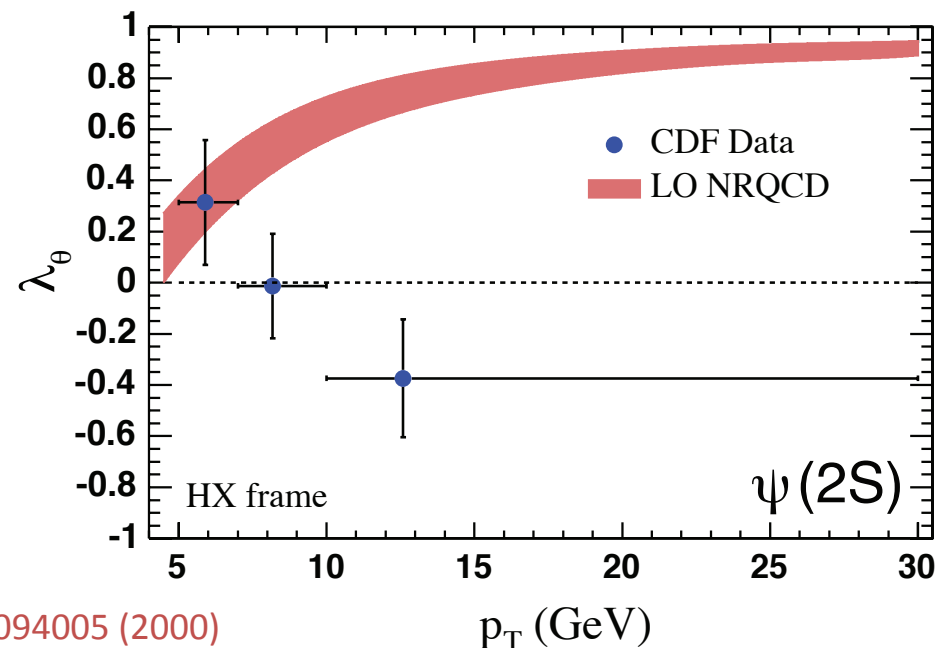


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PRD 62, 094005 (2000)
PRL 99, 132001 (2007)



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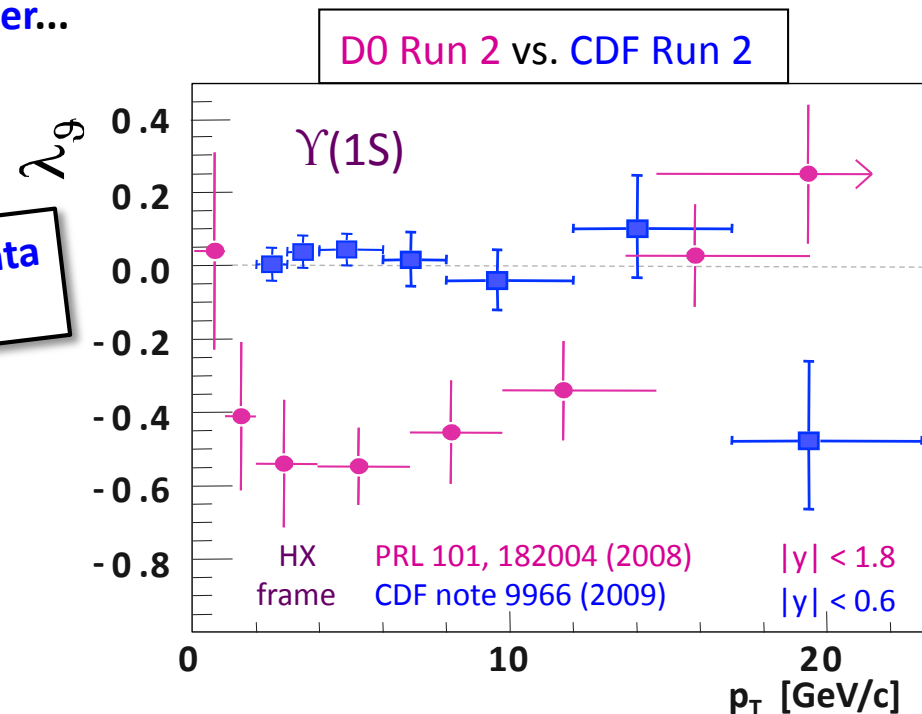
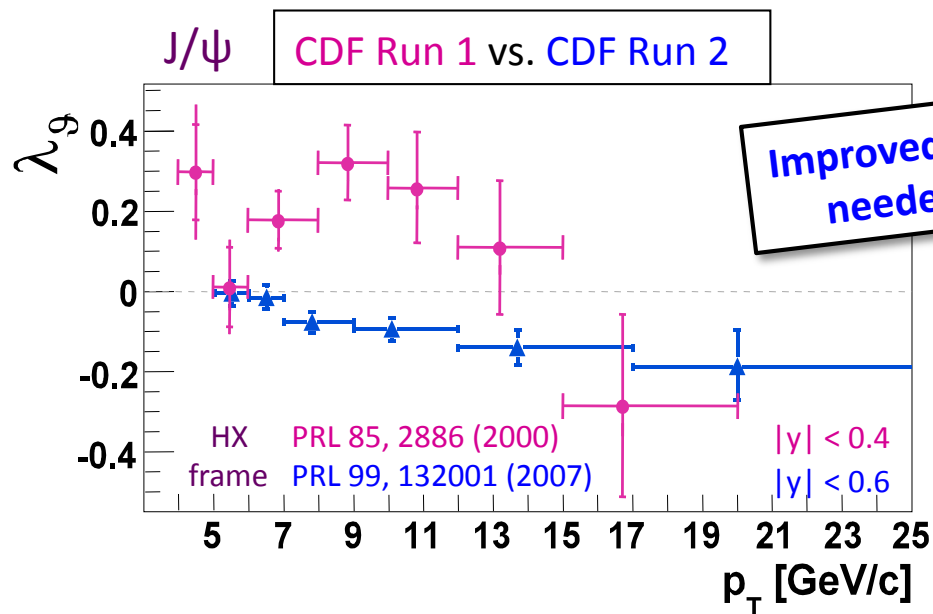


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But the Tevatron results mutually excluded each other...



Quarkonium production in two not-so-easy steps

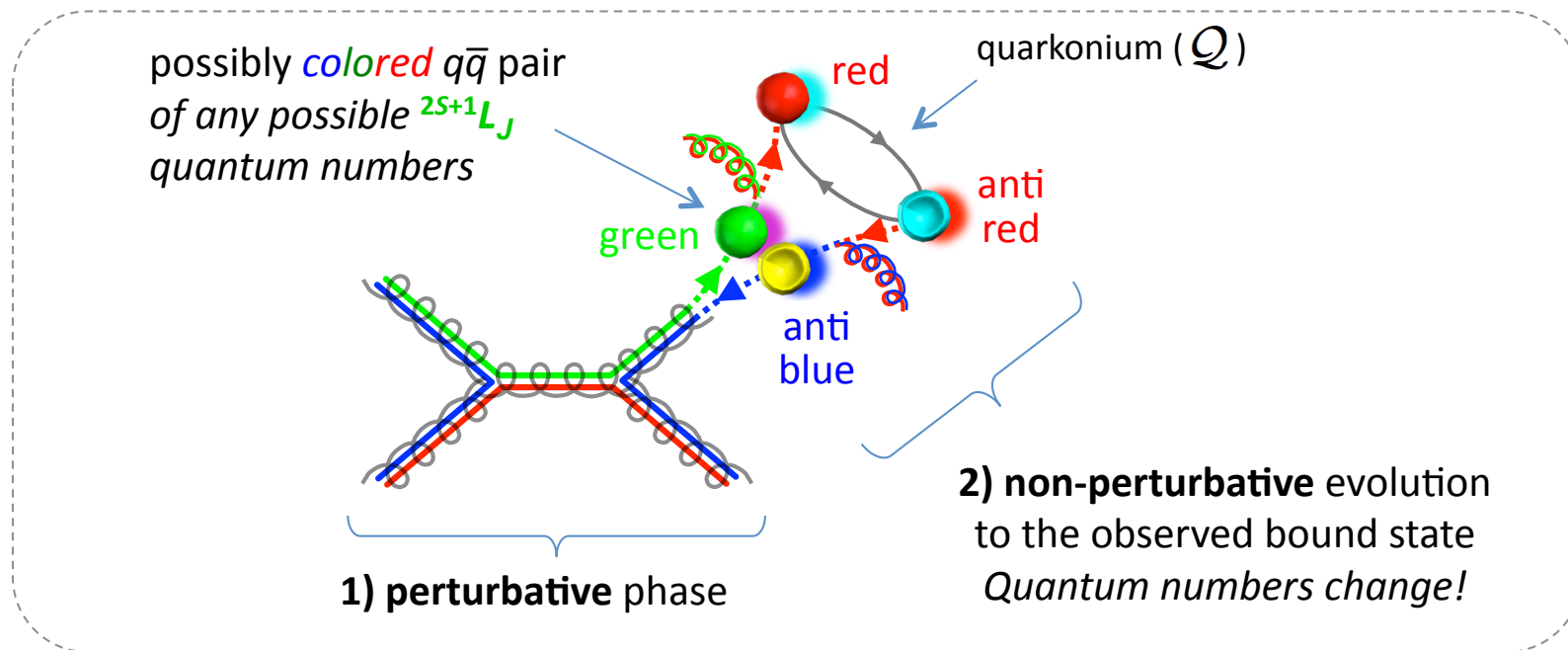
NRQCD is an effective field theory that factorizes quarkonium production in two steps:

- 1) production of the initial quark-antiquark pair (perturbative QCD)
- 2) hadronization of the quark pair into a bound quarkonium state (non-perturbative QCD)

$$\sigma(Q) = \sum_n \sigma[qq\bar{q}(n)] \langle \mathcal{O}^Q(n) \rangle$$

$$\mathcal{n} = {}^{2S+1}L_J^{[C]} \quad , \quad C = 1, 8$$

Quantum numbers of the heavy quark pair
S, L, J = spin, orbital and total ang. momentum



NRQCD predicts the existence of *intermediate color-octet (CO) states* in nature, that subsequently evolve into physical color-singlet (CS) quarkonia by *non-perturbative emission of soft gluons*.

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Short-distance coefficients (SDCs)

- Cross section of partonic processes to form $Q\bar{Q}$ in state n \otimes PDF
- Process-dependent functions of kinematics
- Can be calculated perturbatively (expansion in α_s)

Long-distance matrix elements (LDMEs)

- Probability of $Q\bar{Q}$ in state n to form quarkonium state Q
- Universal constants (independent of kinematics)
 - Determined from fits to experimental data

The LDMEs should follow a **hierarchy in powers of v** , the relative velocity of the quark pair in the quarkonium system \rightarrow **Non-relativistic approximation** ($v^2 \sim 0.3$ for the ψ and ~ 0.1 for the Υ):

\rightarrow Truncation of v -expansion for S-wave states

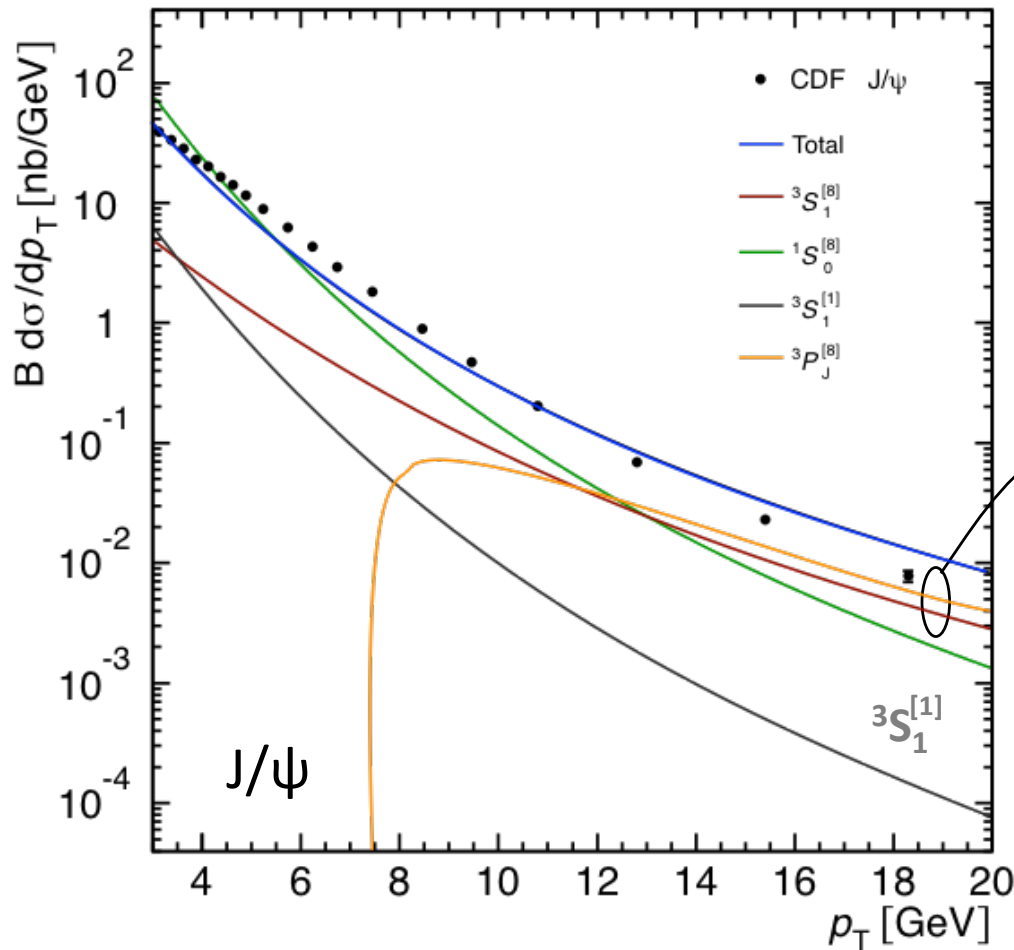
\rightarrow NRQCD includes **4 terms** (intermediate states):

CS term 3S_1 (same n as Q)

CO terms: ${}^1S_0, {}^3S_1, {}^3P_J$

Fitting the theory to the data: a pedagogical example

The J/ψ cross section is fitted adding the (free) 1S_0 , 3S_1 , 3P_J octets to the (fixed) 3S_1 singlet



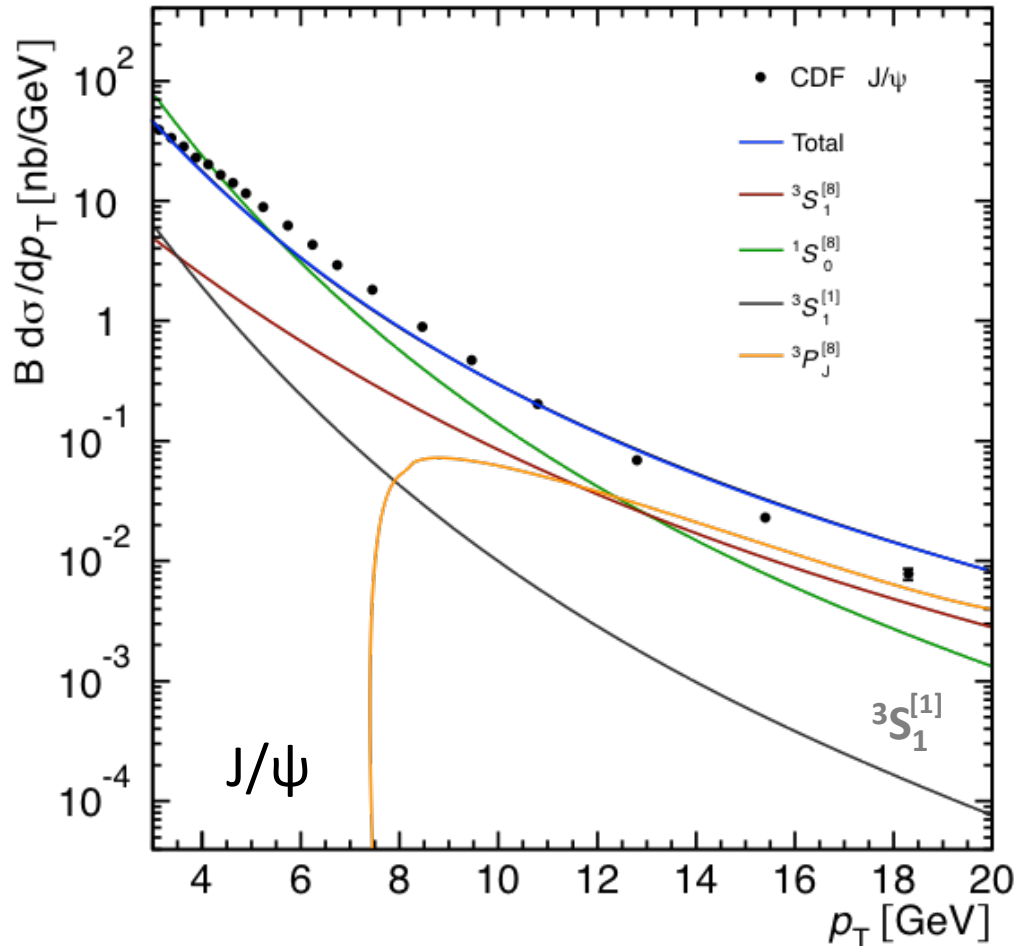
Curves: SDC functions at NLO by M. Butenschön and B. Kniehl PRL 108, 172002 (2012)

The 3S_1 and 3P_J octets dominate at high p_T
 What does this imply for the polarization ?

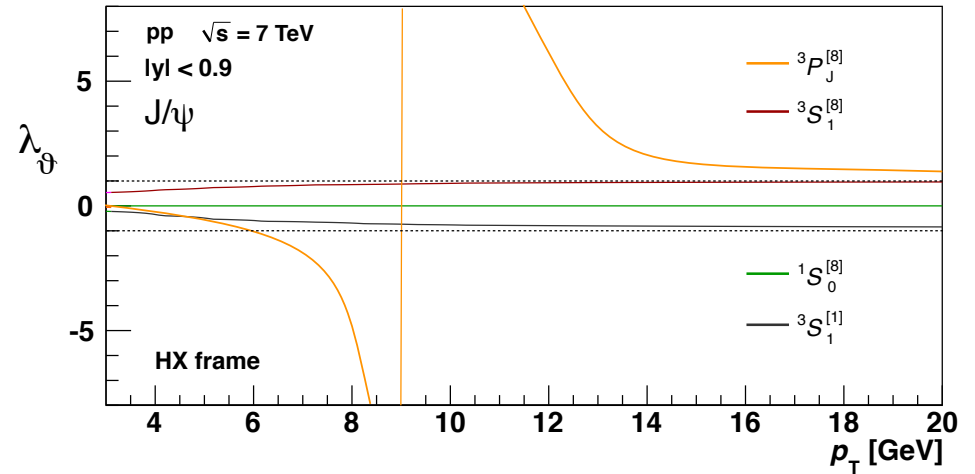
Note:
 the fit starts at $p_T = 3$ GeV
 feed-down not taken into account

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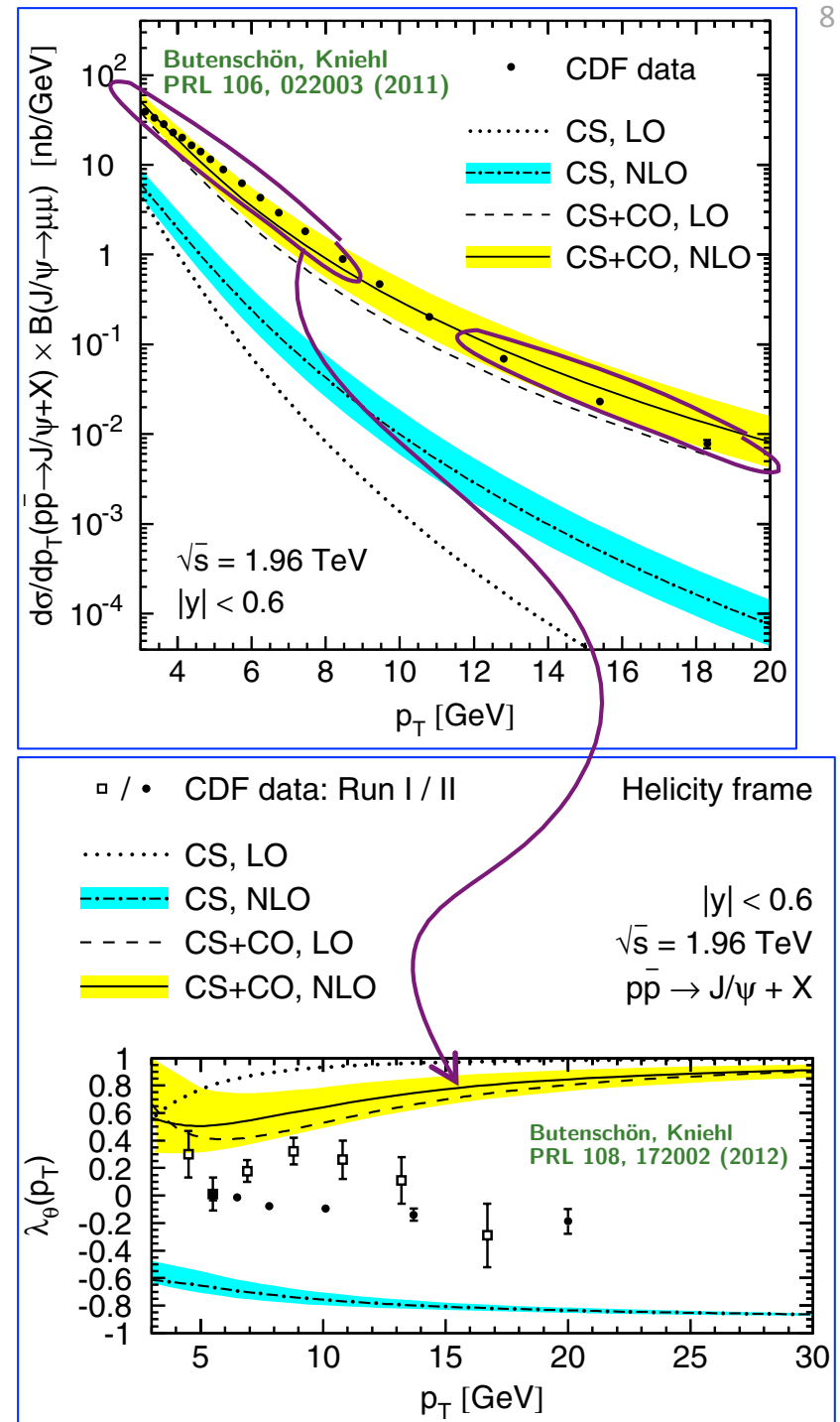


Every term has a specific polarization:
 $^3S_1^{[1]} \rightarrow \lambda_\theta \approx -0.9$ (longitudinal)
 $^1S_0^{[8]} \rightarrow \lambda_\theta = 0$ (isotropic)
 $^3S_1^{[8]} \rightarrow \lambda_\theta \approx +1$ (transverse)
 $^3P_J^{[8]} \rightarrow \lambda_\theta \gg +1$ (“hyper-transverse”)
@NLO, approximations, HX frame

→ This is why NRQCD predicts transverse polarization at high p_T

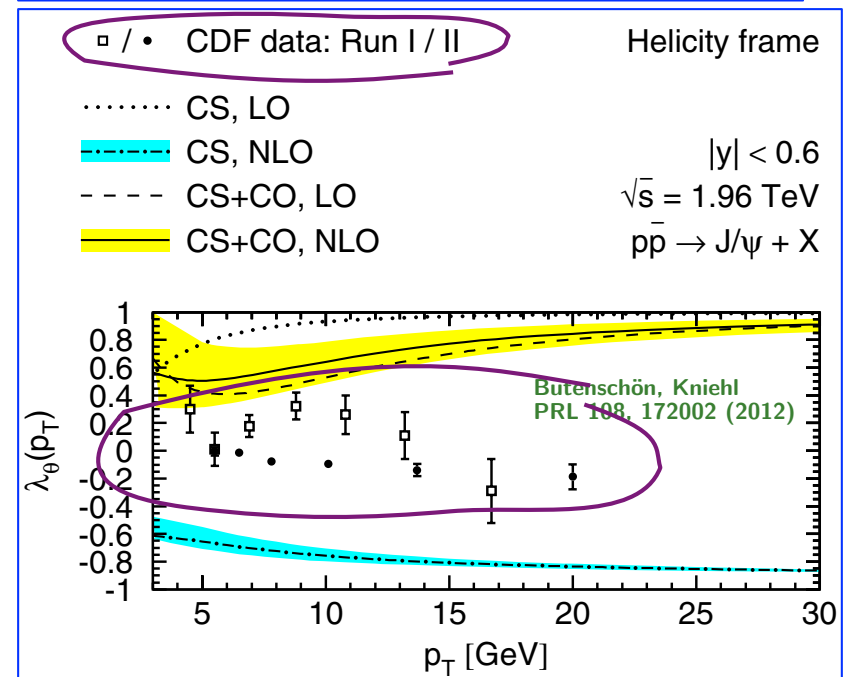
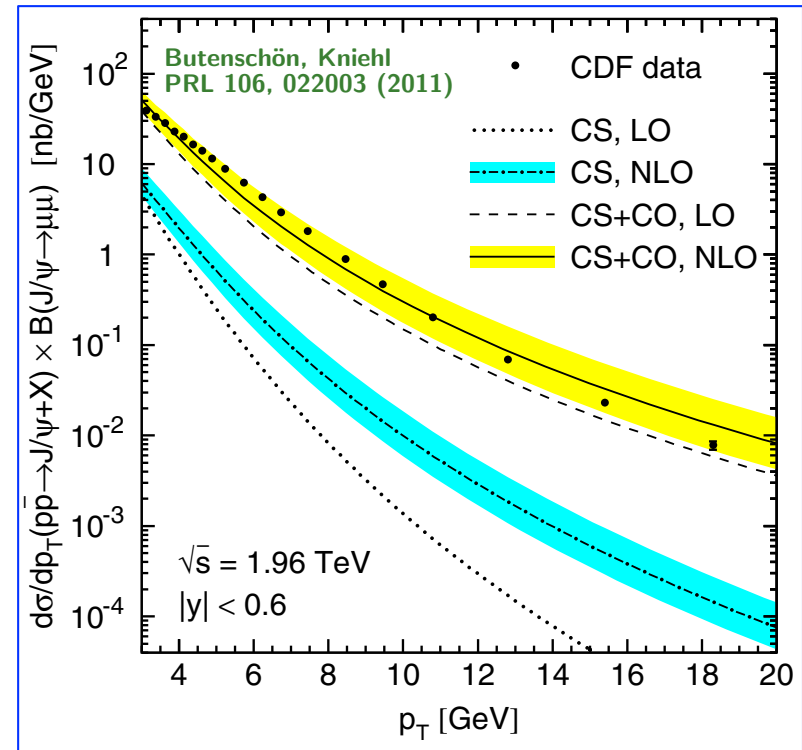
NRQCD vs. pre-LHC data: the executive summary

- The pre-LHC NRQCD analyses fitted the LDMEs from quarkonium cross sections...
 - **CO processes realized in nature!**
 - ...and then predicted the polarizations
 - **Quarkonia transversely polarized at high p_T**
 - The CDF J/ψ and $\psi(2S)$ polarization measurements disagreed with the prediction: **puzzle!**
 - Measured cross sections and polarizations cannot be reproduced **simultaneously**
 - But Tevatron data are inconsistent
 - Feed-down effects blur the J/ψ picture
 - $\psi(2S)$ λ_θ has very large errors
- Theorists remained skeptical regarding the existence of a *fundamental* problem...



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Missions to be accomplished in the LHC-era

1. We need “better” data!

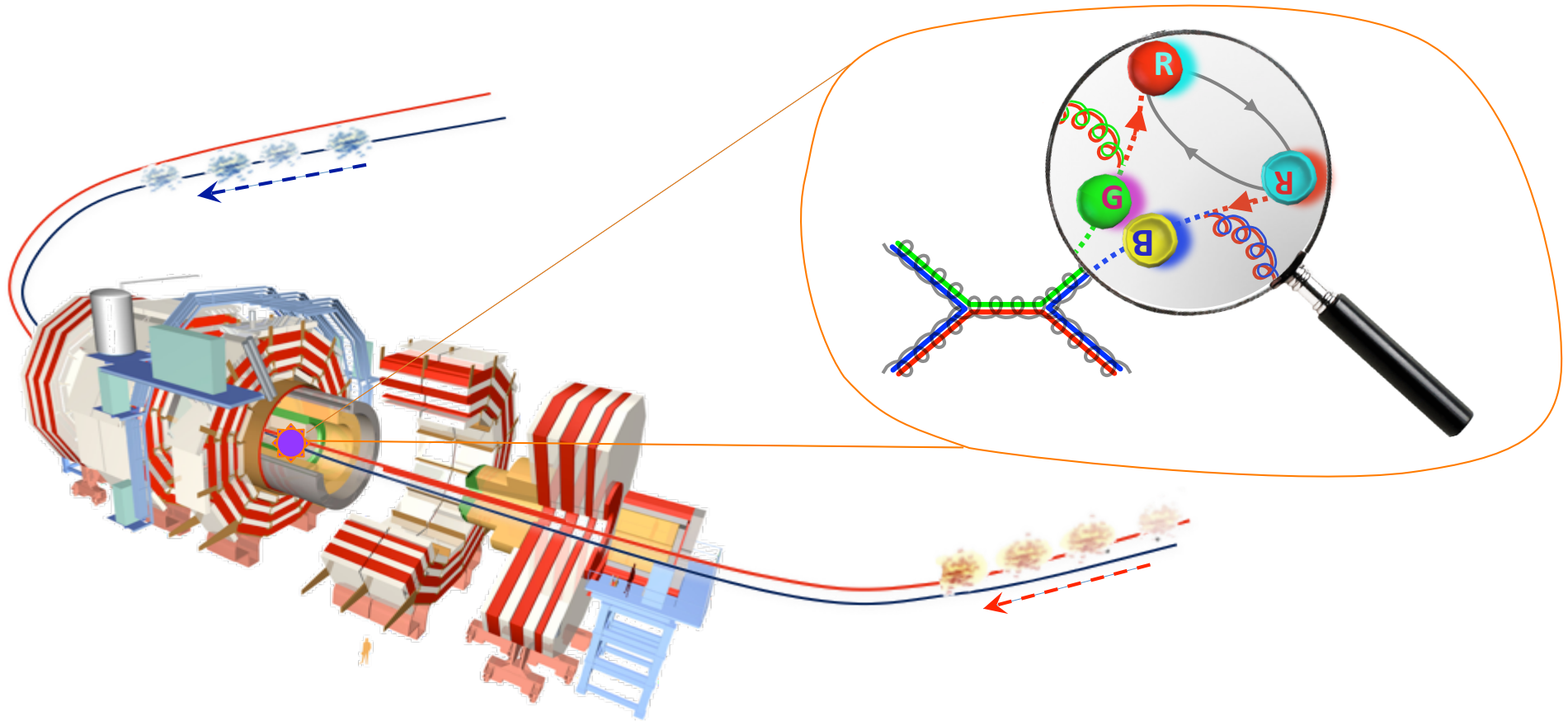
→ Part 2)

- **Quarkonium polarization:**
 - More reliable analysis techniques
 - Consistent experimental picture
 - Polarization of S-wave and P-wave states
- **Quarkonium production cross sections:**
 - Up to the highest possible p_T
 - Cross sections of S-wave and P-wave states

2. We need “better” theory!

→ Part 3)

- Improved understanding of pQCD inputs
- Consistent NRQCD model vs. data comparisons

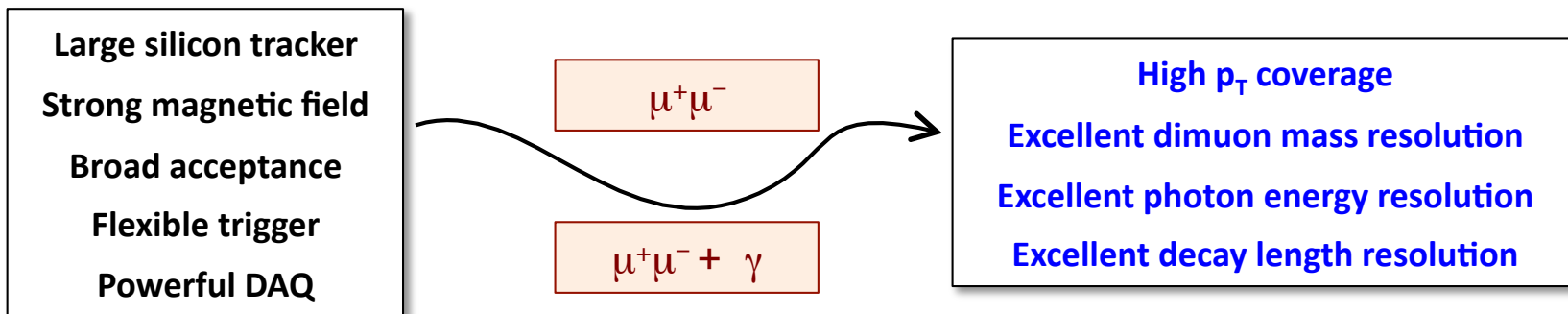
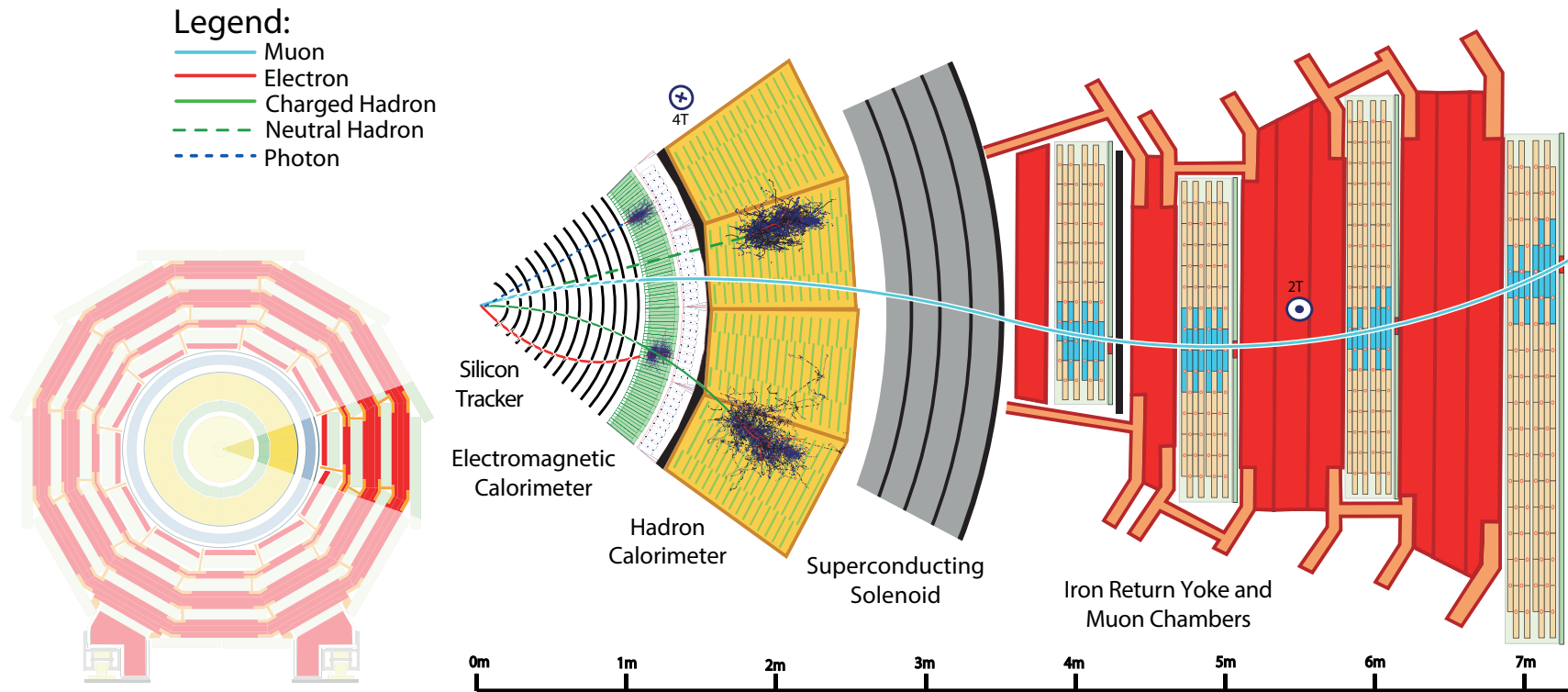


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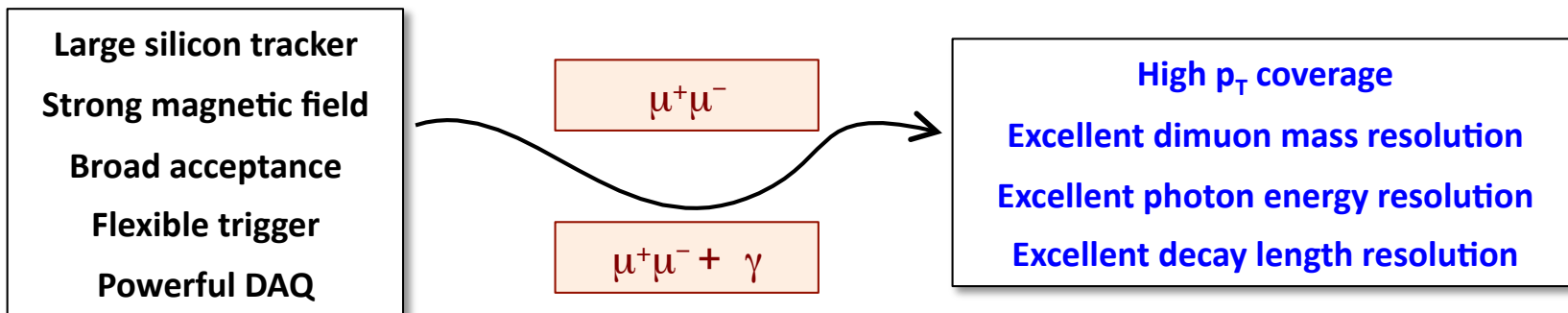
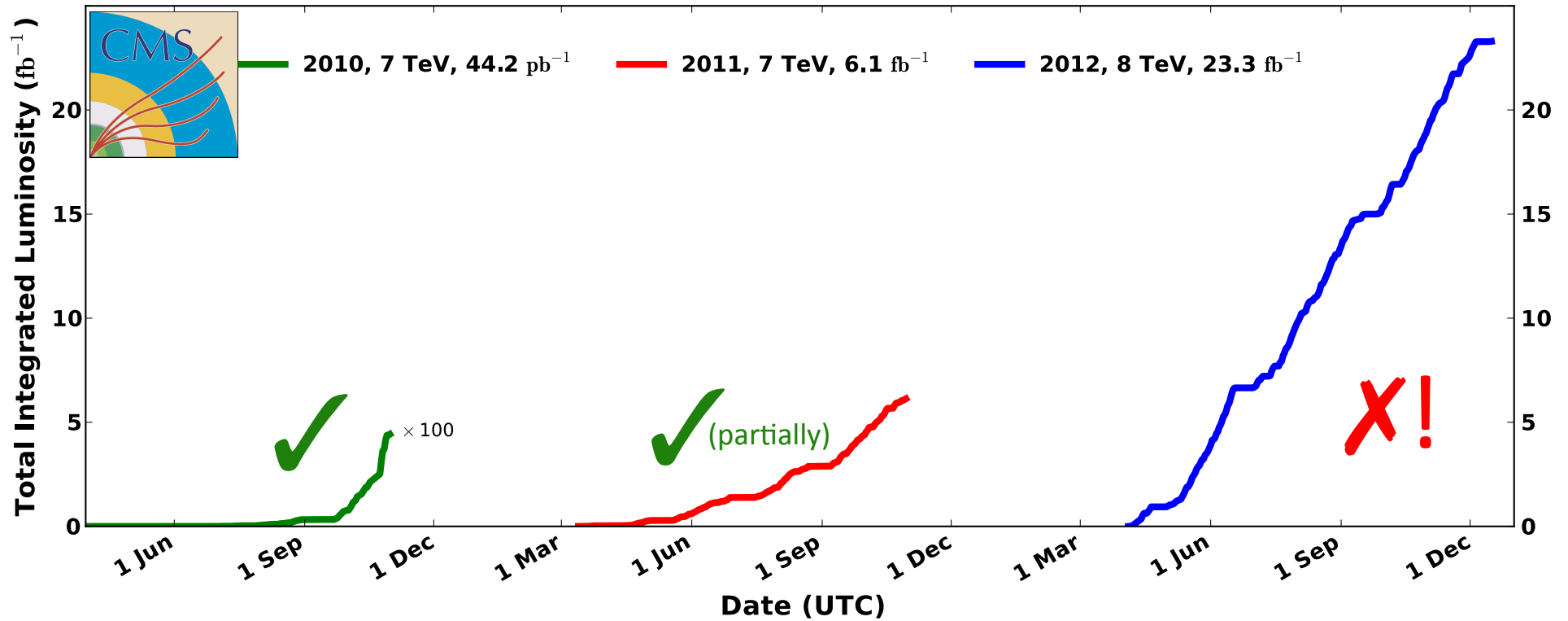
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The CMS detector: quarkonium performance



The CMS detector: quarkonium performance



Charming and beautiful CMS measurements: S states

Very good **dimuon mass resolution**

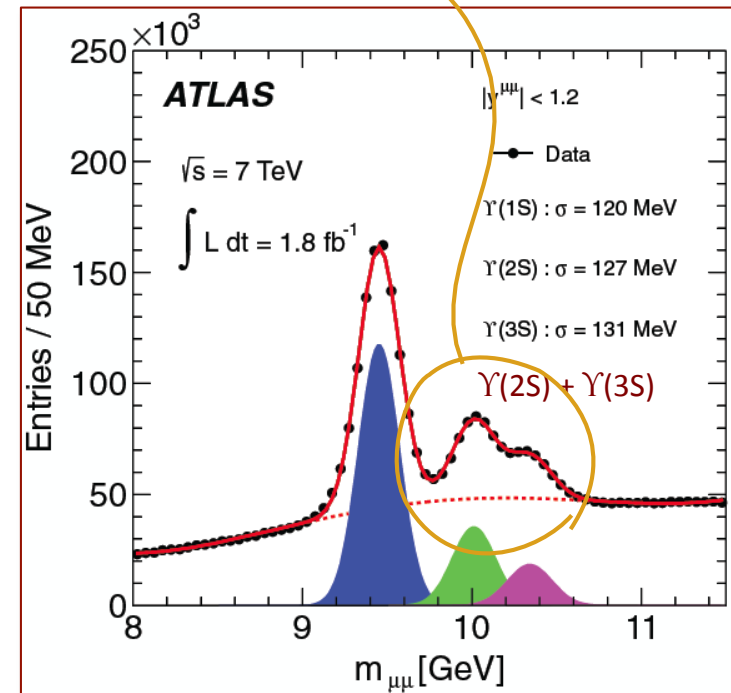
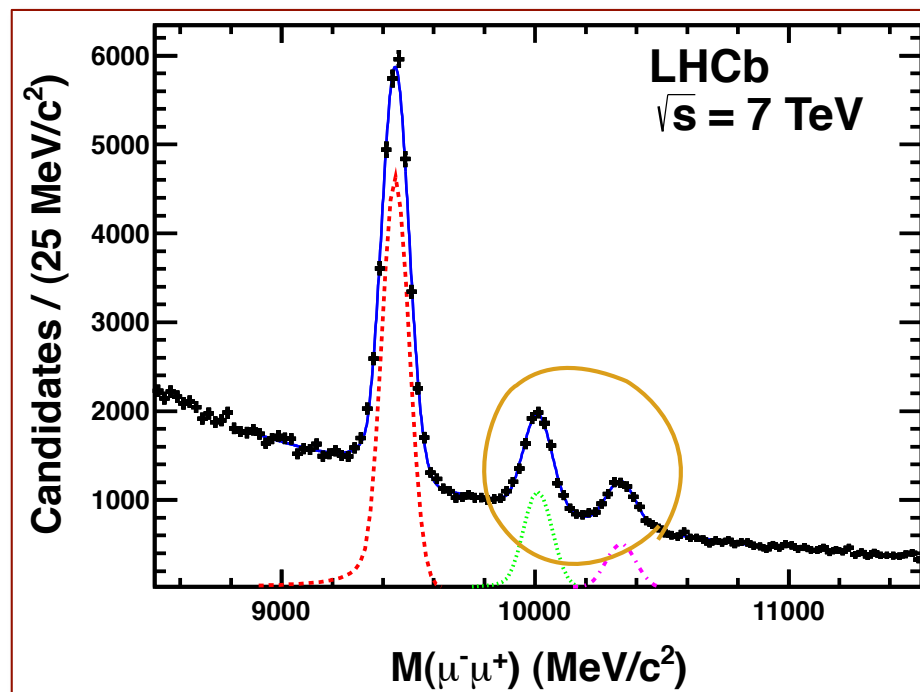
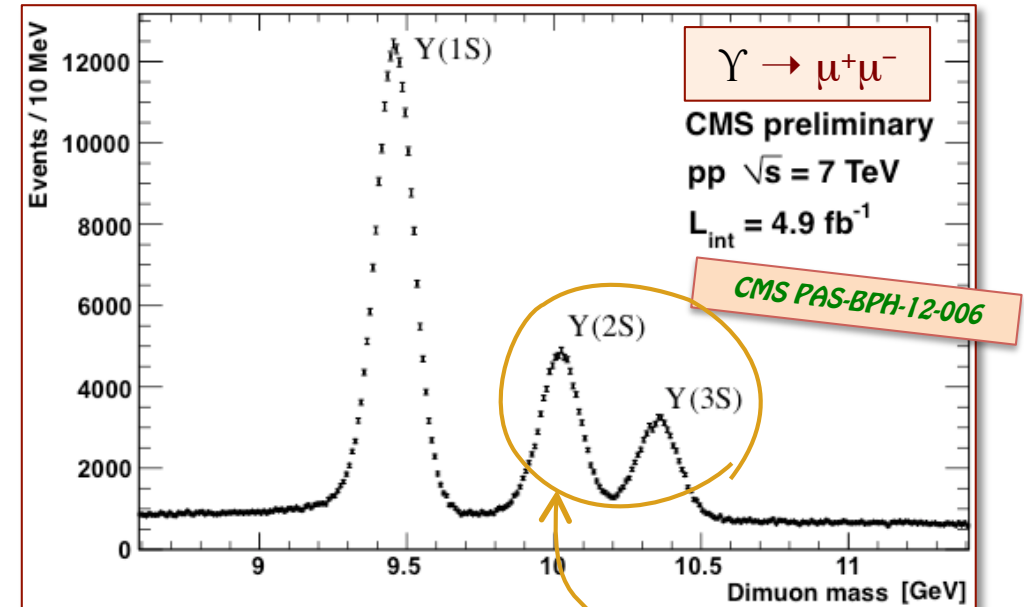
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High p_T dimuon coverage

→ much better than LHCb; similar to ATLAS

Excellent secondary vertexing

→ Crucial to remove non-prompt charmonia



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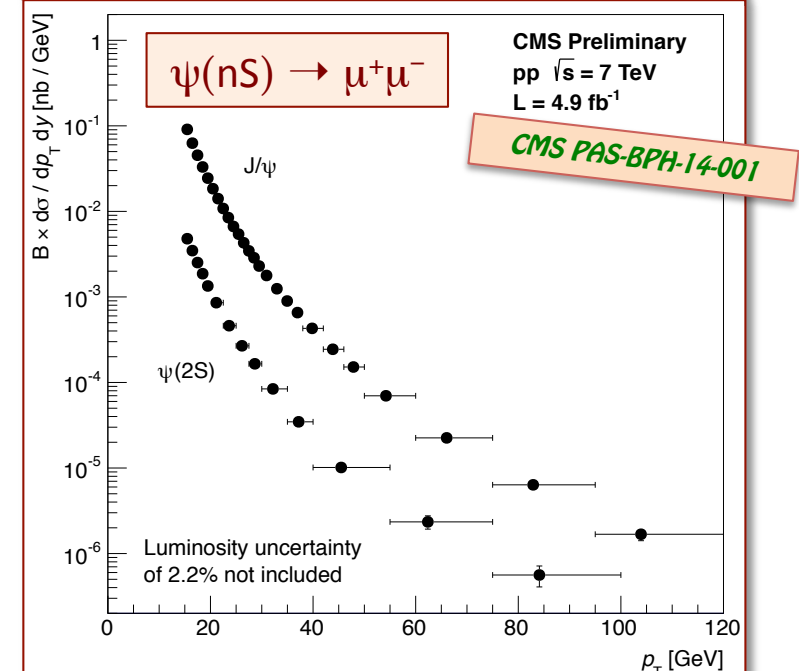
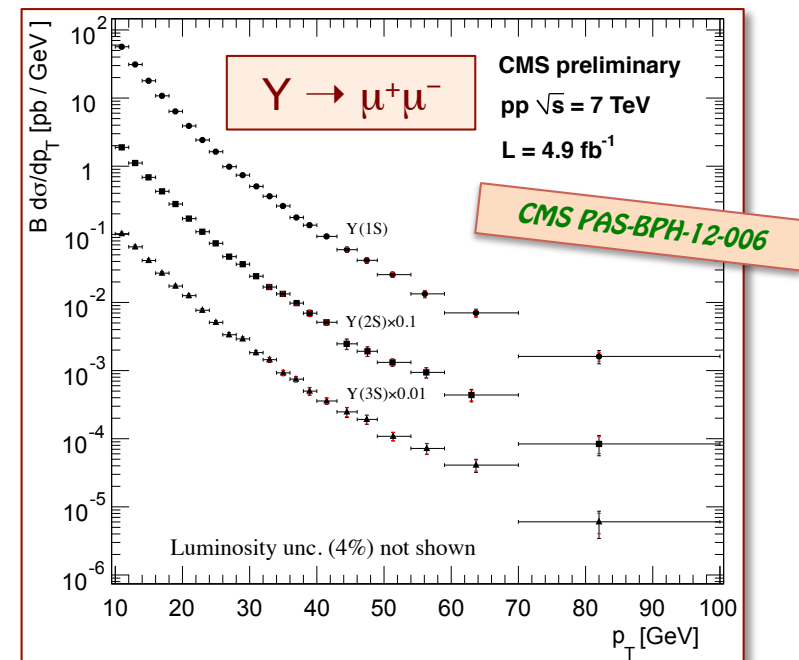
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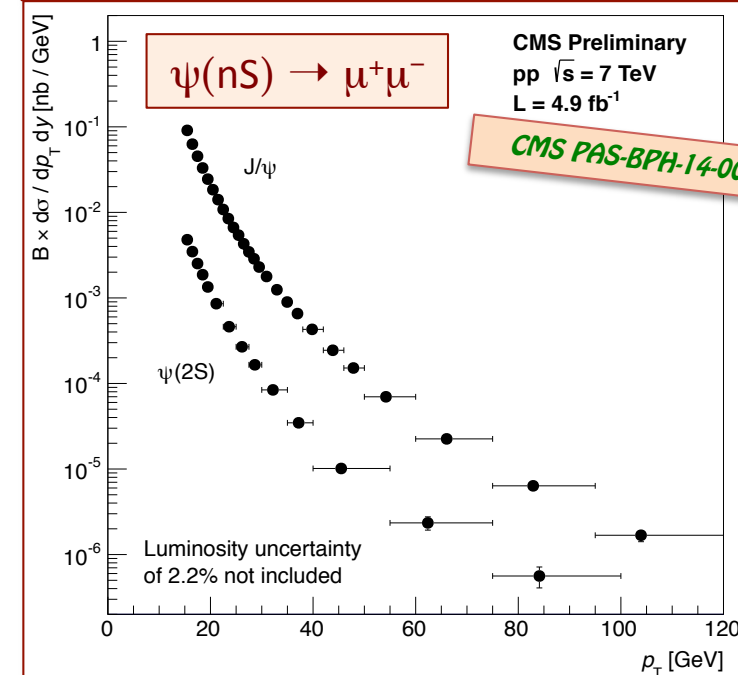
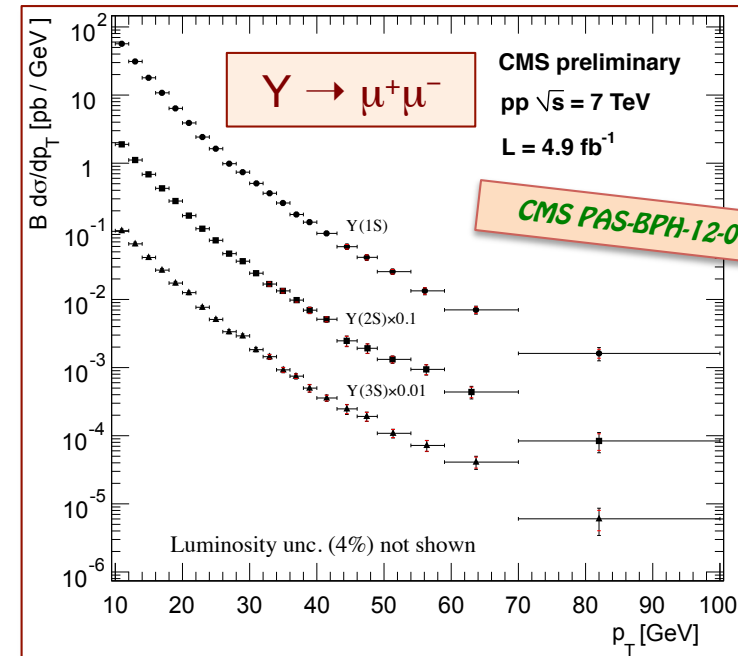
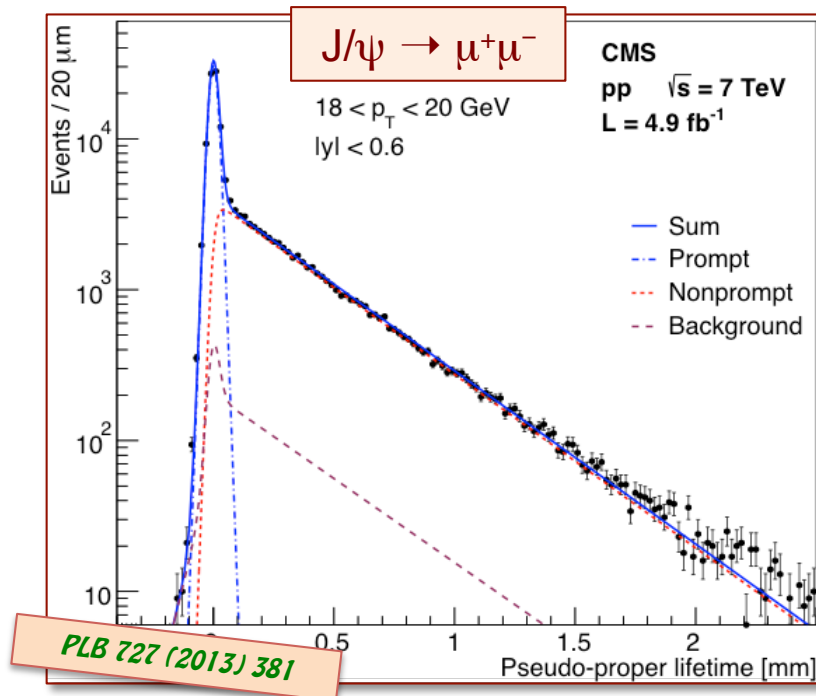
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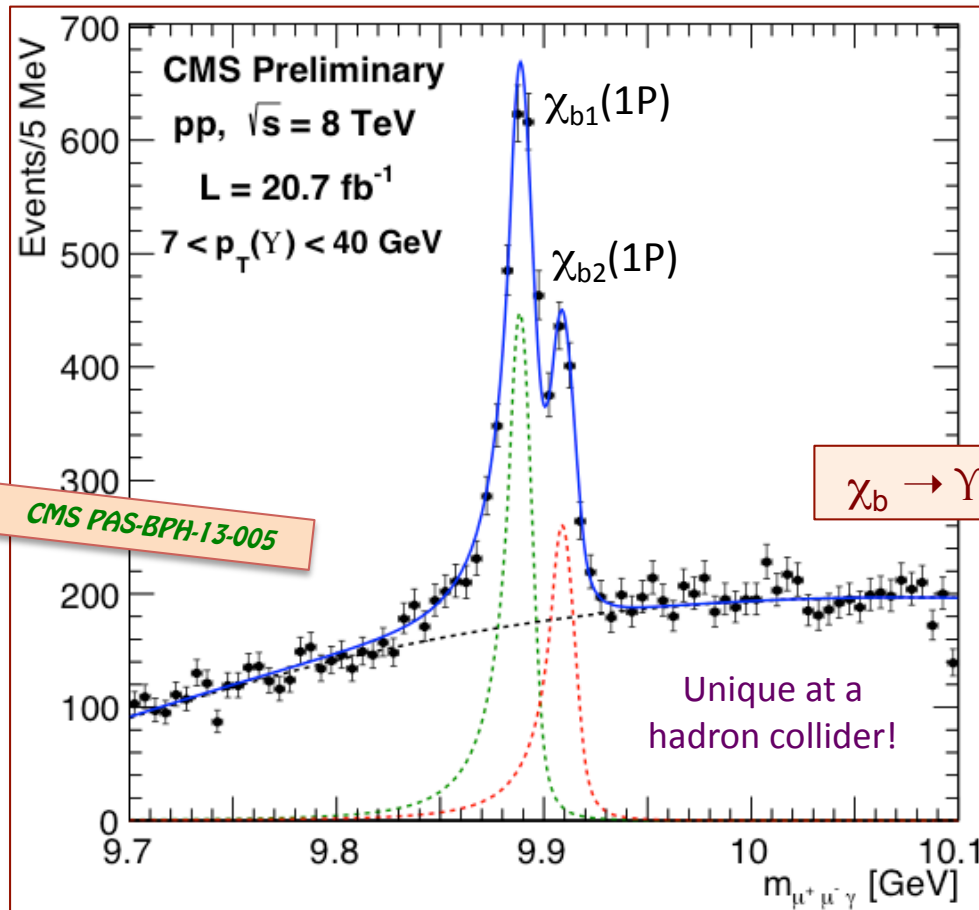


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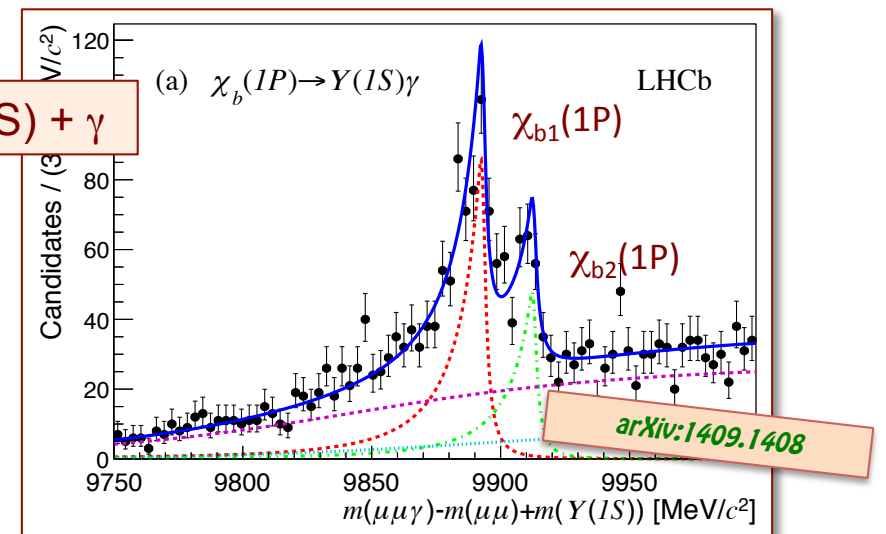
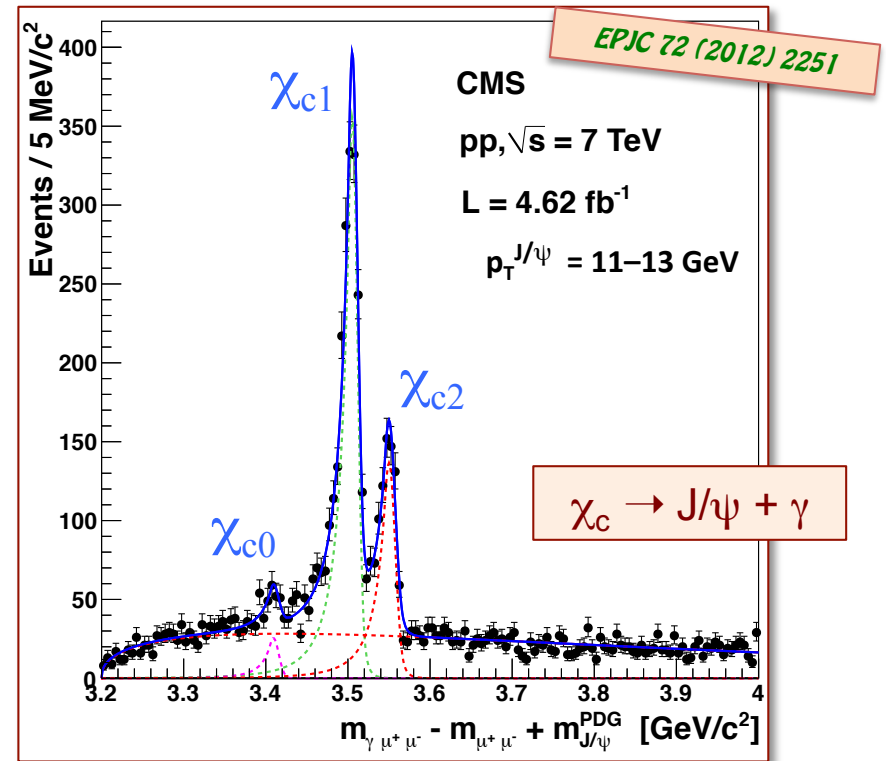
Low energy photon conversions

- „Sub-optimal“ efficiency
- Excellent mass resolution (≈ 5 MeV)

Current CMS results limited to measurements of the χ_{c2} / χ_{c1} and $\chi_{b2}(1P) / \chi_{b1}(1P)$ cross-section ratios



$\chi_b \rightarrow Y(1S) + \gamma$



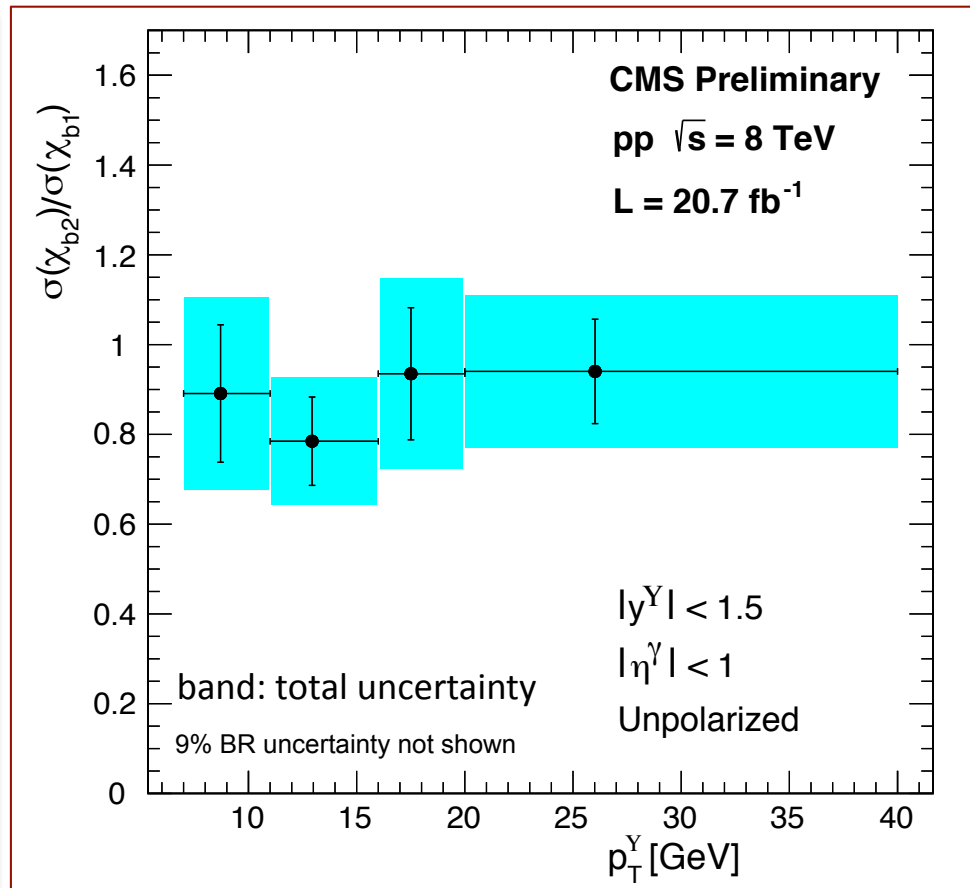
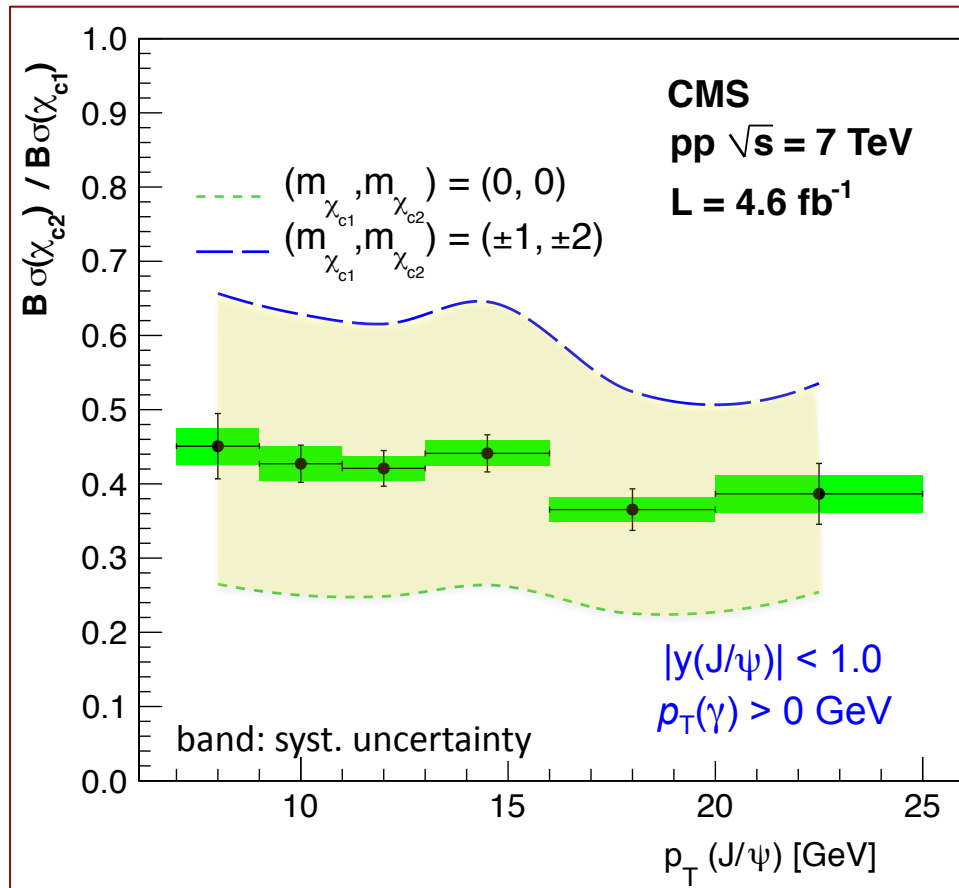
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Future CMS measurements will include „absolute“ cross sections of the P-wave charmonium and bottomonium systems



LHC: Quarkonium cross sections

Differential cross sections at mid-rapidity, for 7 different quarkonium states, measured by CMS and ATLAS, as function of p_T/M (*)

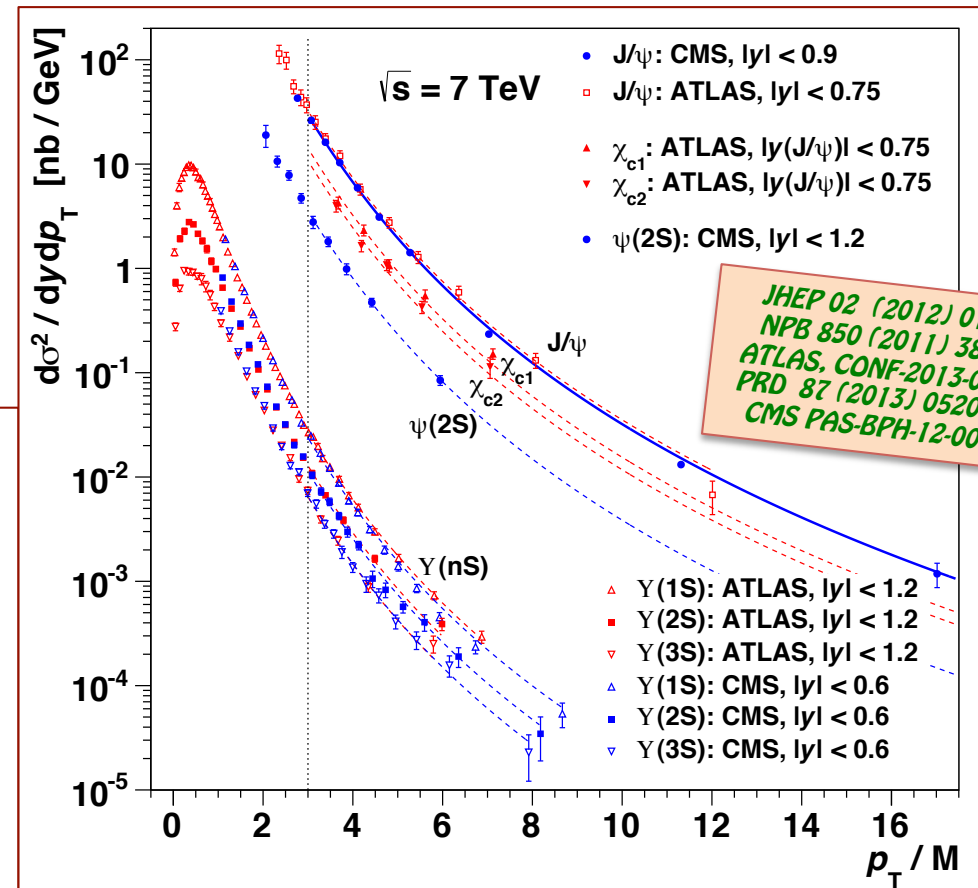
Shapes are well described by a single empirical power-law (for $p_T/M > 3$), common to all considered results (5 S-wave and 2 P-wave states, with highly varying feed-down characteristics)

This strongly suggests that quarkonium production is dominated by 1 single production mechanism, common to all S and P quarkonia

Compilation by P. Faccioli *et al.*,
arXiv:1403.3970 (2014)

solid: fit to CMS J/ψ data
dashed: replicas with adjusted normalizations

(*) p_T is mass-rescaled to equalize the kinematic effects of different average parton momenta and phase spaces



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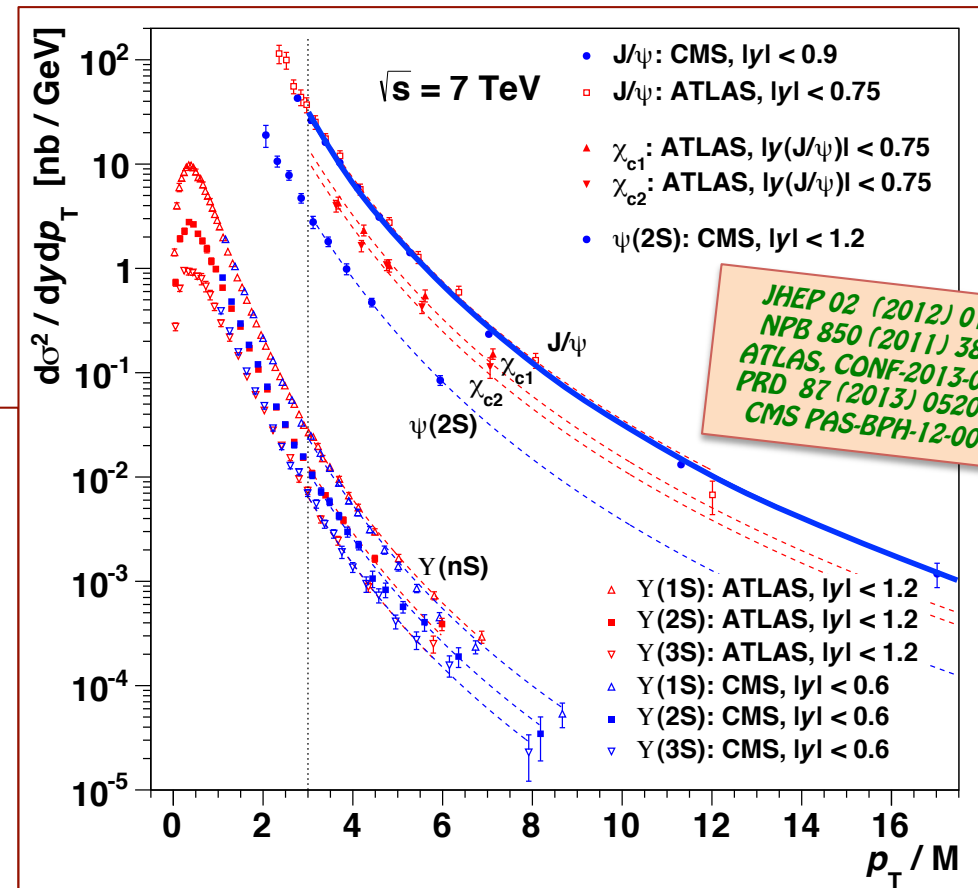
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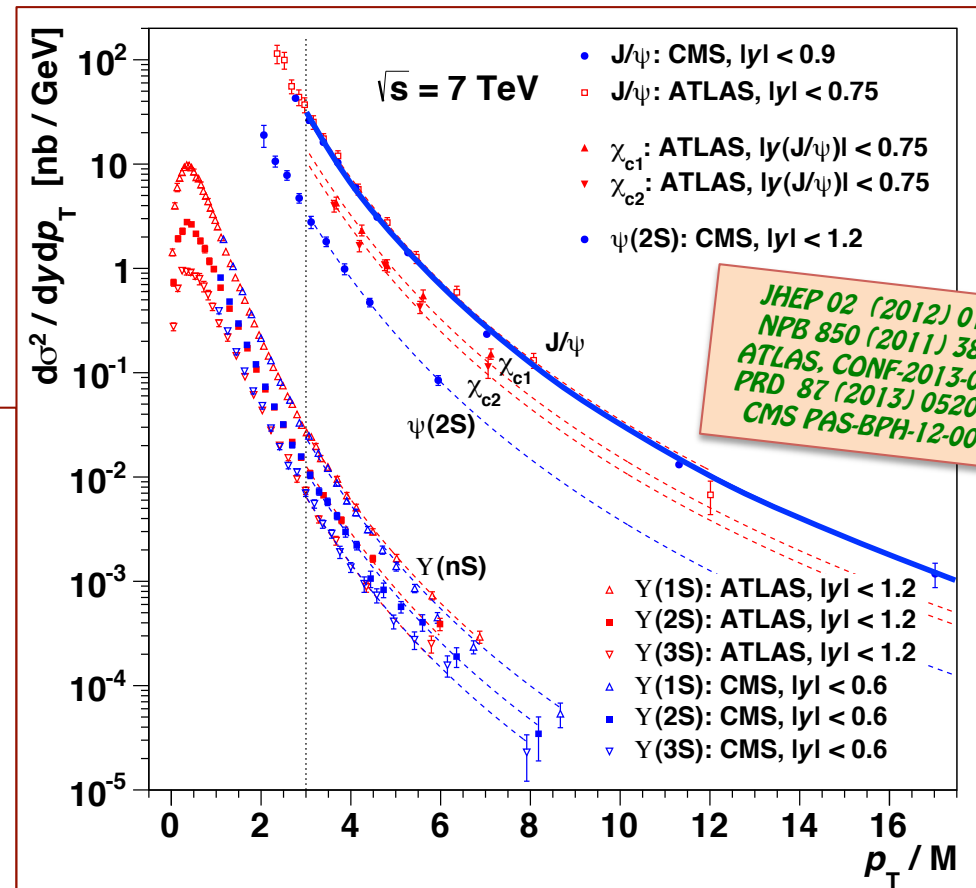
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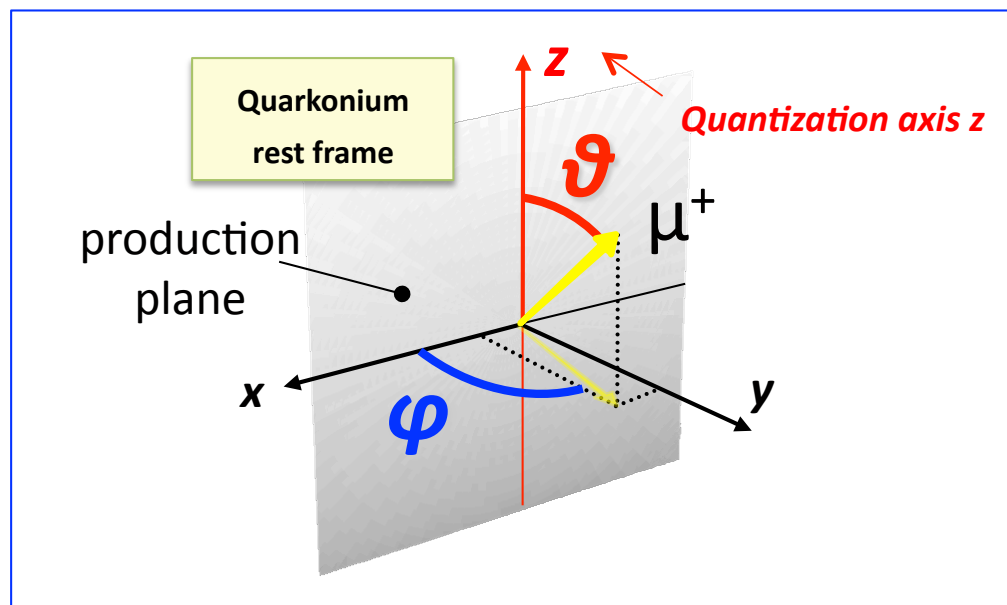
CMS: Quarkonium polarization analyses

Quarkonium polarizations are measured from the **angular decay distributions** in dimuon decays

We measure the full angular distribution and report the λ_θ , λ_ϕ and $\lambda_{\theta\phi}$ polarization parameters (in 3 frames) for five S states, vs. p_T and in several $|y|$ ranges.

We further measure the frame-invariant parameter $\tilde{\lambda}$

The underlying continuum background is removed using the invariant mass distribution;
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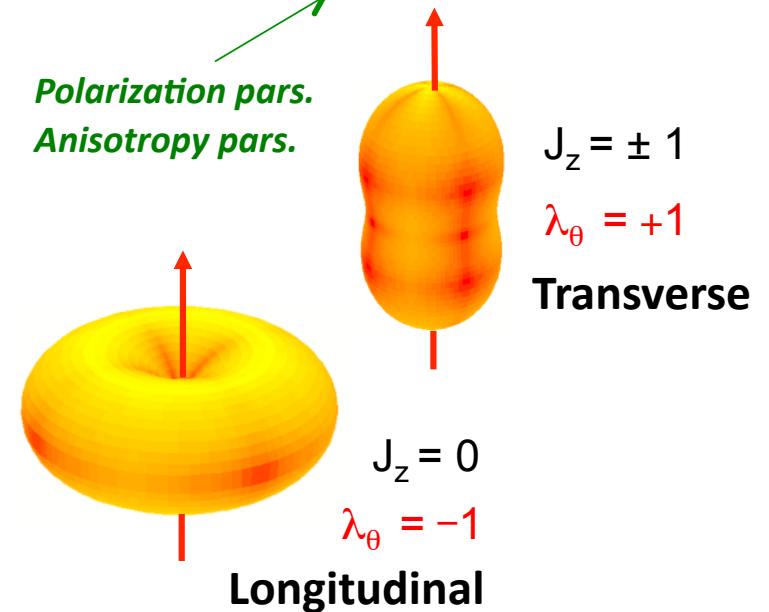
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EPJC C69 (2010) 657

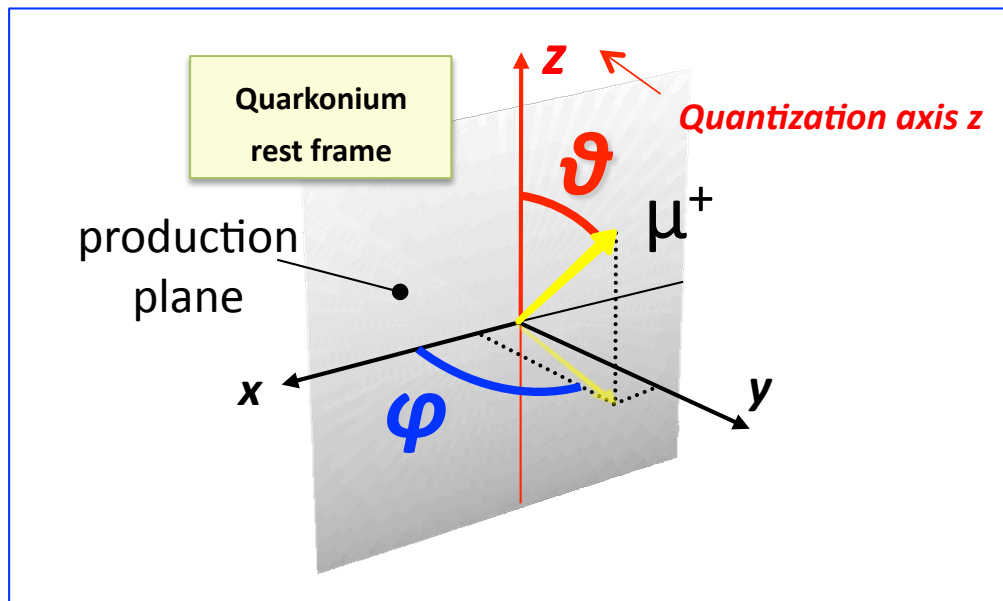
$$\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos \phi$$

Polarization pars.
Anisotropy pars.



Frame-invariant:

$$\tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}$$

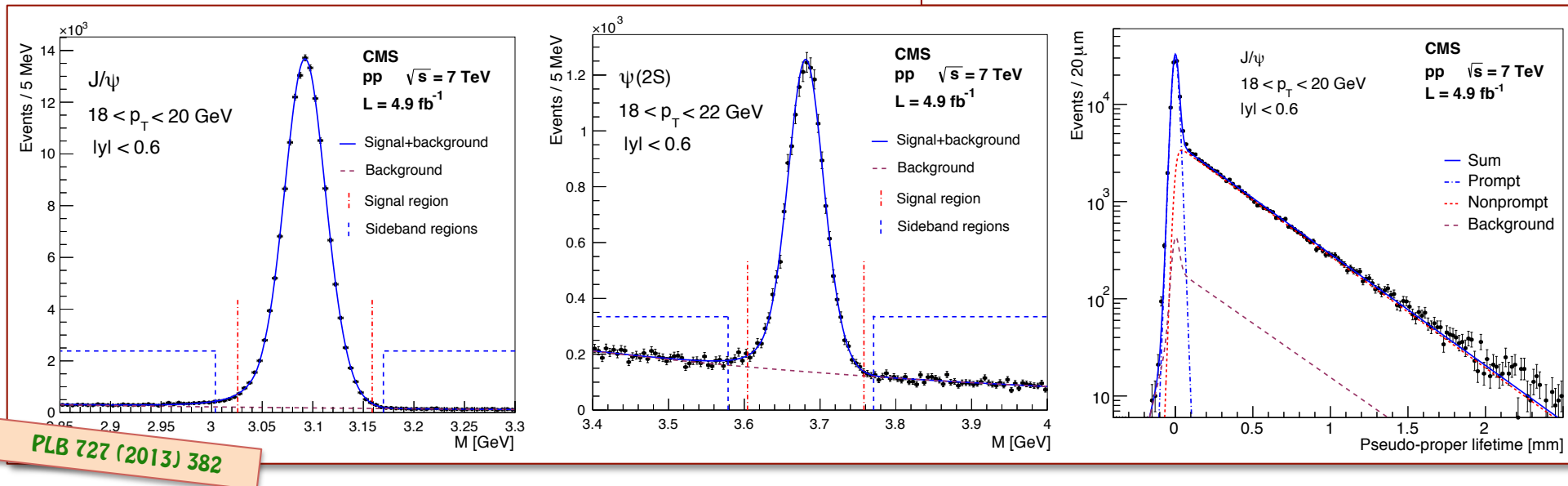
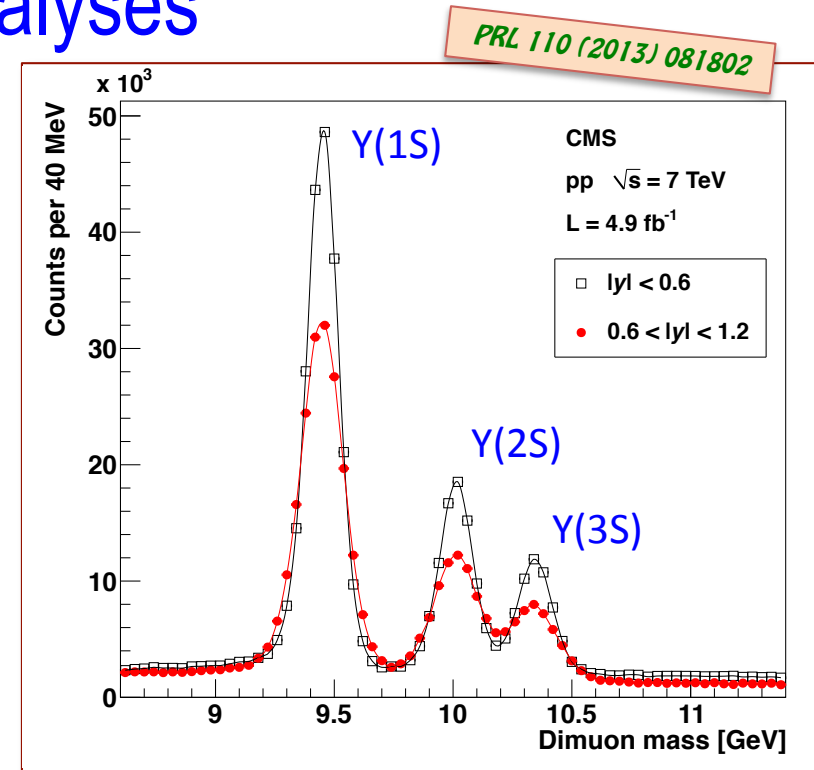


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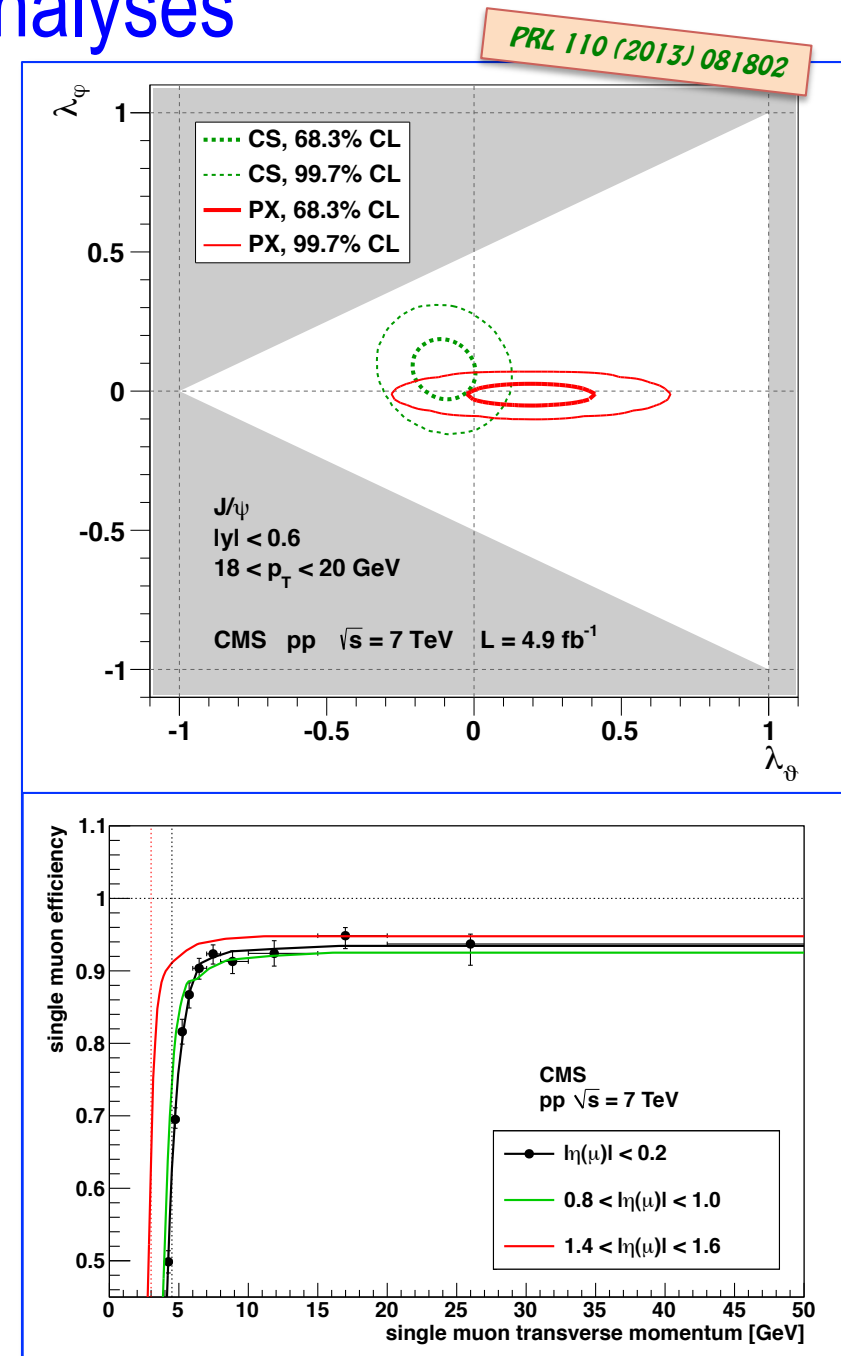
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We calculate the multi-dimensional **posterior probability density** as result of the analysis

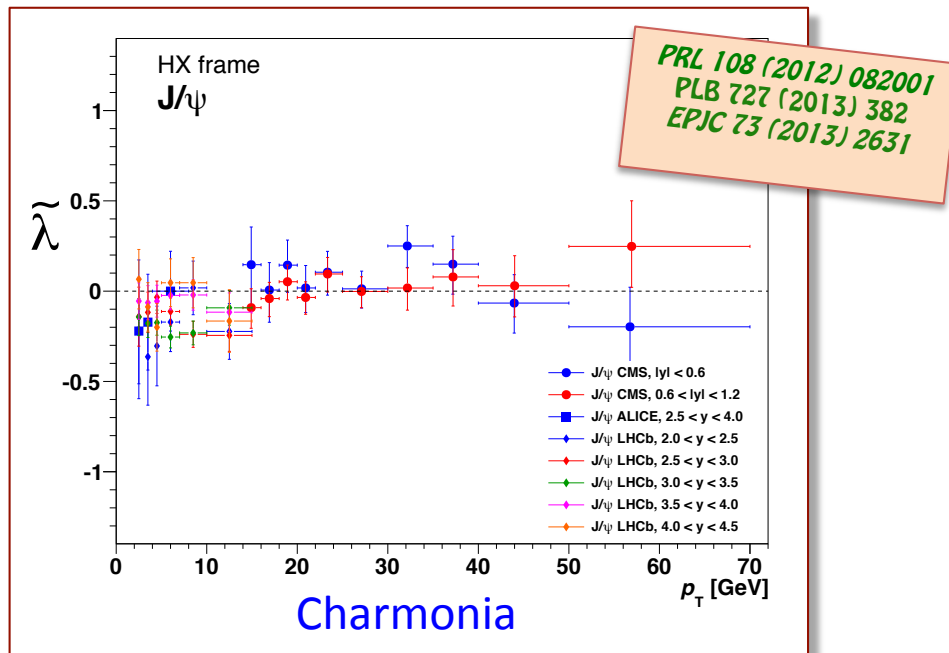
Main experimental challenges:

- ✧ reliable **background modeling** (sidebands)
- ✧ precise mapping of **(di)muon efficiencies** (T&P)

Uncertainties are dominated by systematics at low p_T and by statistics at high p_T



LHC: Quarkonium polarization results

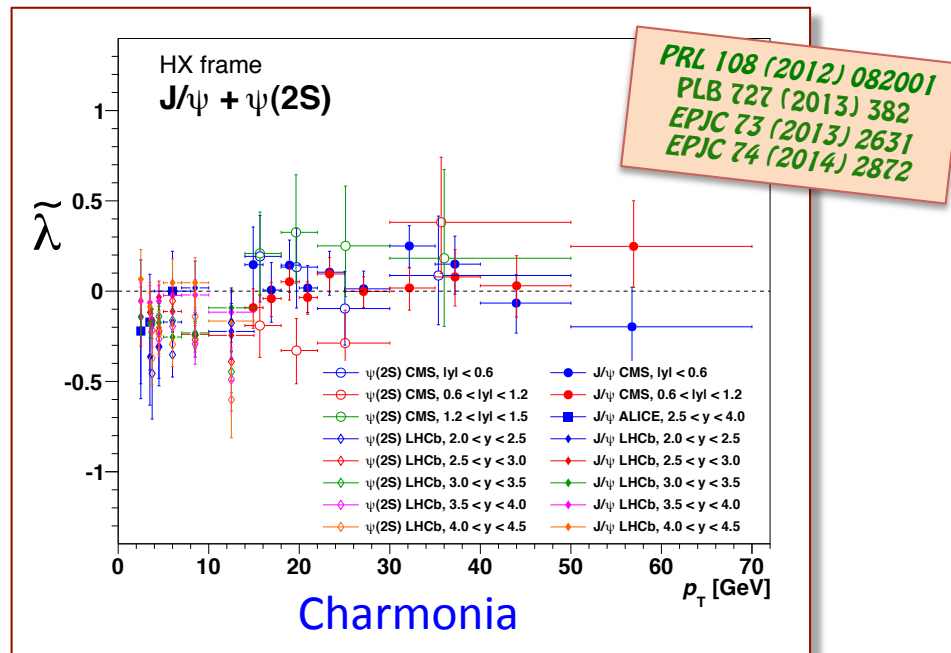


Good consistency between CMS, LHCb, ALICE and CDF. Previous experimental inconsistencies overcome due to novel and more robust analysis techniques (*EPJC 69 (2010) 657*).

No strong polarizations seen in any of the measurements

- no dependencies on p_T or rapidity
- no strong changes between S-states with very different P-wave feed-down characteristics
- no evident differences between charmonium and bottomonium states

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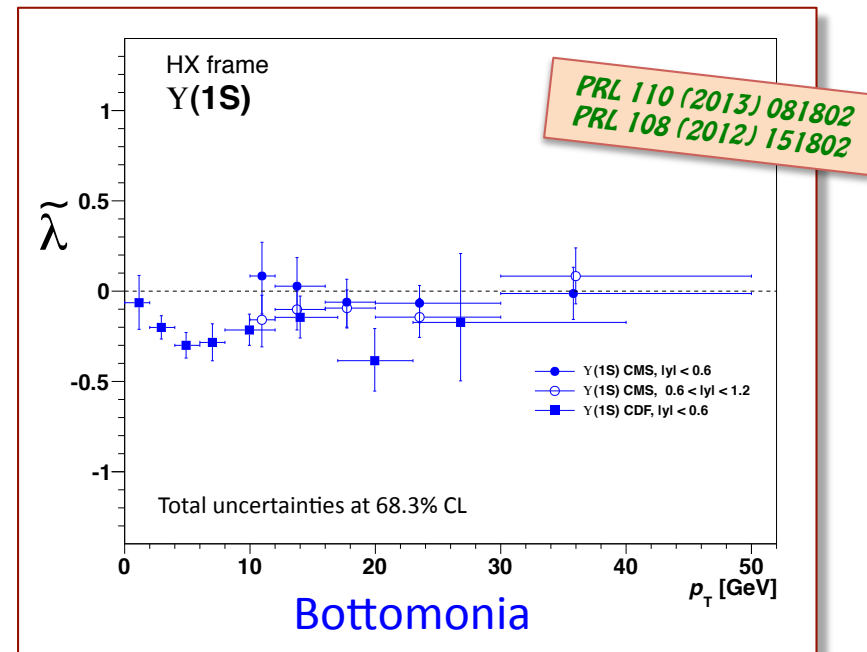
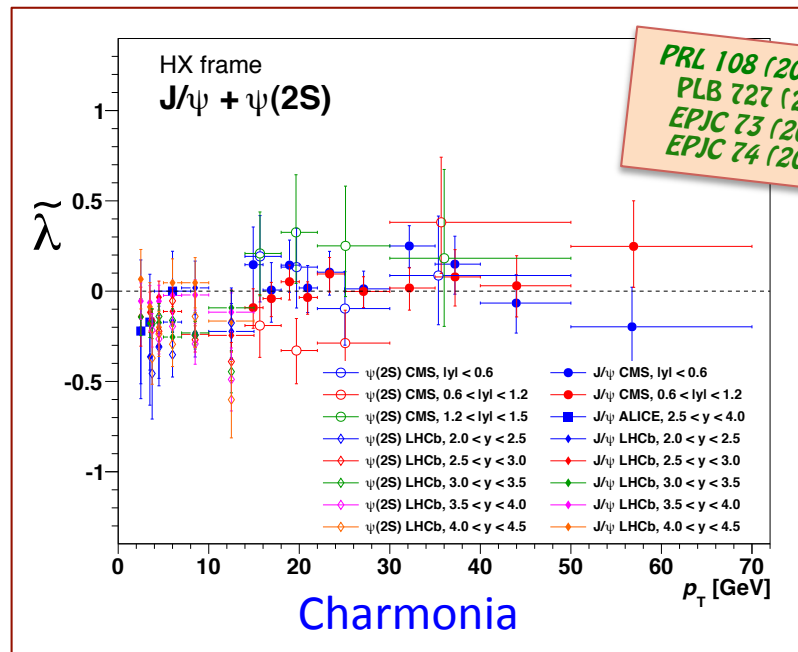


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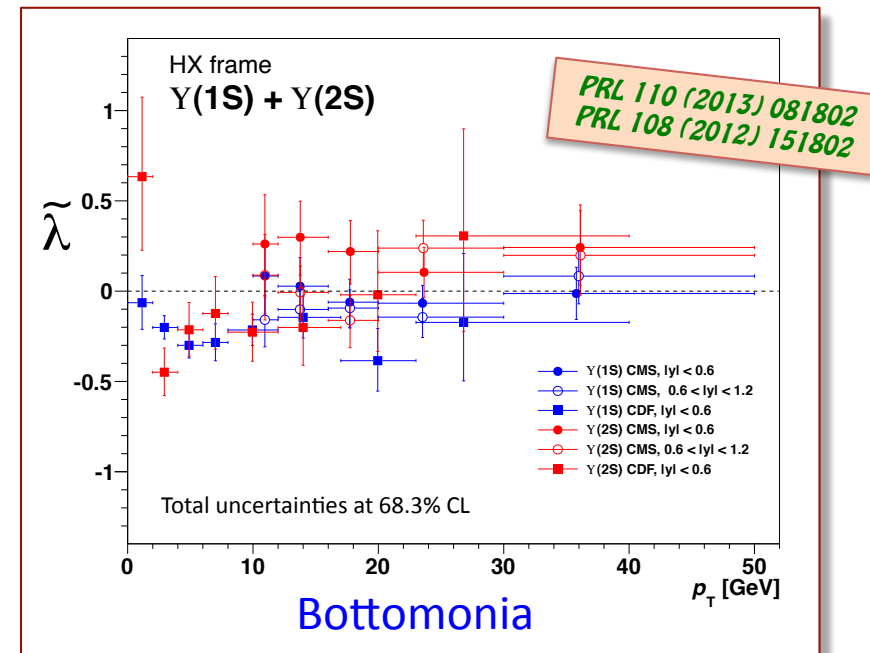
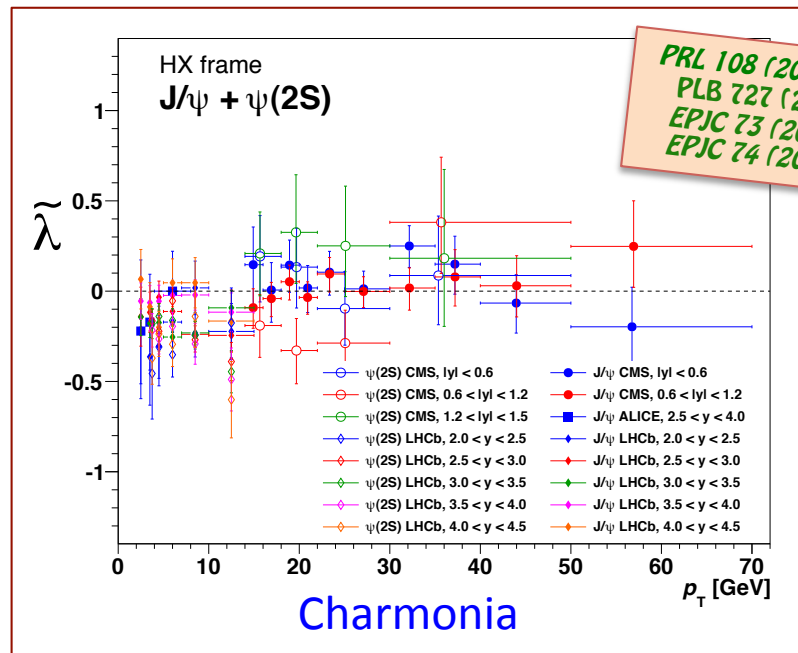


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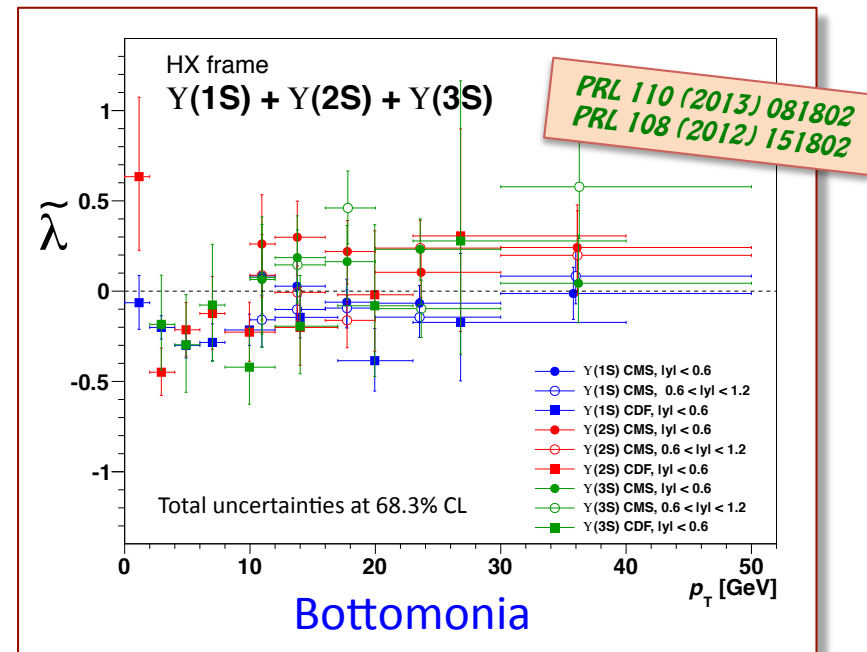
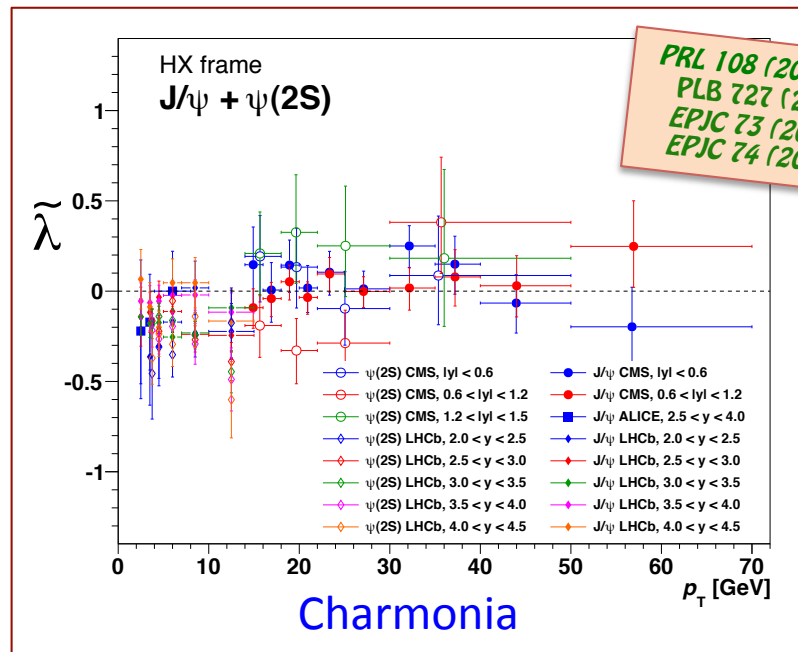


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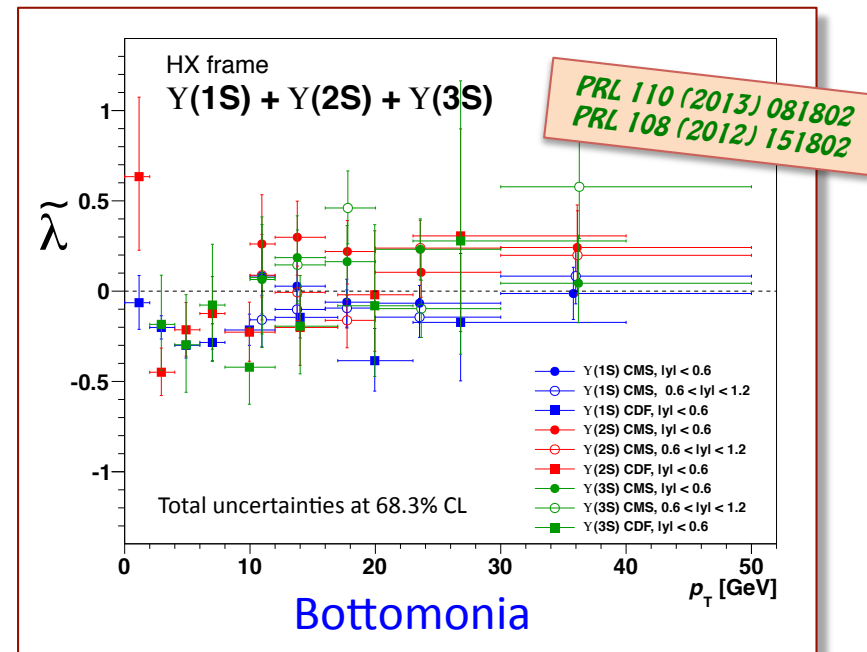
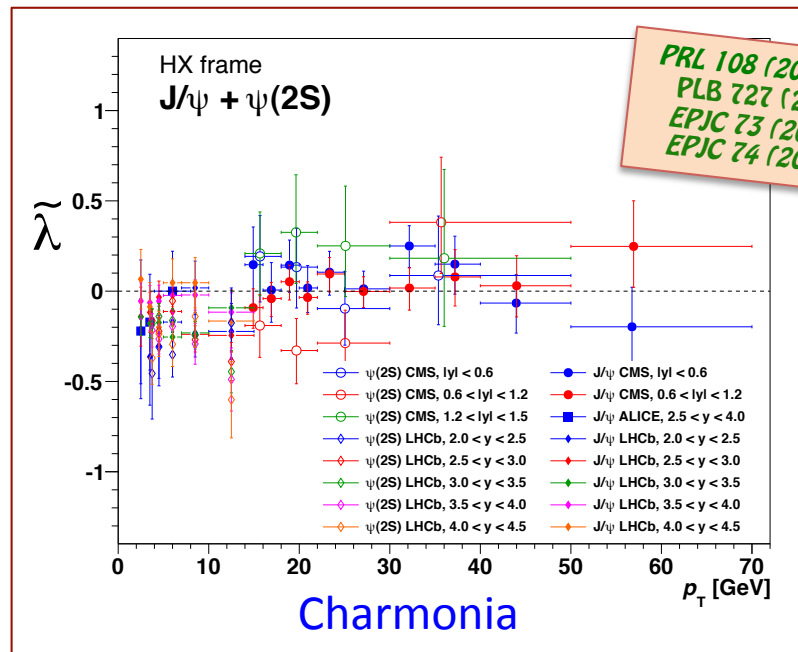


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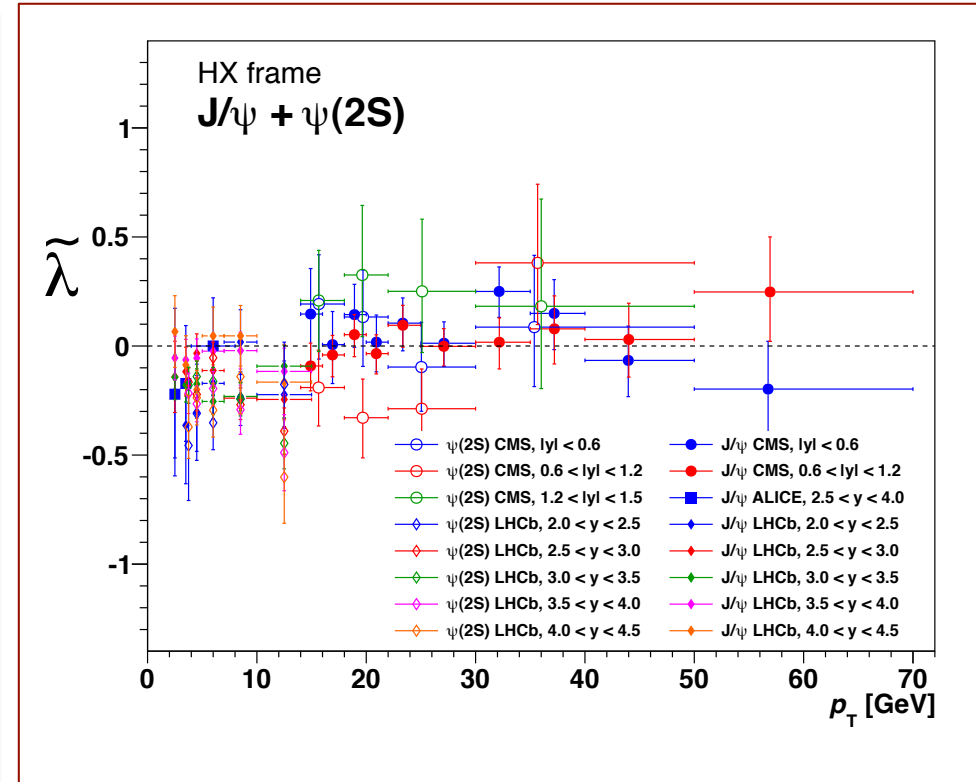
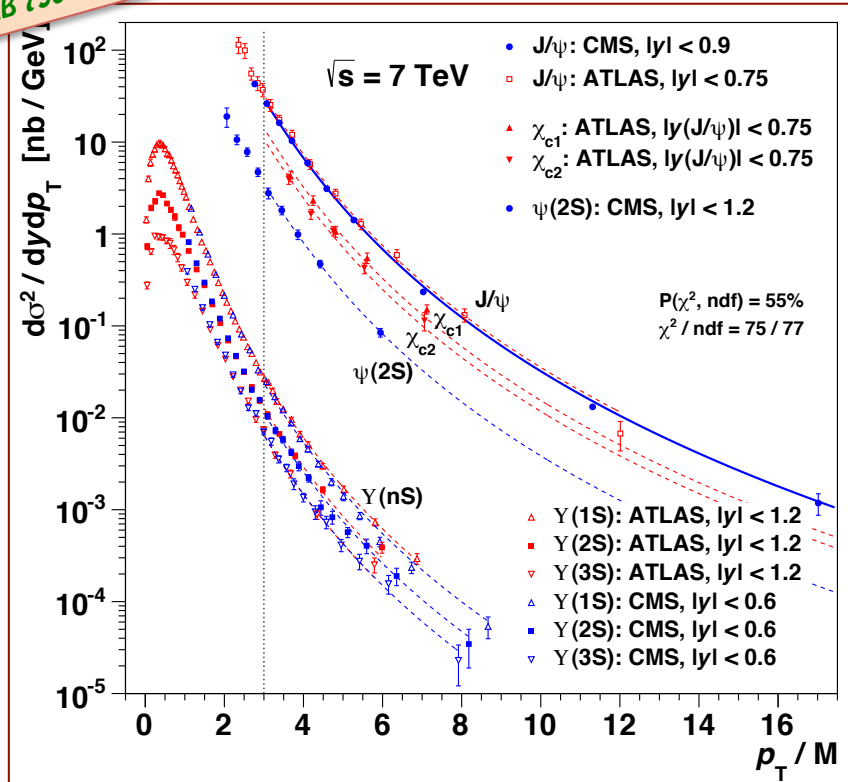
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Lessons from the LHC: Two data-driven observations

PLB 736 (2014) 98

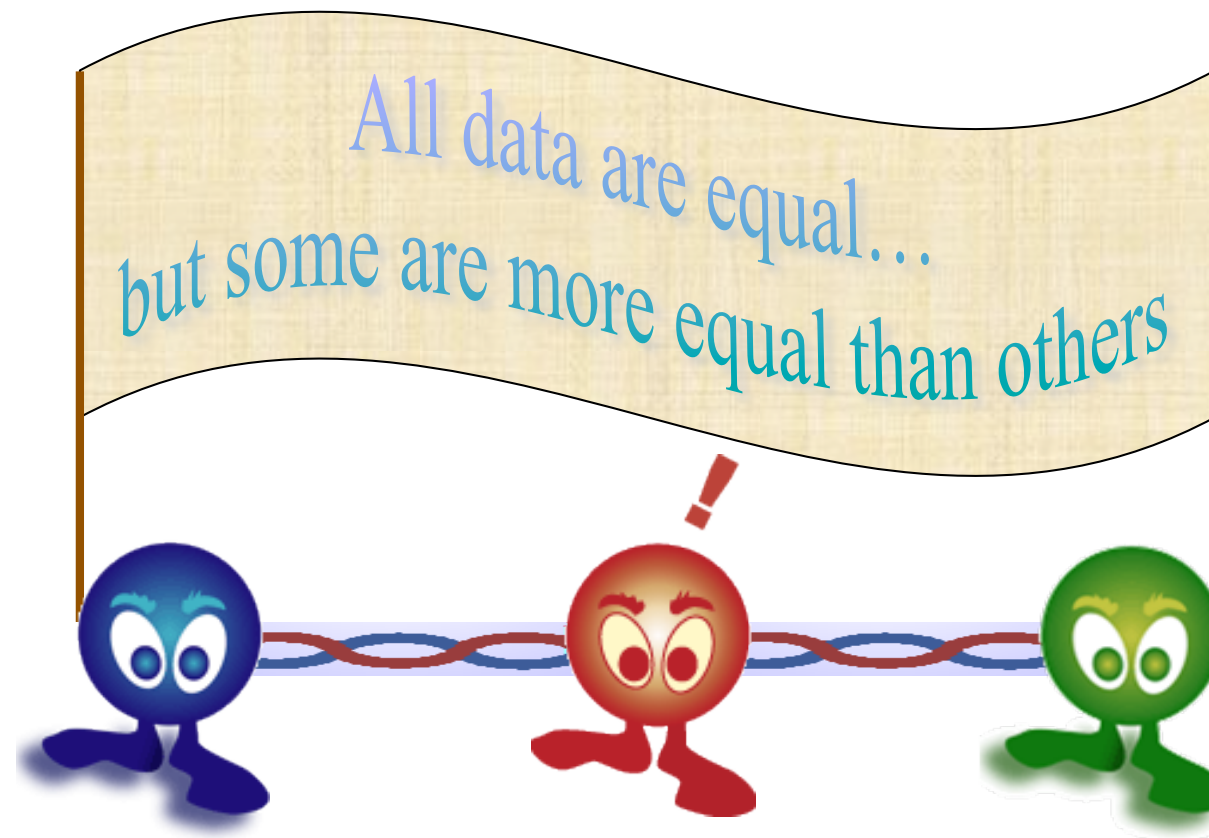


1. Cross section data

- „ p_T/M scaling”: All quarkonium states are produced in a very similar way
- Likely dominated by **one color octet** mechanism

2. Quarkonium polarization data

- All S-wave quarkonia are produced unpolarized
- The dominating CO contribution is suspected to be the **unpolarized 1S_0** ^[8] term

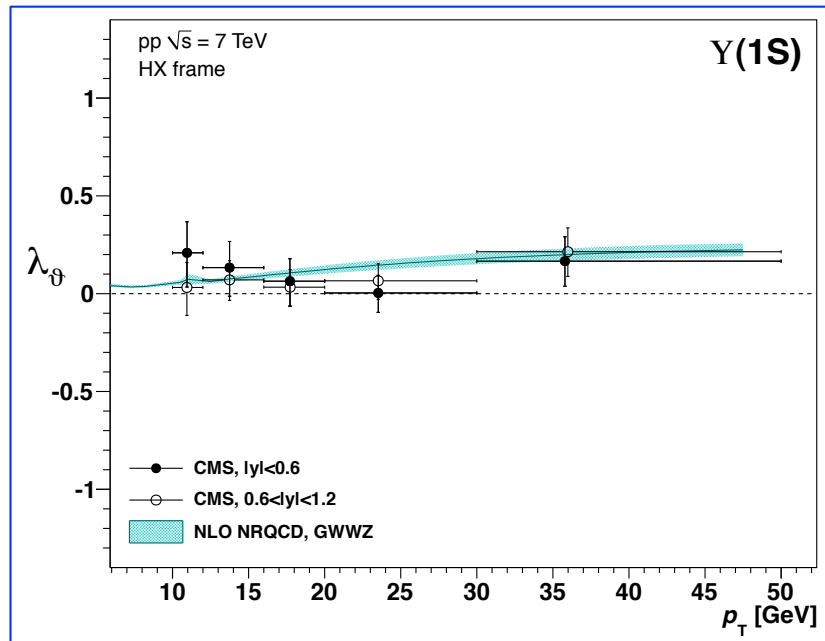


- 1) Motivation & introduction: the pre-LHC puzzles
- 2) Quarkonium production measurements at CMS

3) Interpretation of the results: a polarized perspective

LHC-era state-of-the-art NRQCD analyses: $Y(nS)$

PRL 112 (2014) 032001

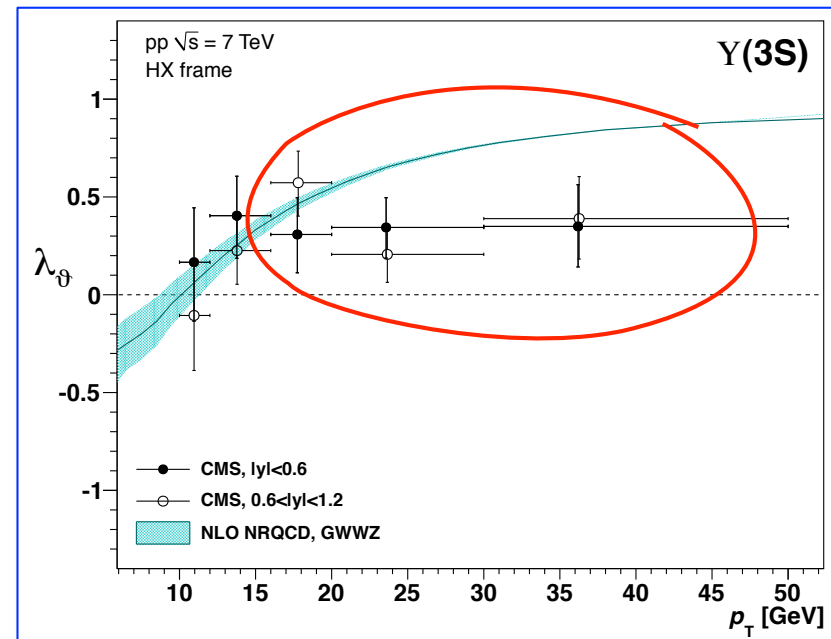
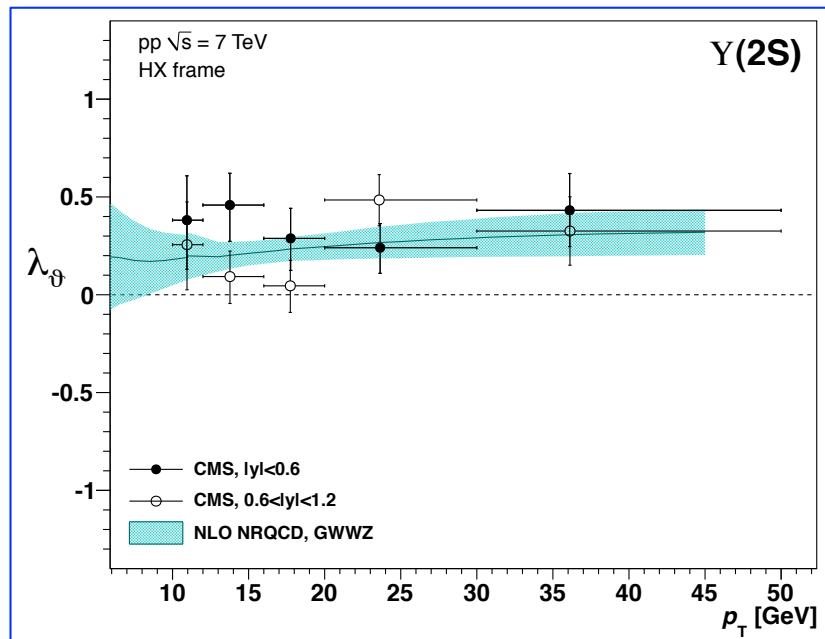


GWWZ Fit: Hadroproduction data, including CMS $Y(nS)$ polarization results, to fit the LDMEs

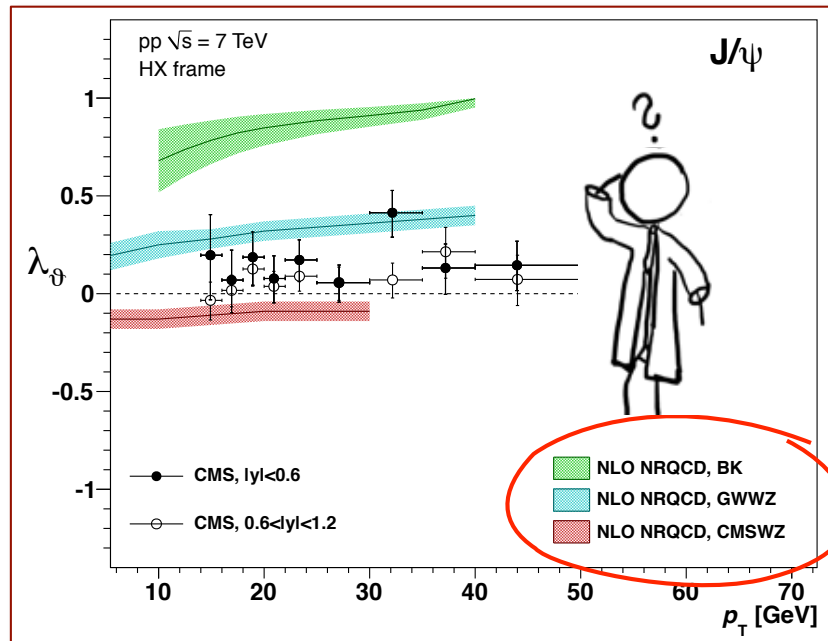
The $Y(1S)$ and $Y(2S)$ predictions include the feed-down decays of P-wave states, while the $Y(3S)$ is assumed to be 100% directly produced

The *unknown* feed-down fractions and polarizations of the P states give the model the freedom needed to fit the $Y(1S)$ and $Y(2S)$ data

Kinematic region: $p_T > 8$ GeV



LHC-era state-of-the-art NRQCD analyses: $\psi(nS)$



BK Fit: Hadro- and photoproduction data,
not including polarization data,
not including feed-down decays to fit the LDMEs
Kinematic region: $p_T > 3$ GeV

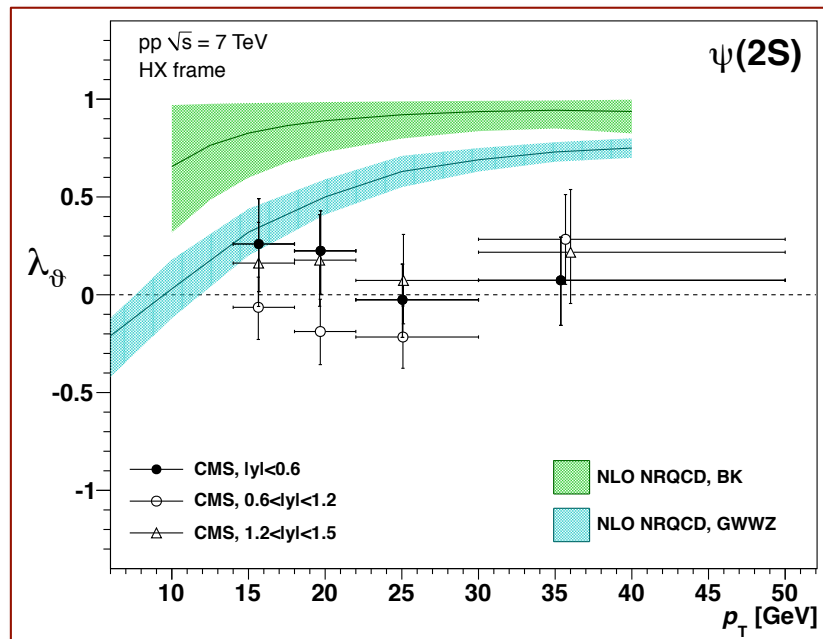
PRL 108 (2012) 172002

GWWZ Fit: Hadroproduction data,
not including polarization data,
including feed-down decays to fit the LDMEs
Kinematic region: $p_T > 7$ GeV

PRL 110 (2013) 042002

CMSWZ Fit: Hadroproduction data,
including polarization data,
not including feed-down decays to fit the LDMEs
Kinematic region: $p_T > 7$ GeV

PRL 108 (2012) 242004



State-of-the-art NRQCD analyses

- Starting from compatible pQCD inputs
- Various differences in the LDME fit
- Contradictory results
- **Completely different physics conclusions!**

NLO NRQCD \neq NLO NRQCD...?
What is going on?

A matter of NRQCD validity domain?

- The *crucial* hypothesis of NRQCD: **factorization**

It is well known that factorization and pQCD calculations are only valid for $p_T \gg M$

Most NRQCD analyses use data down to rather low values of p_T / M

GWWZ :	$\psi(nS)$	$p_T / M > 2$	$\Upsilon(nS)$	$p_T / M > 0.8$
CMSWZ :	$\psi(nS)$	$p_T / M > 2$		
BK :	J/ψ	$p_T / M > 0.95$		

Implications of these choices have not been tested!

Problem: Lowest- p_T data points have smallest uncertainties → determine the LDME fit results
 → Are the fitted LDME values very sensitive to the exact value of $p_{T,\min}$?

The high- p_T reach of the LHC measurements allows us to progressively exclude the lowest- p_T data

→ Search for the **domain of validity of NRQCD** calculations!

Towards better NRQCD global fits

PLB 736 (2014) 98

“Technical” choices:

Cross sections and polarizations are *simultaneously* used in the fit

Experimental **correlated uncertainties** (e.g. luminosity) and **polarization-dependent acceptances** are accounted for, correlating the individual observables and measurements (**never done before**)

Theoretical uncertainties are accounted for directly in the fit, as difference between LO and NLO calculations, correlating the individual quarkonium states (**never done before**)

The pQCD inputs are taken from the NLO calculations of Butenschön and Kniehl

Strategic choices:

Only LHC measurements are used; earlier results were ambiguous, incomplete or at too low p_T

The analysis is restricted to the $\psi(2S)$ and $\Upsilon(3S)$ data, to minimise the number of free LDMEs; we neglect the $\chi_b(3P)$ feed-down contamination in the $\Upsilon(3S)$

To get more reliable results, the “wild” 3P_J ^[8] octet is not included in the initial fits
When we include it, the fit quality does not improve and the results are not affected

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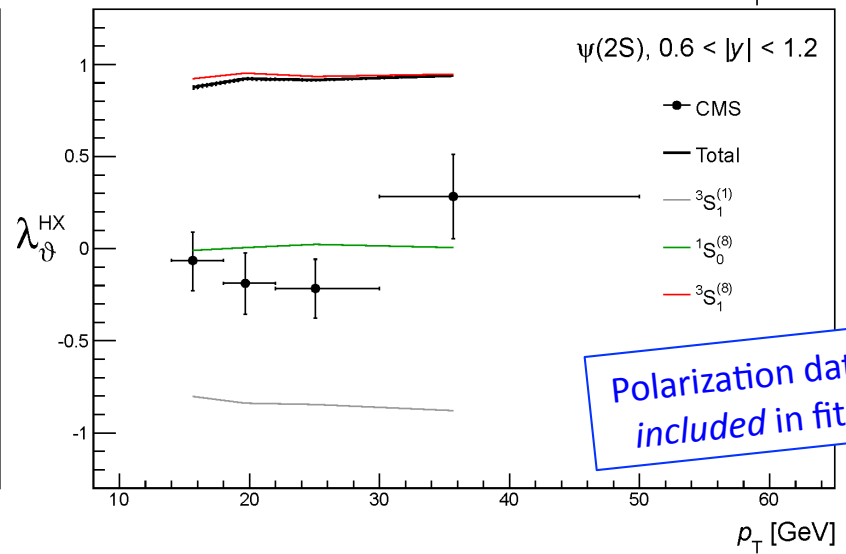
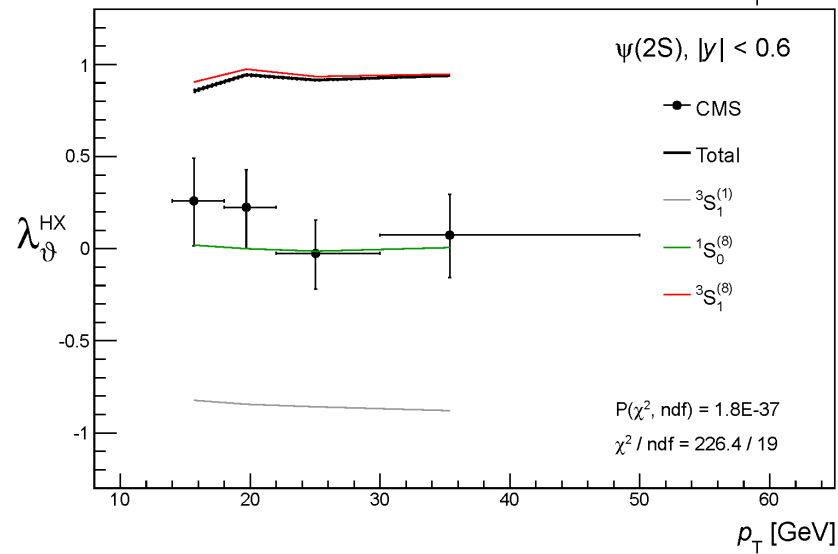
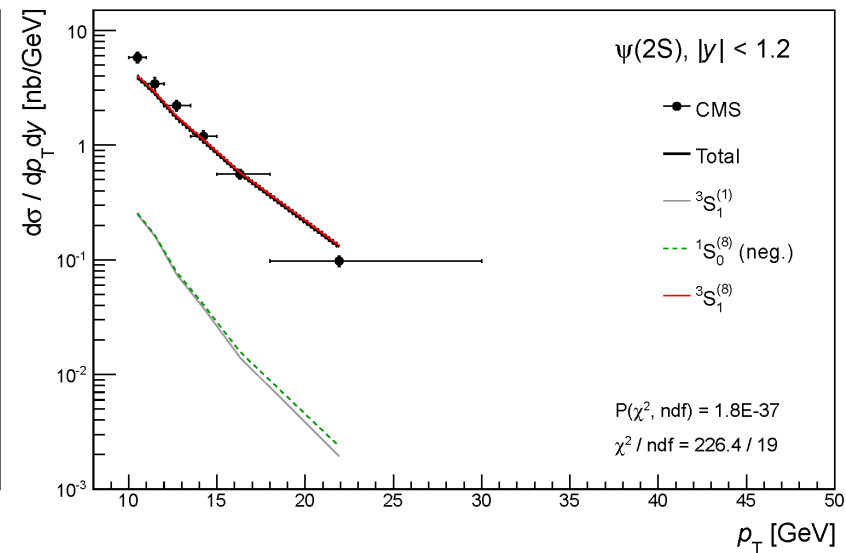
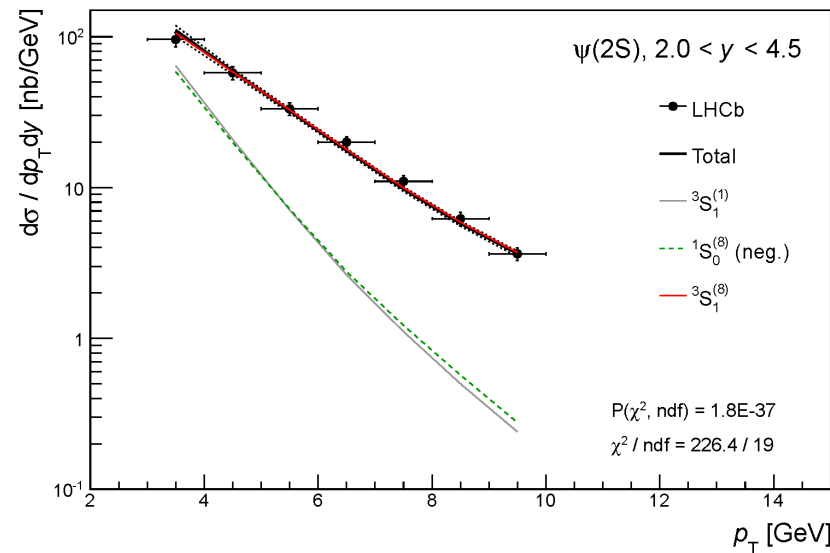
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Illustration of a $\psi(2S)$ fit, starting from 3 GeV

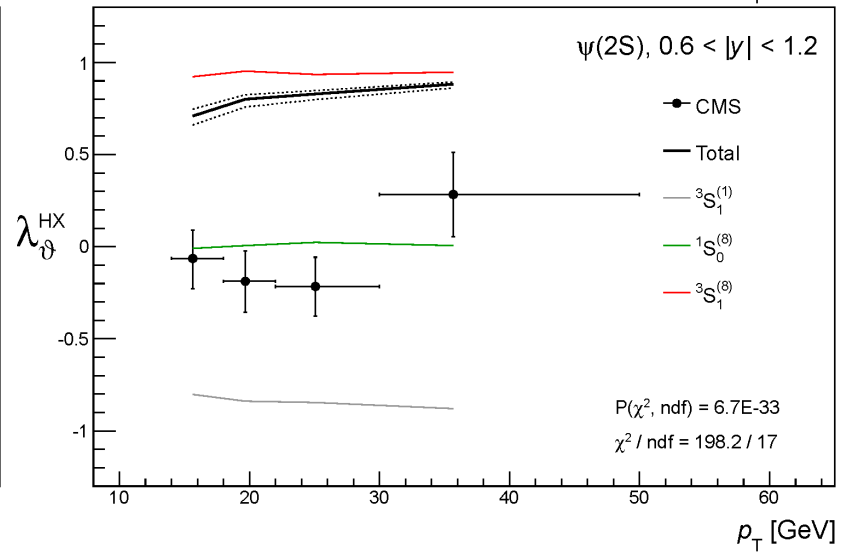
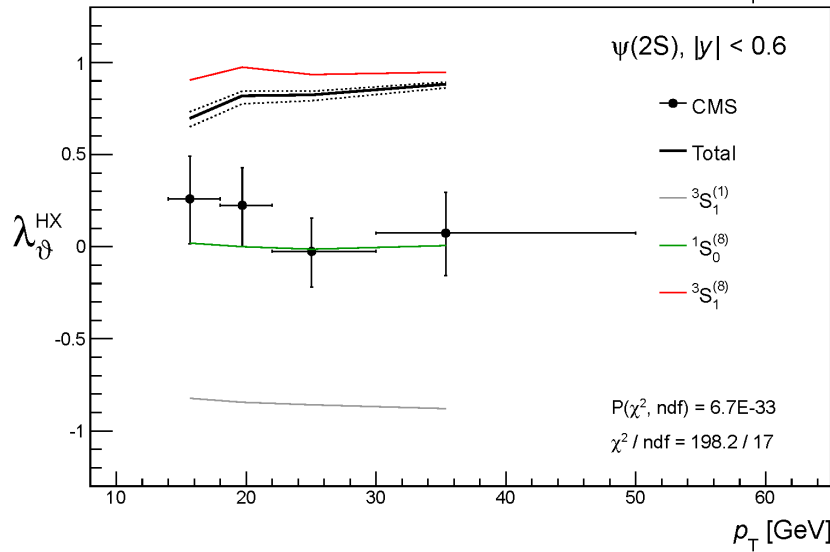
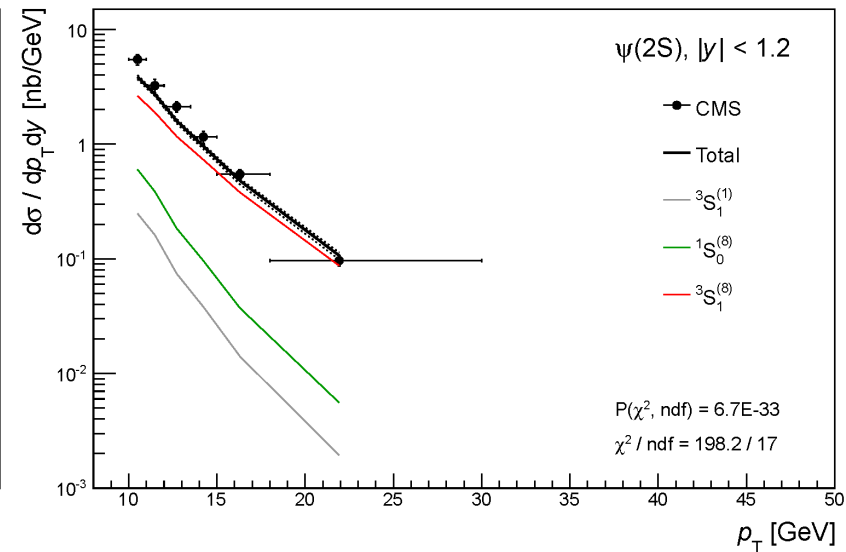
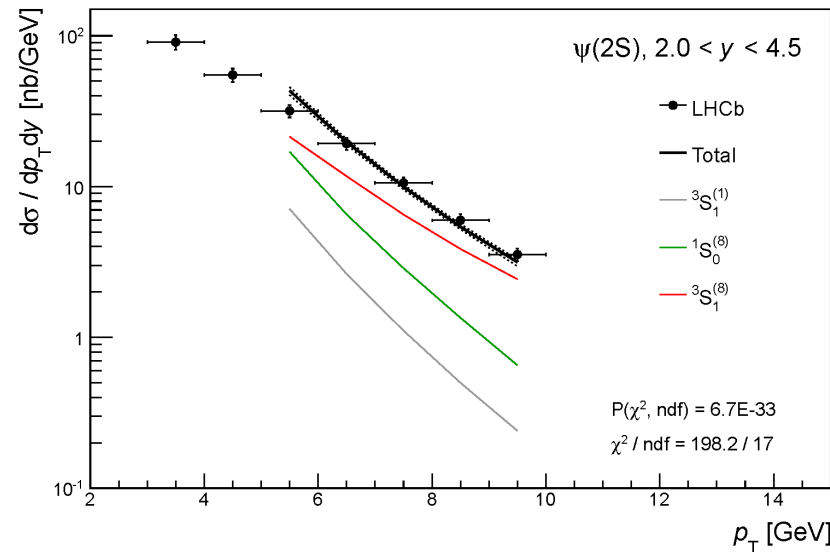


Polarization data
included in fit!

- $^3S_1^{[8]}$ dominates
- $^1S_0^{[8]}$ small and *negative*

$P(\chi^2)$ **1.8E-37**

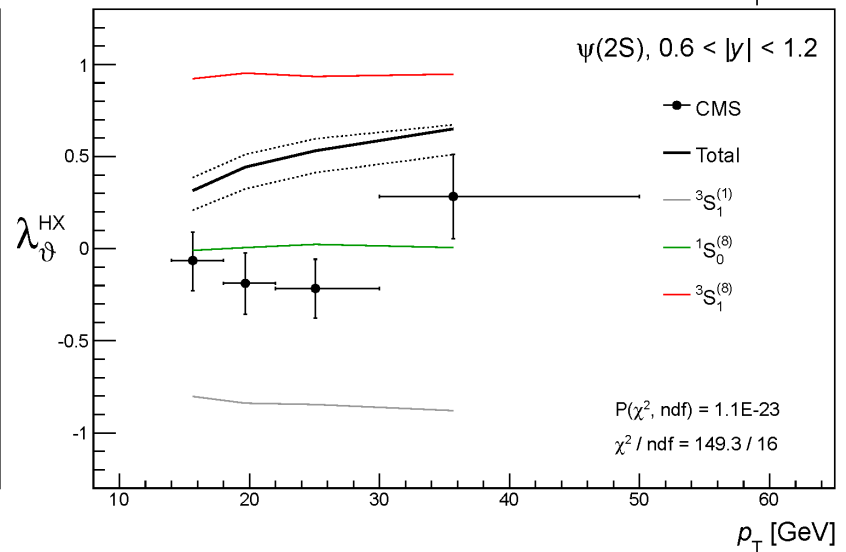
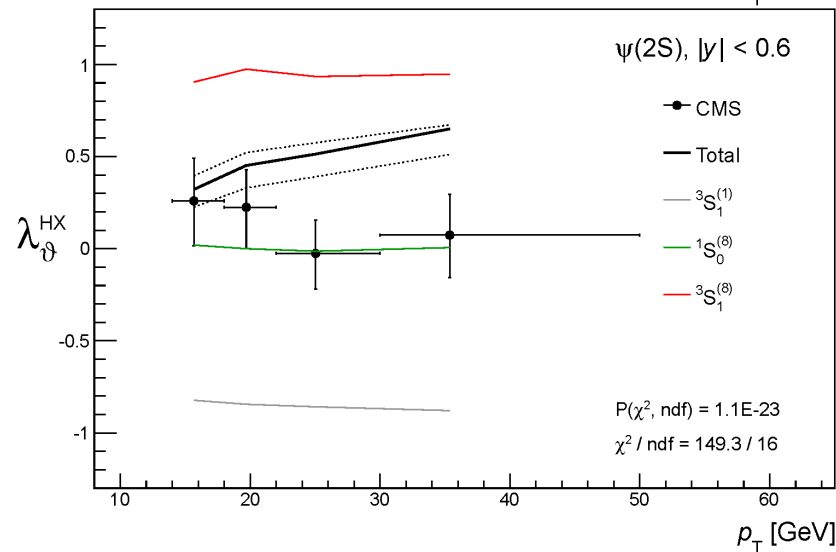
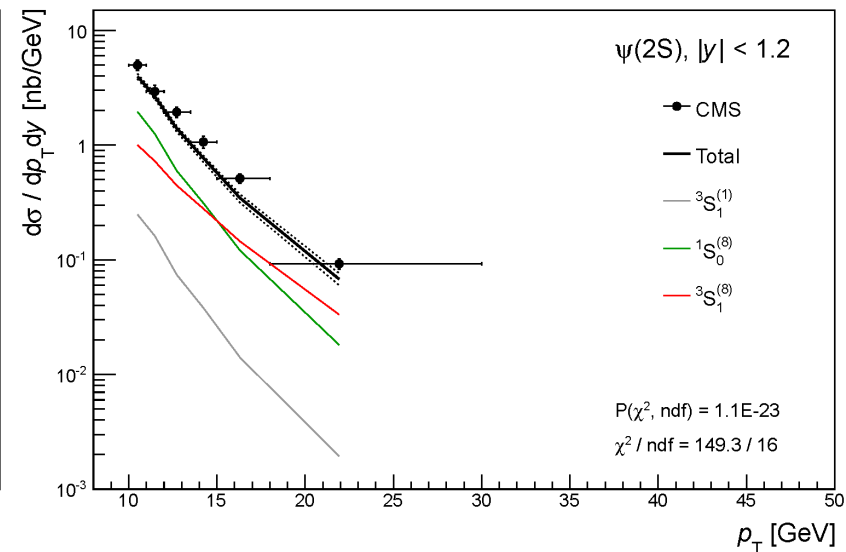
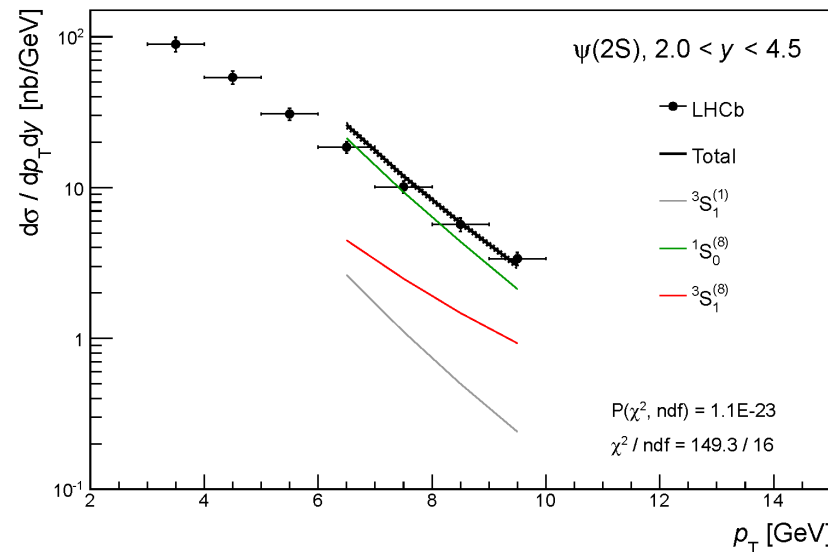
Illustration of a $\psi(2S)$ fit, starting from 5 GeV



- $^3S_1^{[8]}$ loses importance
- $^1S_0^{[8]}$ becomes positive

$P(\chi^2)$ **0.67E-32**

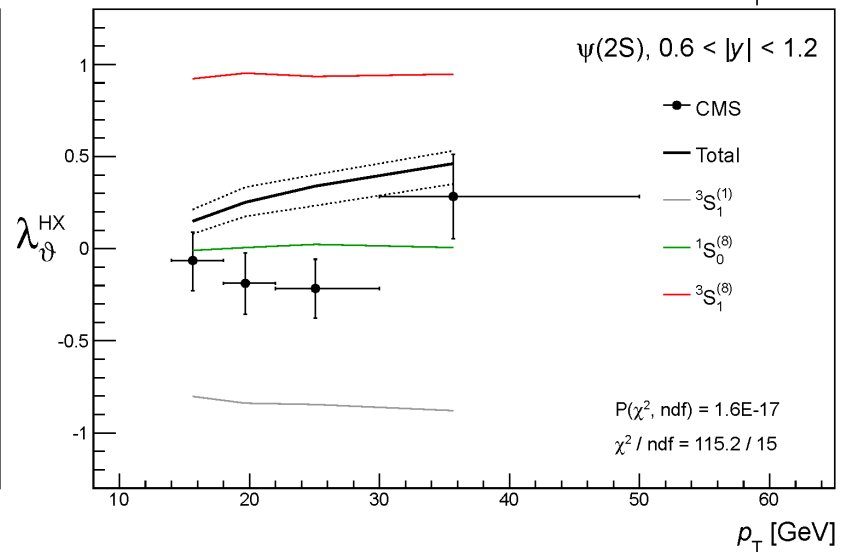
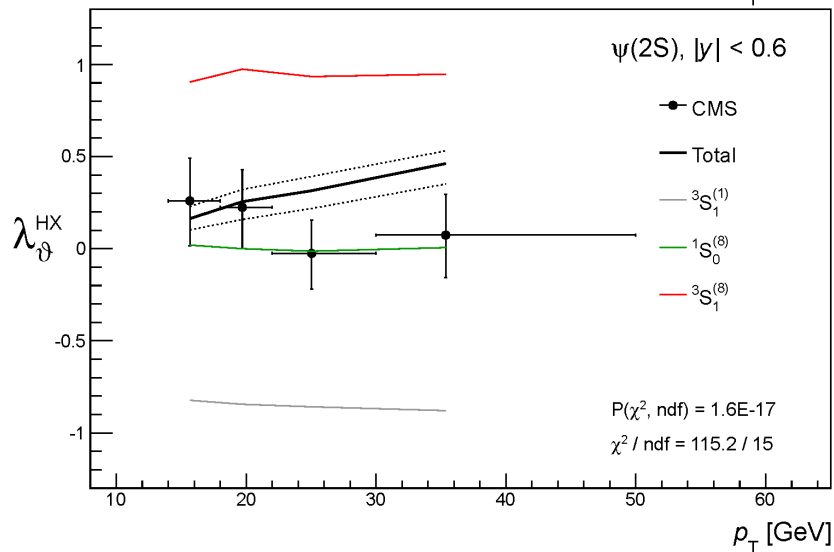
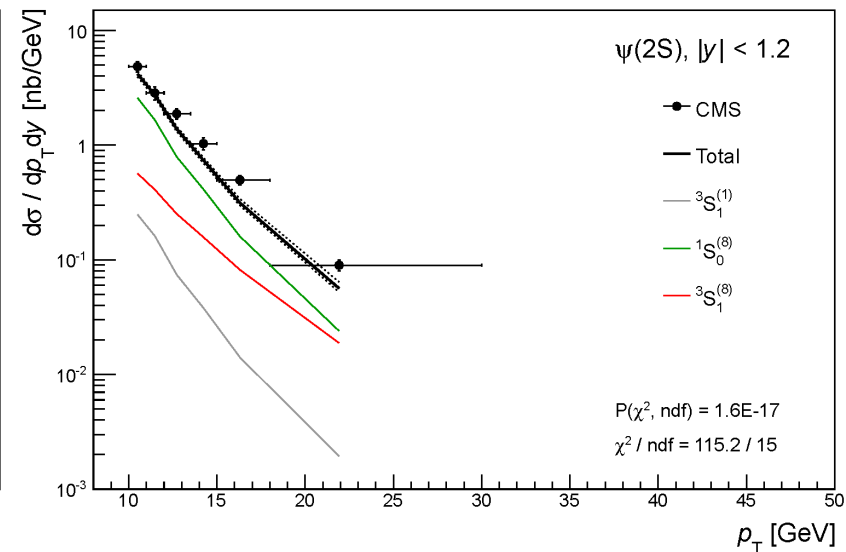
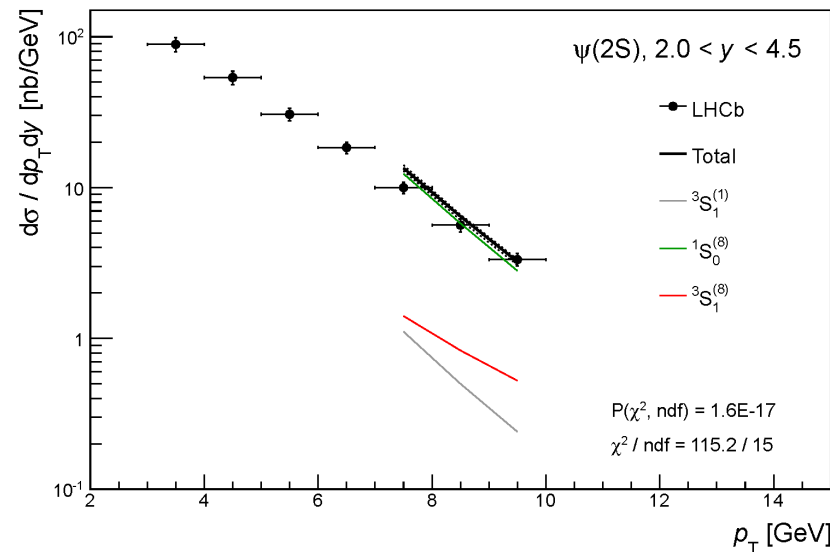
Illustration of a $\psi(2S)$ fit, starting from 6 GeV



- $^3S_1^{(8)}$ and $^1S_0^{(8)}$ start exchanging their roles
- polarization is decreasing

$P(\chi^2)$ **1.1E-23**

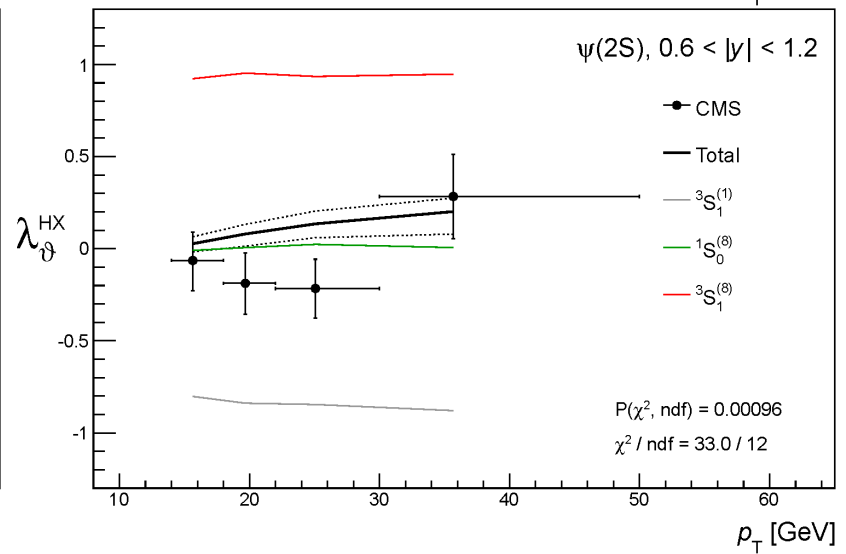
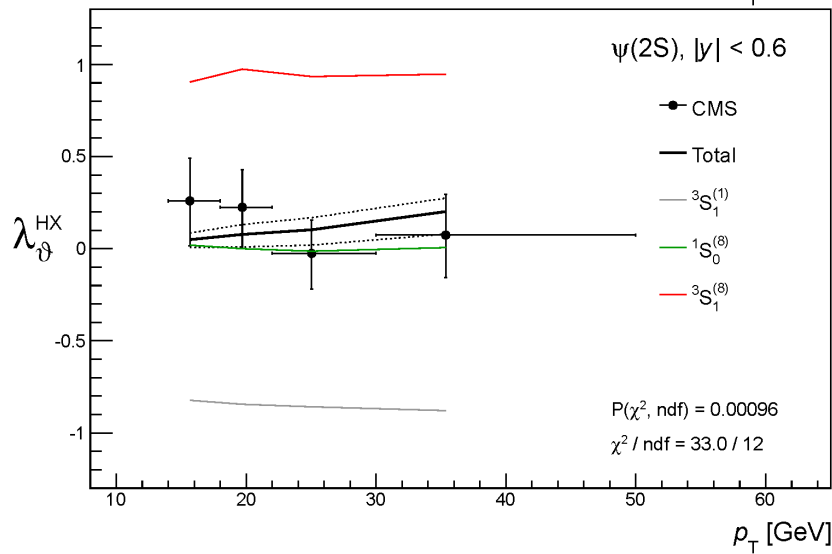
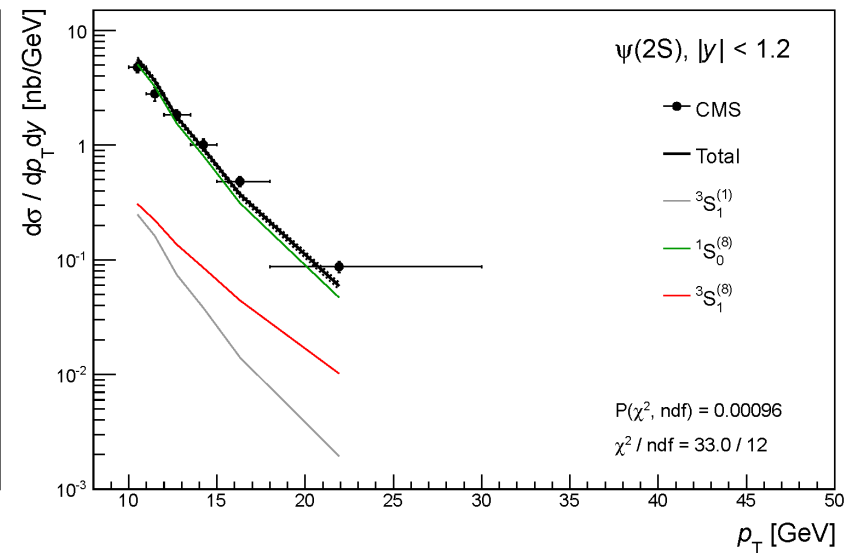
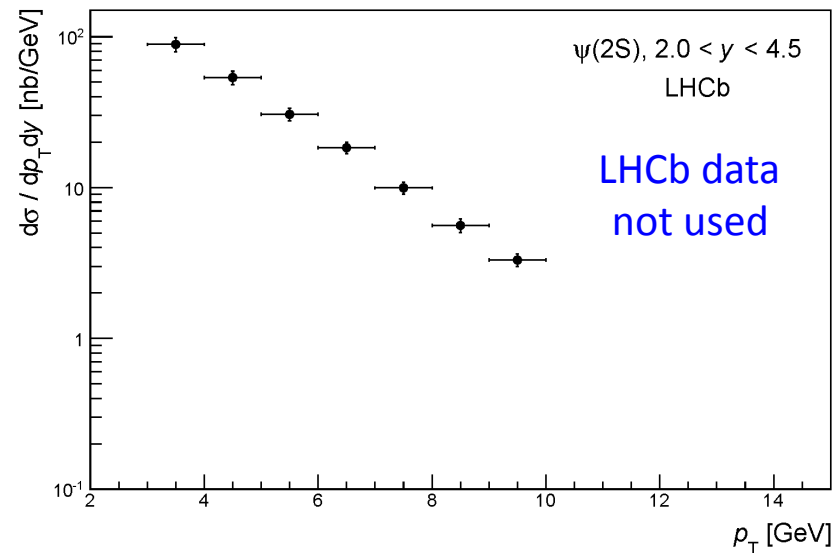
Illustration of a $\psi(2S)$ fit, starting from 7 GeV



- $^1S_0^{(8)}$ is now the main contribution
- polarization gets closer to the data

$P(\chi^2)$ **1.6E-17**

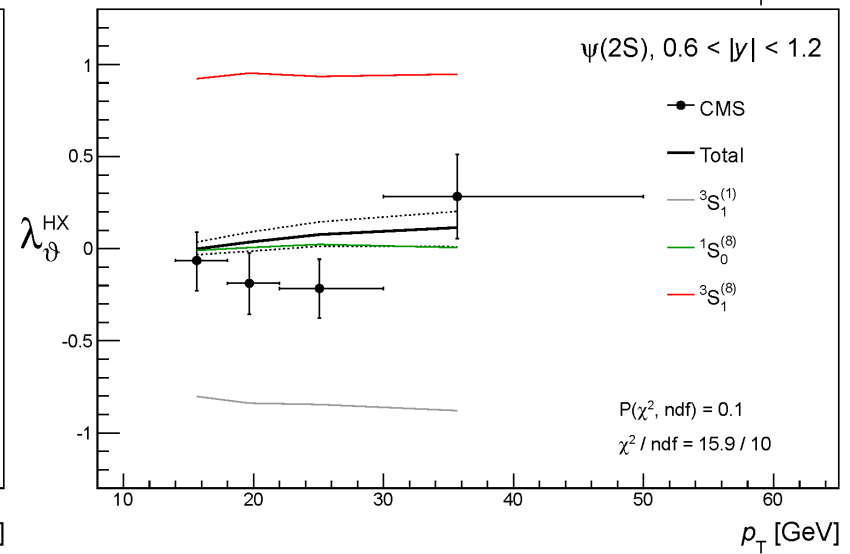
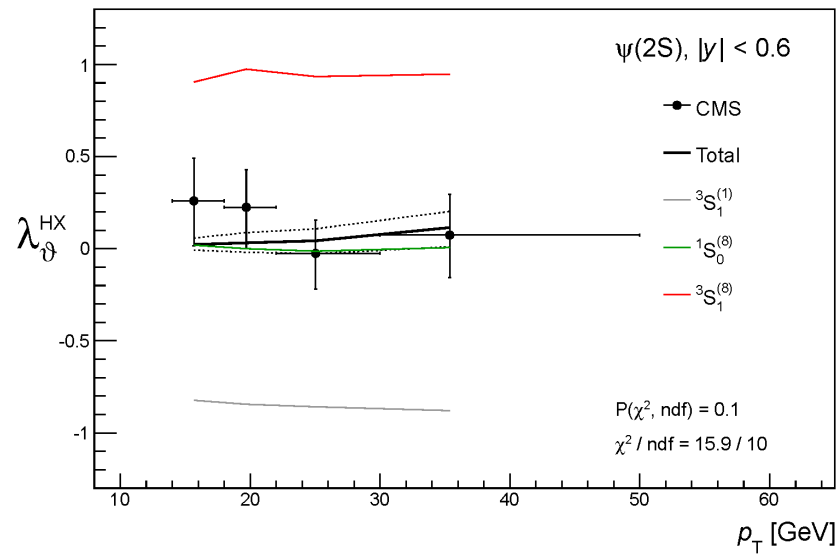
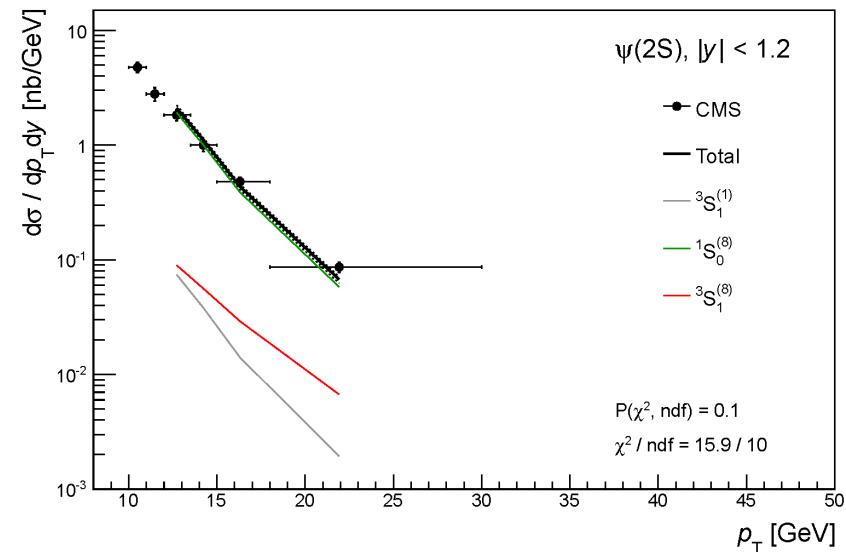
Illustration of a $\psi(2S)$ fit, starting from 10 GeV



- $^1S_0^{(8)}$ dominates
- polarization agrees with the data

$P(\chi^2)$ **0.96E-3**

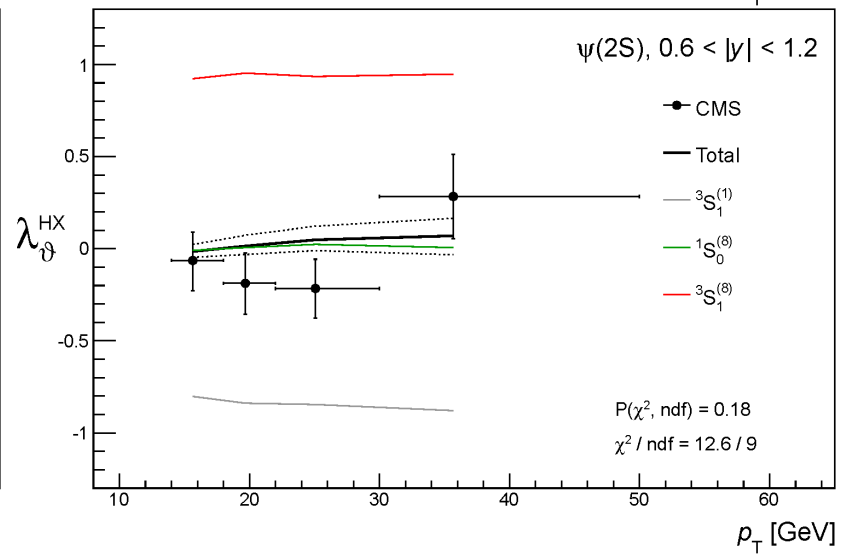
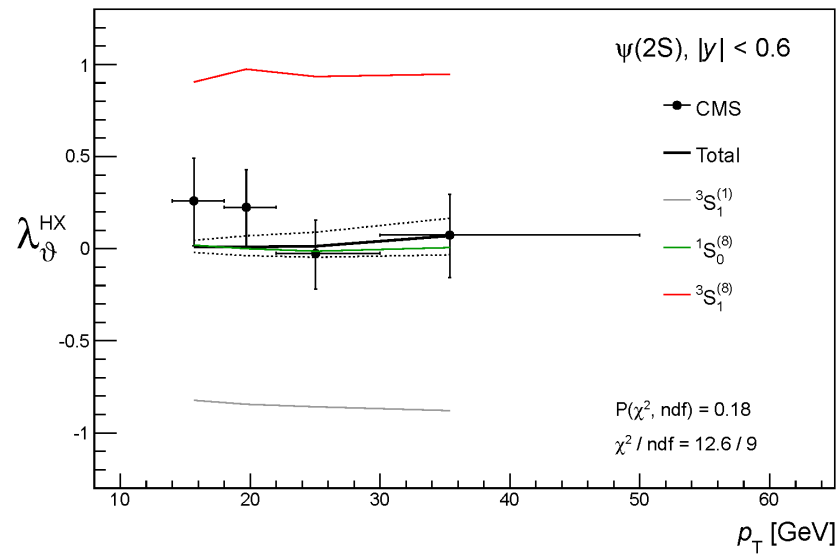
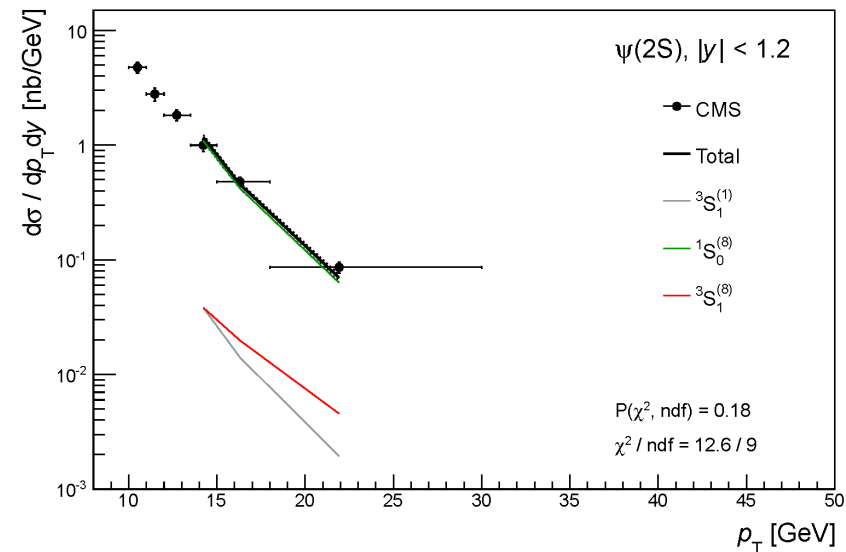
Illustration of a $\psi(2S)$ fit, starting from 12 GeV



- $^1S_0^{(8)}$ dominates
- polarization agrees with the data

$P(\chi^2)$ 10%

Illustration of a $\psi(2S)$ fit, starting from 13 GeV



- $^1S_0^{(8)}$ dominates
- polarization agrees with the data

$P(\chi^2)$ **18%**

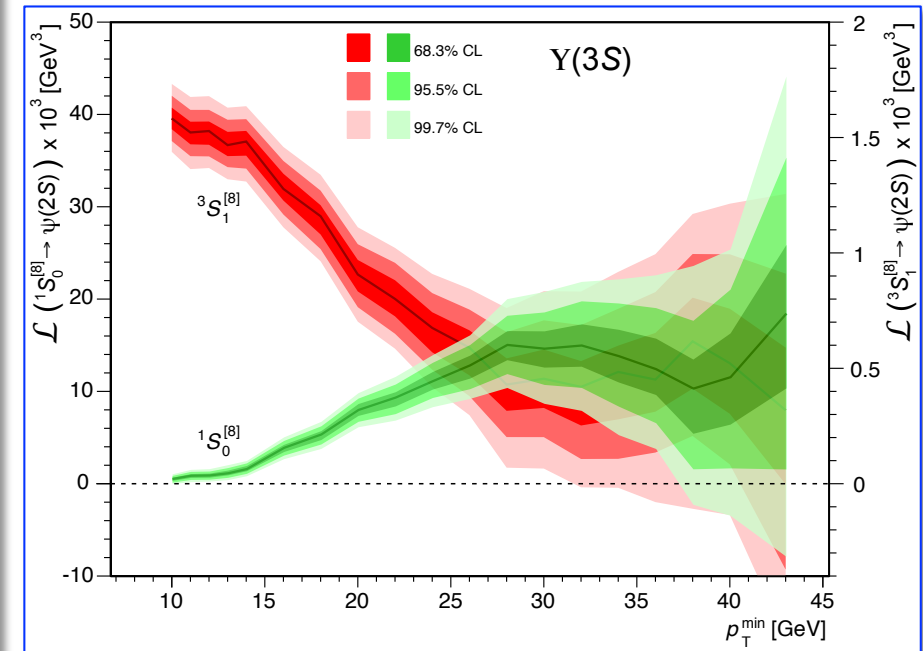
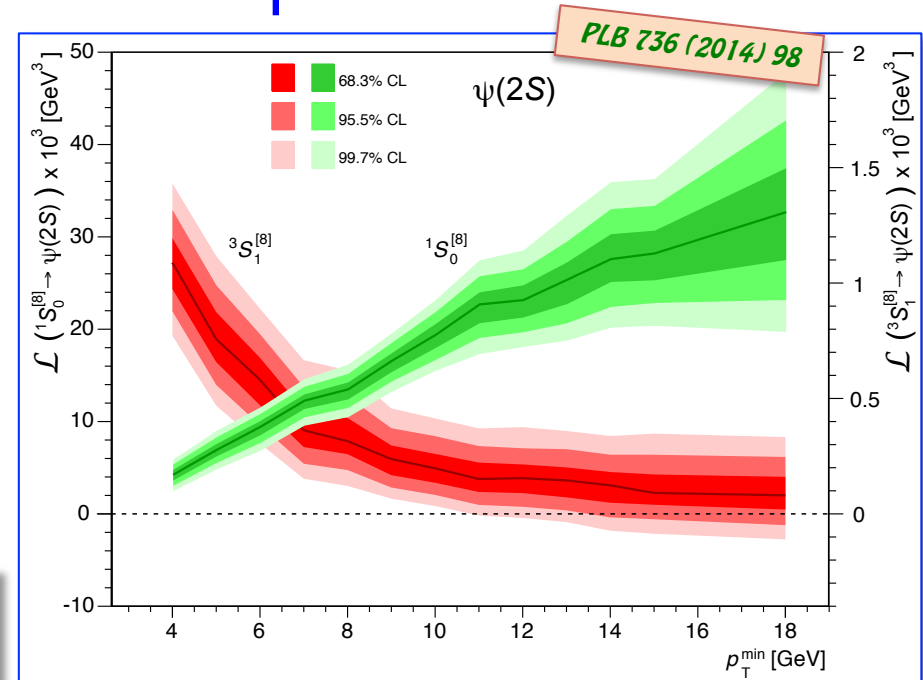
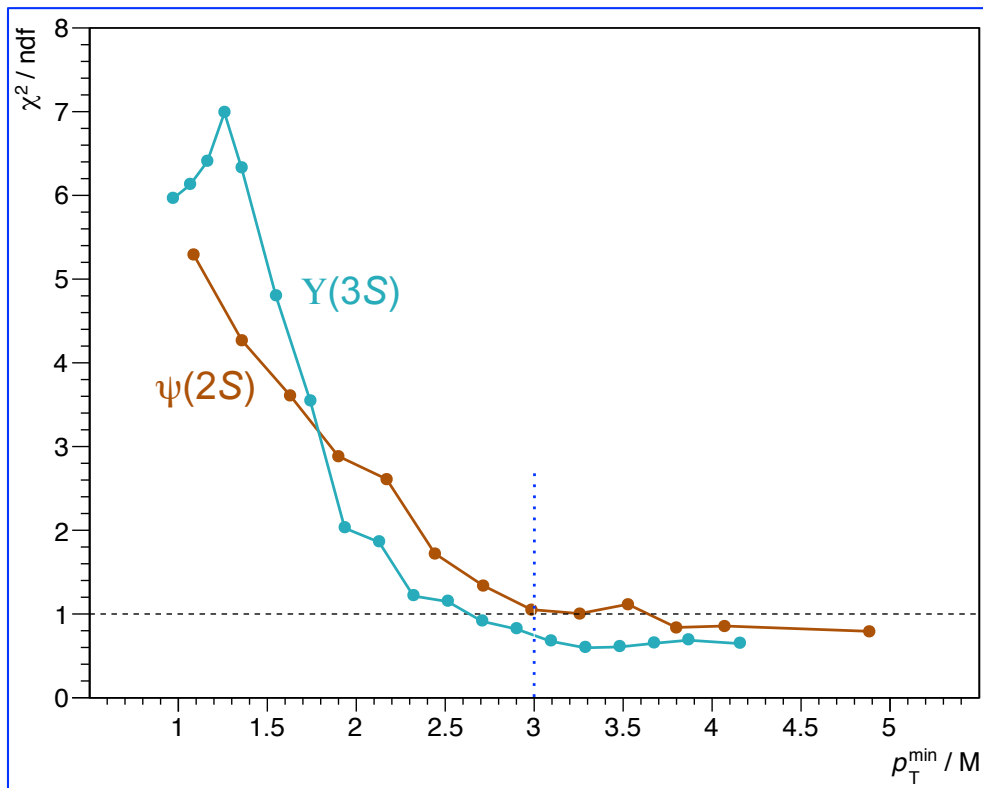
All data are equal but some are more equal than others

Fit quality improves drastically when removing low p_T/M data

For $p_T/M > 3$ the fit gets stable

Good description of $\psi(2S)$ and $Y(3S)$ cross sections and polarizations

And the $^3S_1^{[8]}$ dominance turns into $^1S_0^{[8]}$ dominance



The end of the quarkonium polarization puzzle

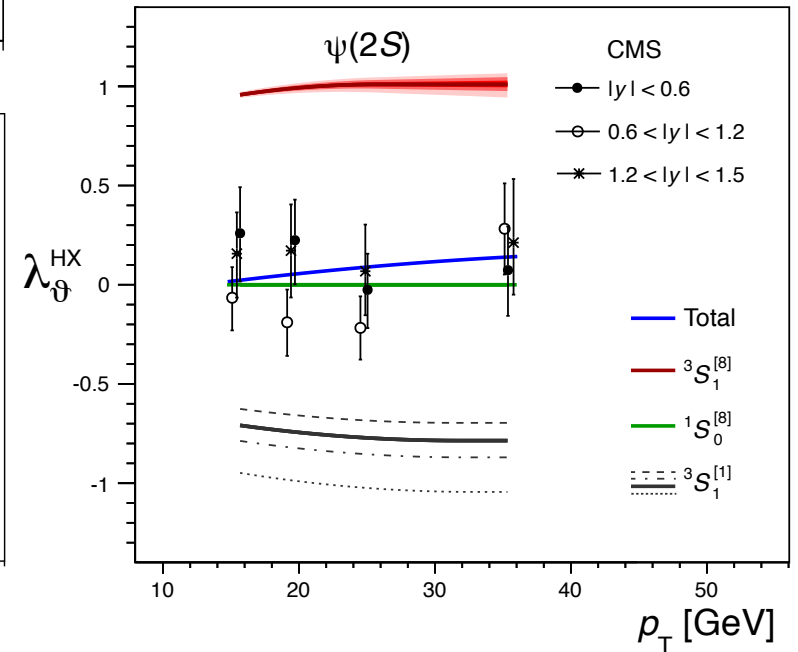
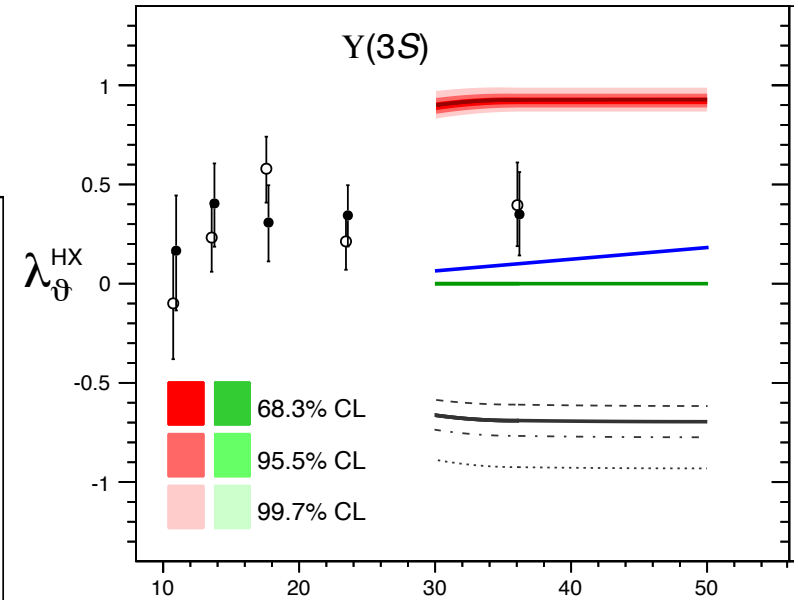
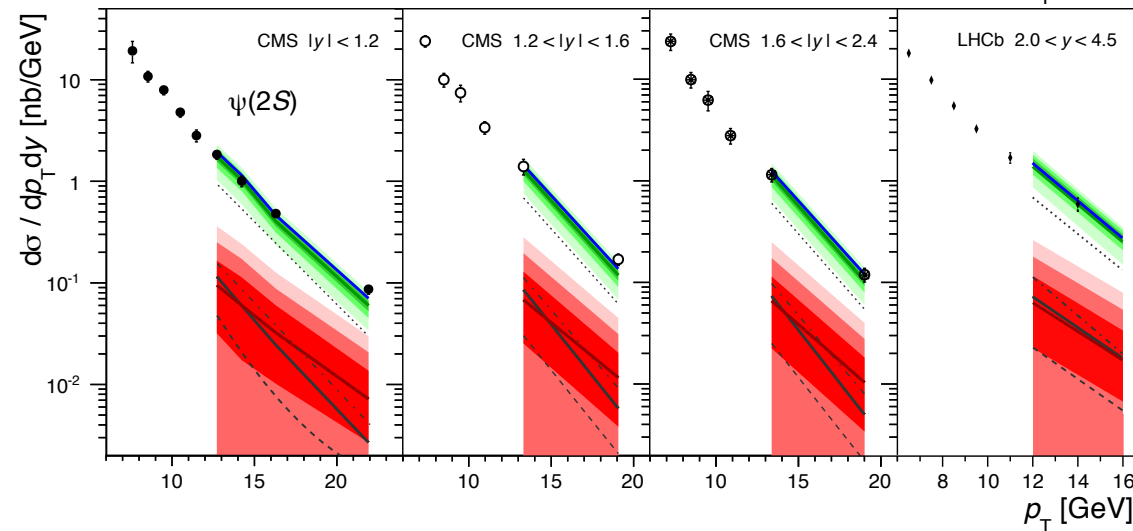
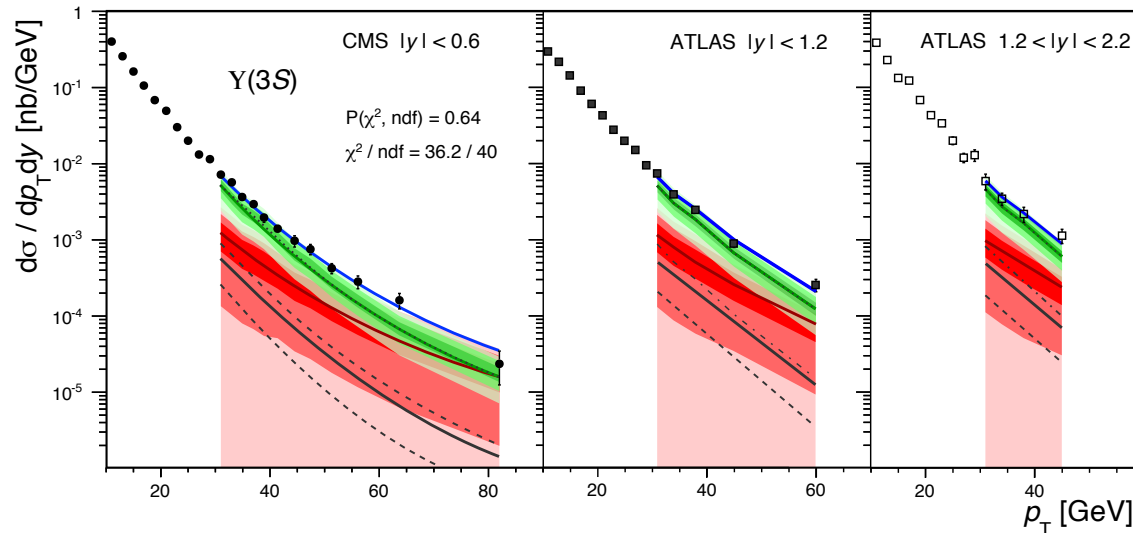
Moving polarization data to the center of the study

Looking only inside the NRQCD domain of validity

→ Color channels $^3S_1^{[1]}$, $^3S_1^{[8]}$ and $^3P_J^{[8]}$ are negligible

→ The $^1S_0^{[8]}$ octet dominates

PLB 736 (2014) 98

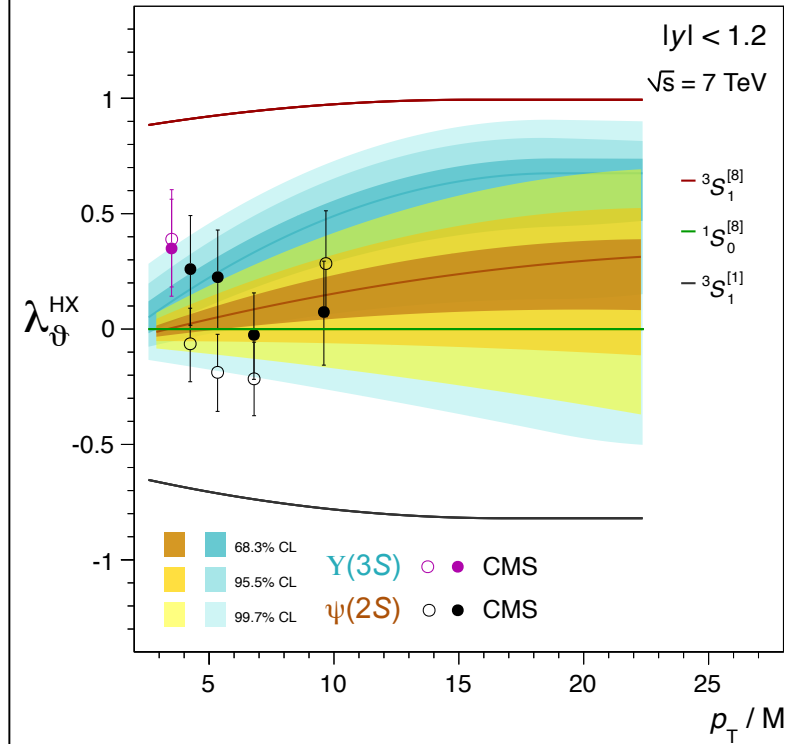
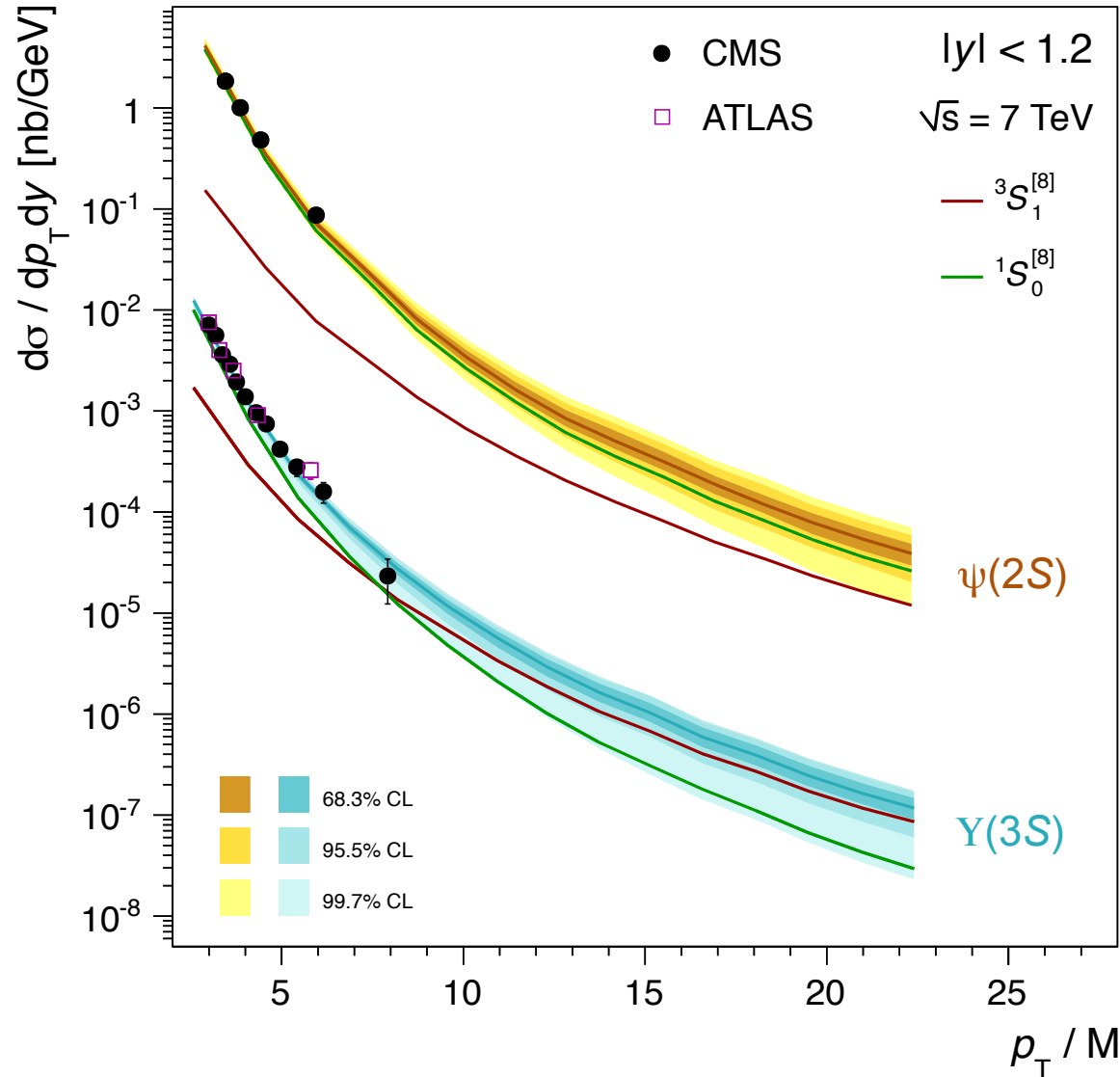


What happens at higher p_T ?

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The $^3S_1^{[8]}$ term could be dominant at higher p_T/M values than currently covered

At very high p_T , $Y(3S)$ should tend to be transversely polarized (**but:** neglected the $\chi_b(3P)$ decays...)



Measurements with 2012 CMS data will vastly improve the accuracy of the results and extend their p_T reach

An unexpected hierarchy

According to NRQCD velocity-scaling rules, LDMEs are of similar magnitude:

$$^1S_0 \approx ^3S_1 \approx ^3P_J$$

It is remarkable to see that the $^3S_1^{[8]}$ LDME is less than 6% of the $^1S_0^{[8]}$ LDME, at 95% CL

The $^3S_1^{[8]}$ transition is practically forbidden

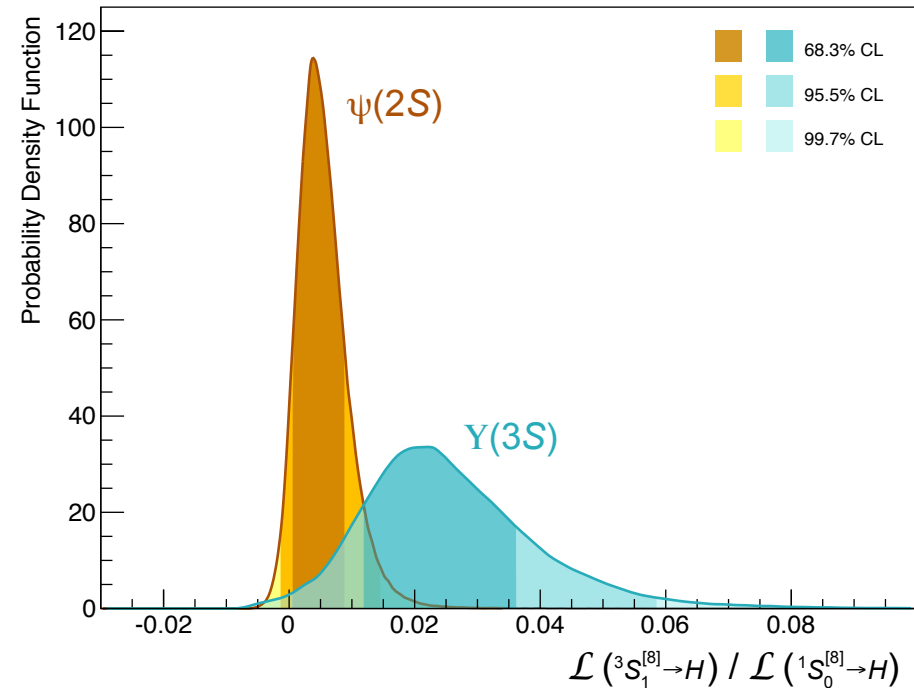
Cross check: fits including the $^3P_J^{[8]}$ octet \rightarrow Small (and *negative*...) contribution:
 \rightarrow the fit quality is not improved, the results not affected

This analysis suggests a strong internal hierarchy between the three LDMEs, for the $\psi(2S)$ and $Y(3S)$:

$$^1S_0 \gg ^3S_1 \gg ^3P_J$$

These are non-trivial observations, important to understand how the quarks interact with each other
 \rightarrow the QQbar bound states are preferably formed from two quarks of:

- 1) different colours (rather than in an already neutral configuration)
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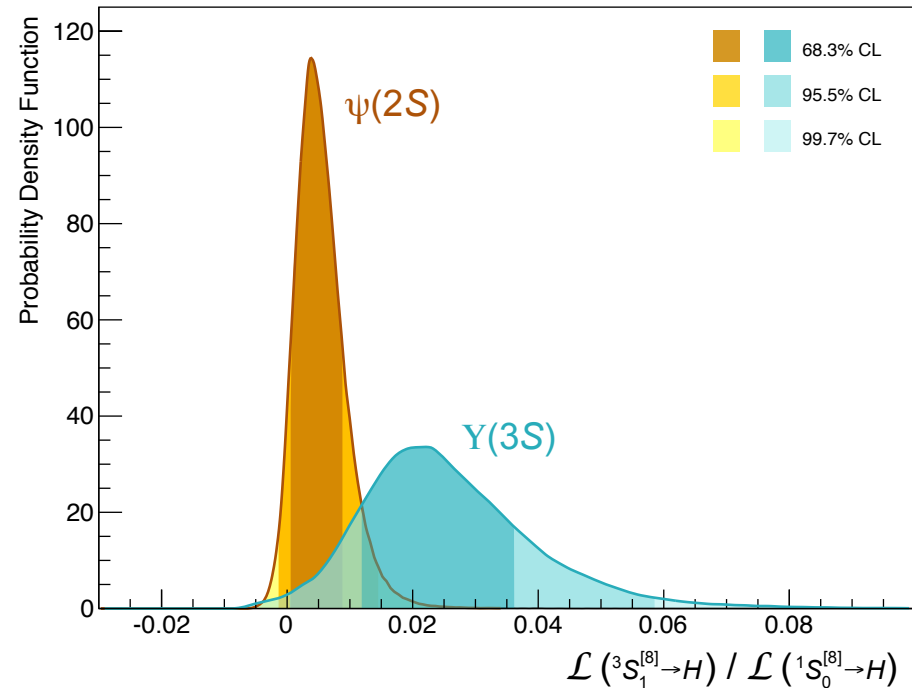
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All quarkonia are equal (?)

Can we generalize these findings?
 Are these new hierarchies valid for all quarkonia?
 ...and even for **hadrons in general**?

The $\psi(2S)$ and $\Upsilon(3S)$ LDMEs are independent free parameters in the fit
 → Consistent with being identical:

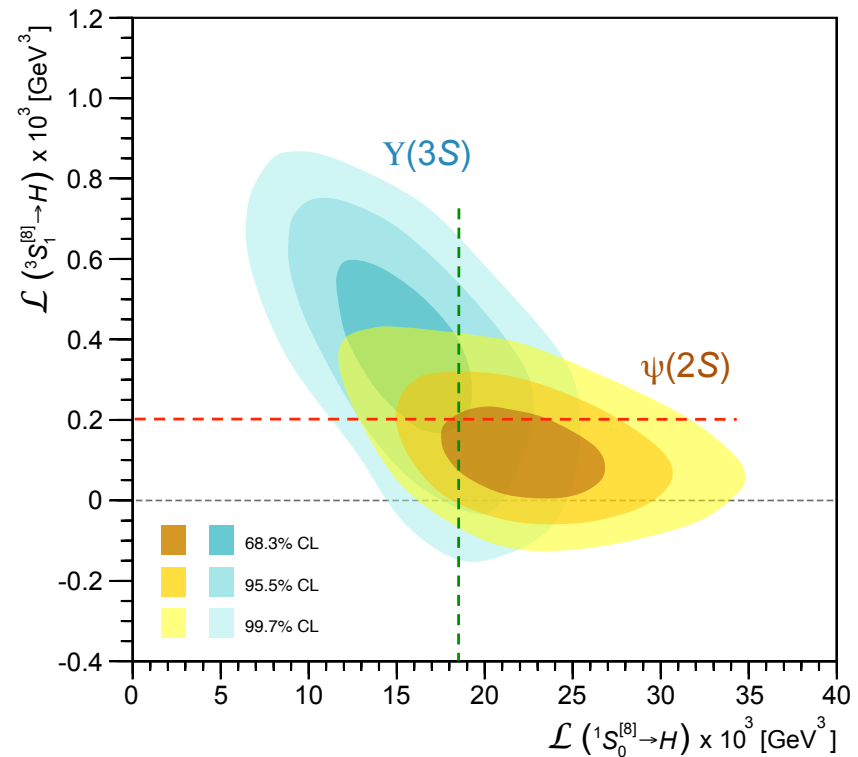
$$\begin{aligned} \mathcal{O}(^1S_0^{[8]}, \psi(2S)) &= \mathcal{O}(^1S_0^{[8]}, \Upsilon(3S)) = 0.0185 \text{ GeV}^3 \\ \mathcal{O}(^3S_1^{[8]}, \psi(2S)) &= \mathcal{O}(^3S_1^{[8]}, \Upsilon(3S)) = 0.0020 \text{ GeV}^3 \end{aligned}$$

Analysis work in progress:

“All-charmonium” global phenomenological interpretation

- Simultaneous fit of all LHC data of J/ψ , $\psi(2S)$, χ_{c1} , χ_{c2} cross sections and polarizations
- Including all feed-down cascades
- LDMEs of P-wave states + direct J/ψ
- Missing experimental input: χ_{cJ} polarizations

“All-bottomonium” global analysis requires more data from the LHC: $\chi_{bJ}(nP)$

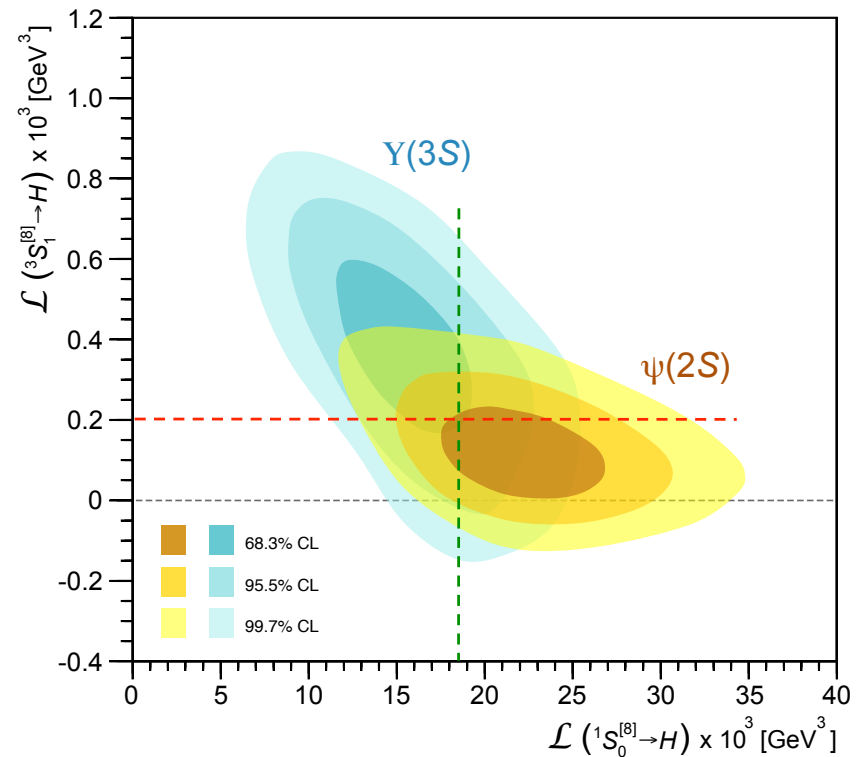


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Tevatron-era: Quarkonium production through color singlet: not enough! Color octet required

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\rightarrow **Cross section data:** p_T/M - scaling, „all quarkonia produced similarly, through one CO process“

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Same pQCD inputs \rightarrow very different results!

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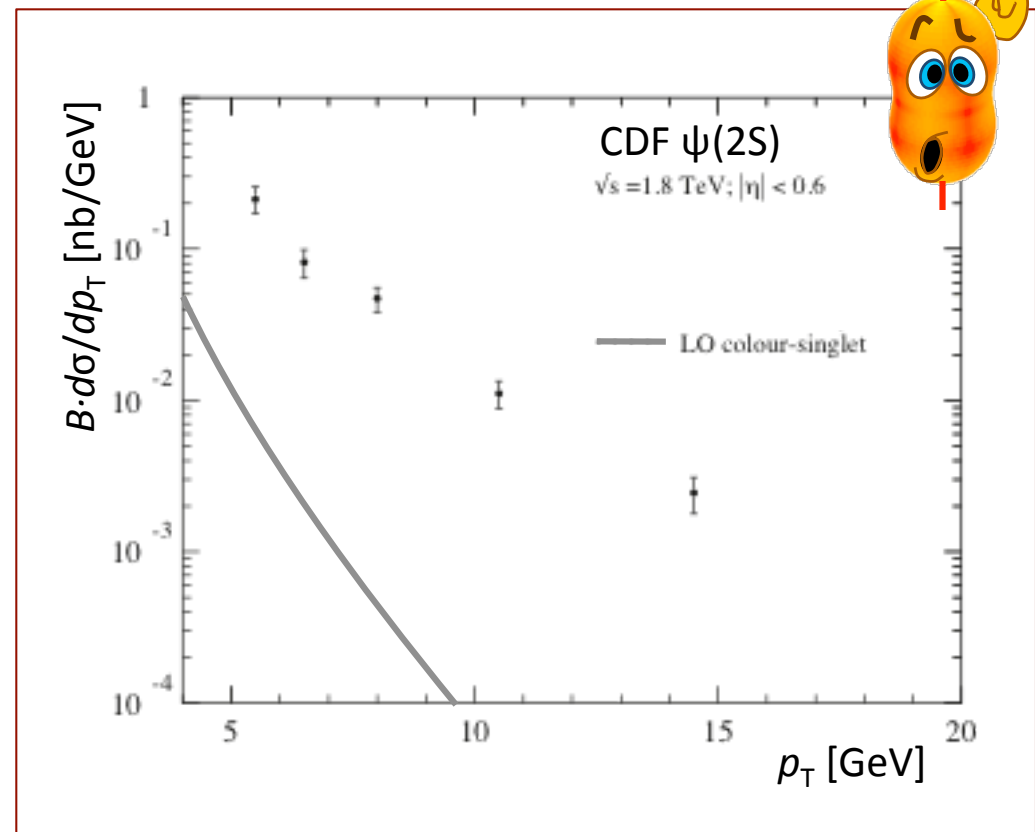
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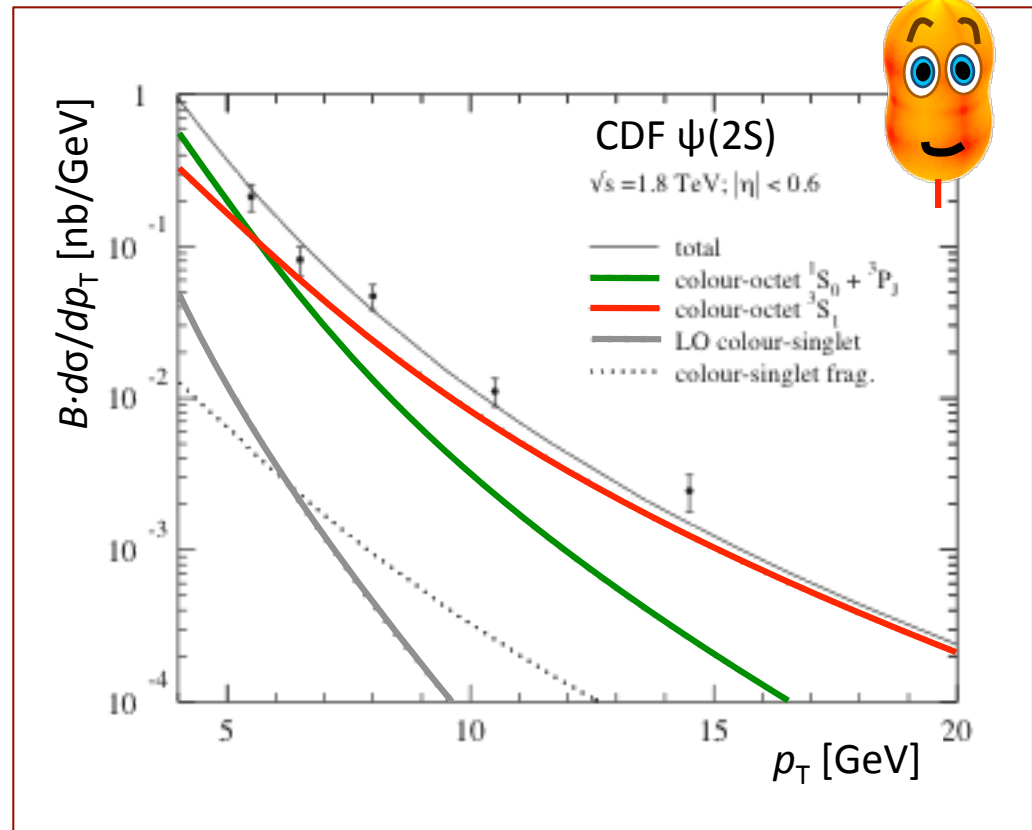
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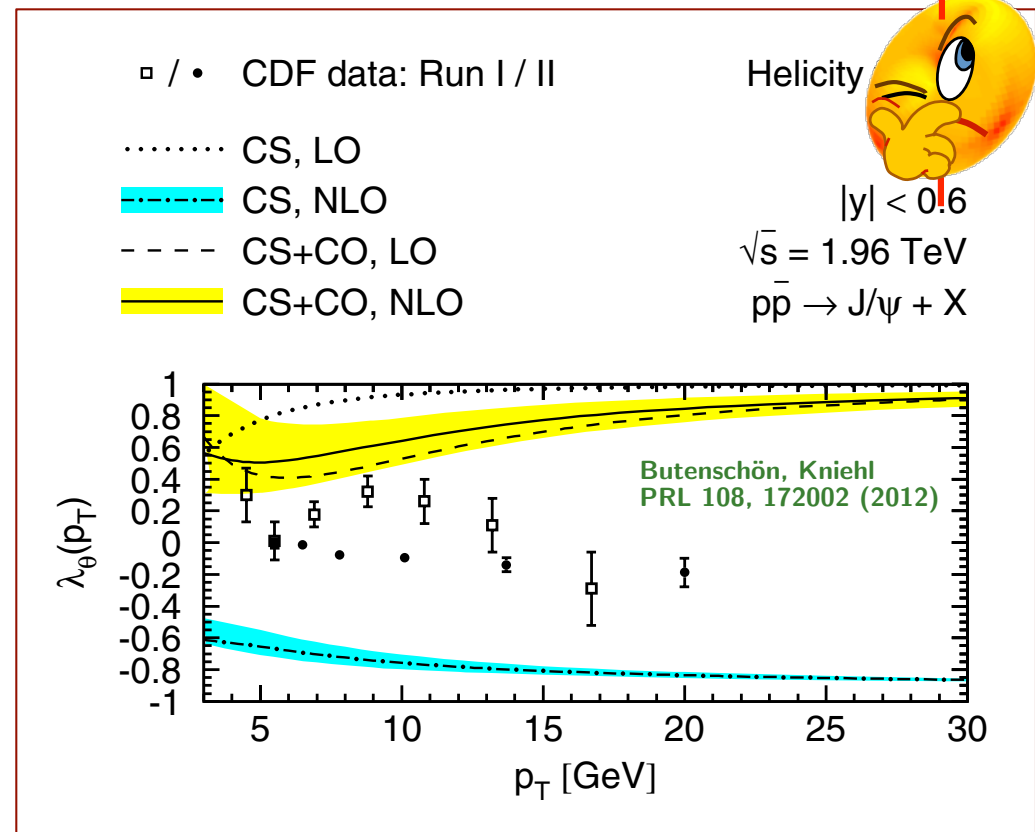
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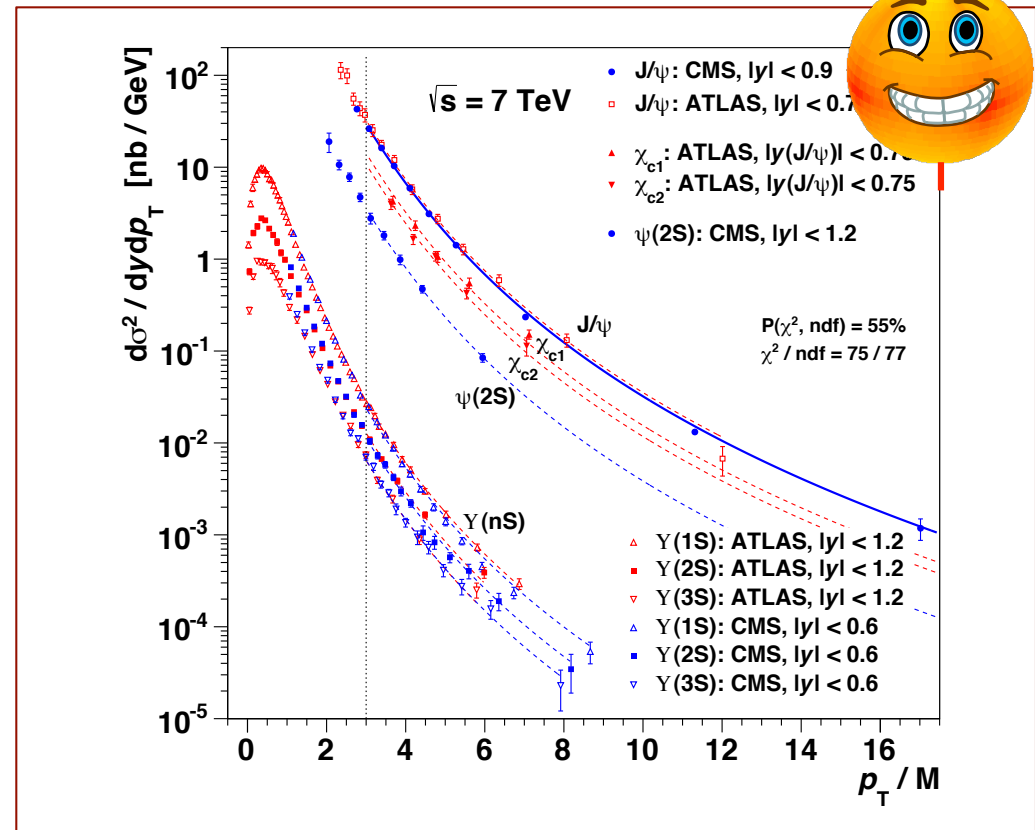
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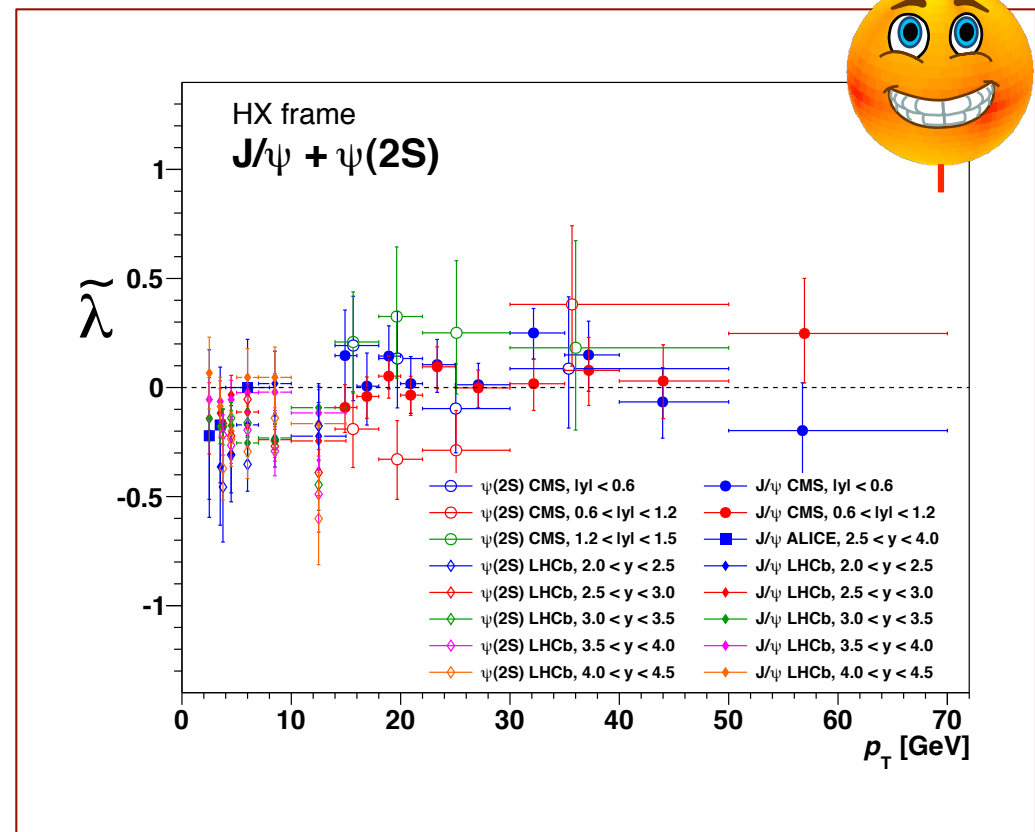
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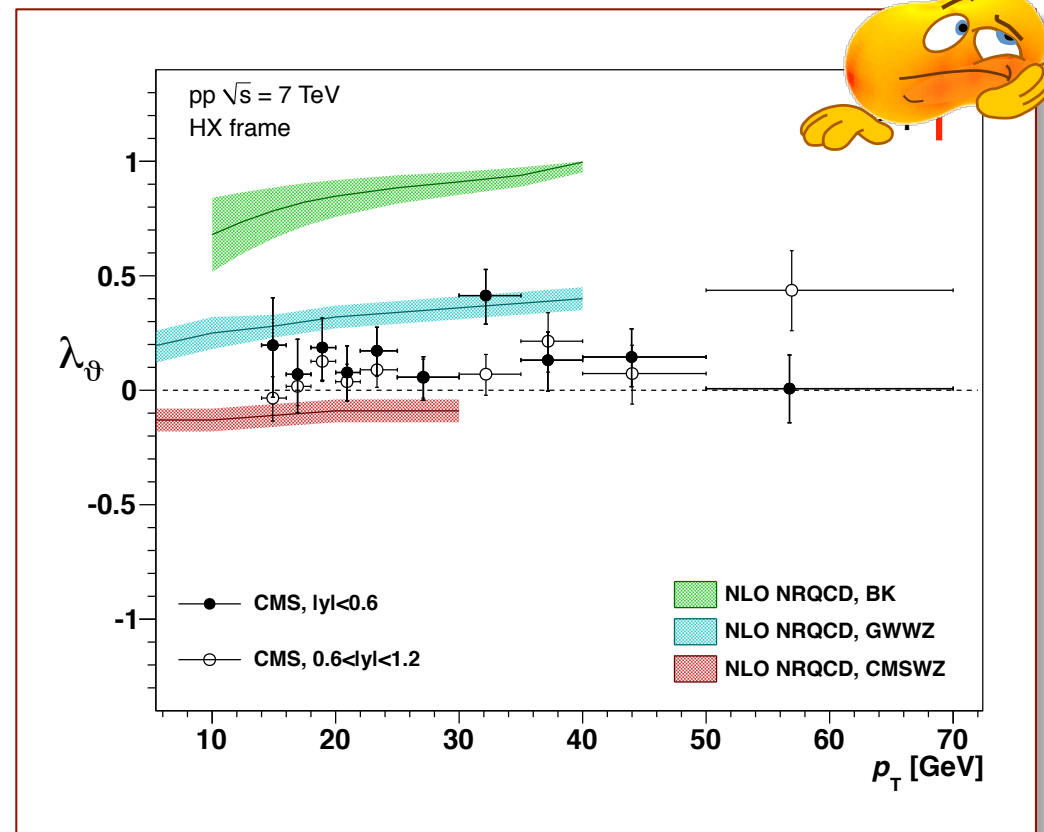
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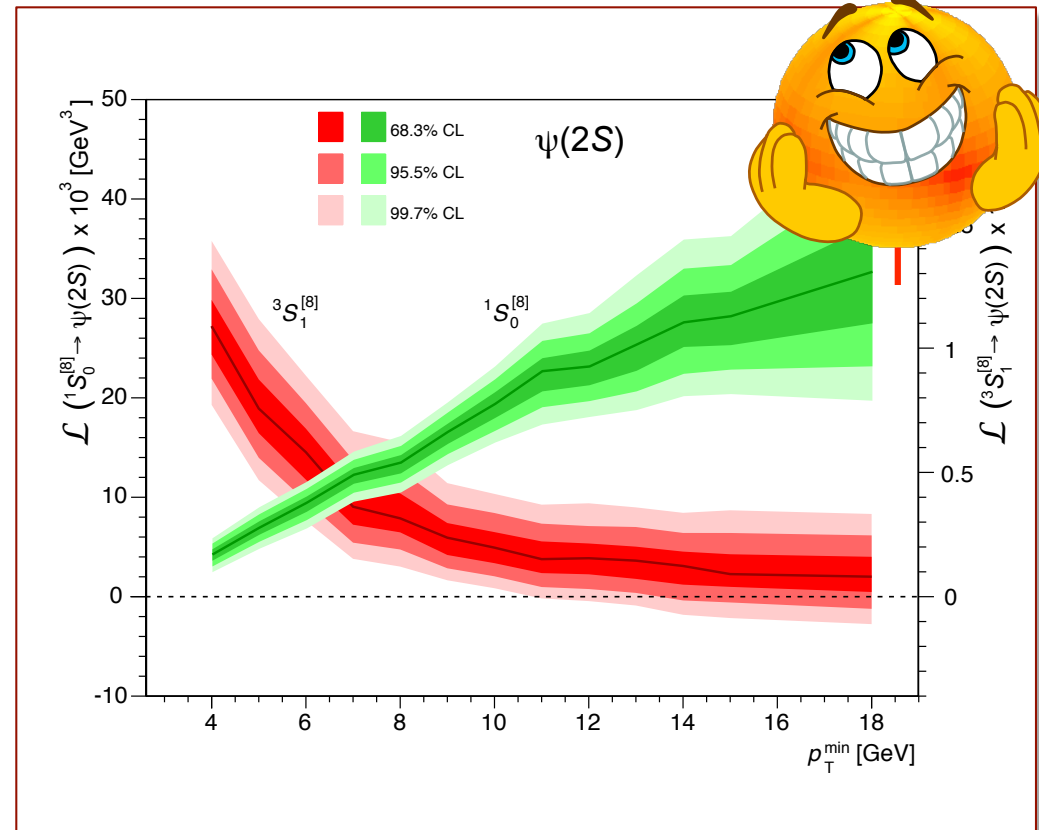
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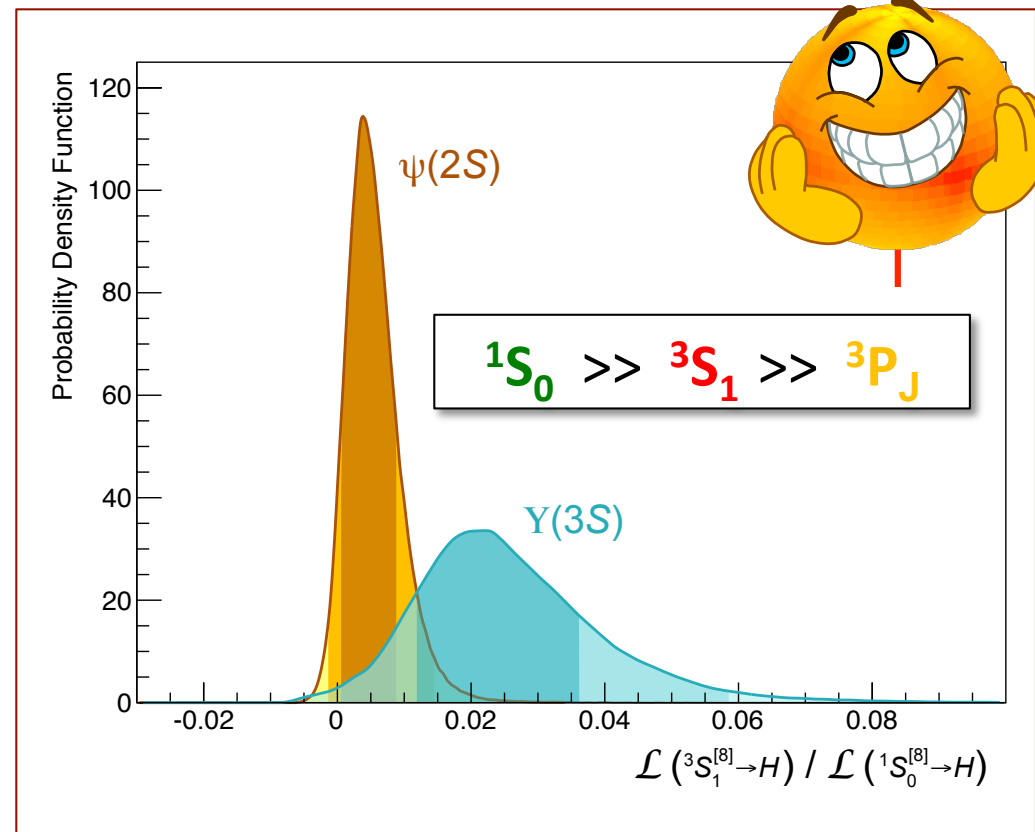
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What's next?

New hierarchies found in $\psi(2S)$ and $\Upsilon(3S)$ data have to be tested on other states (S- and P-wave)

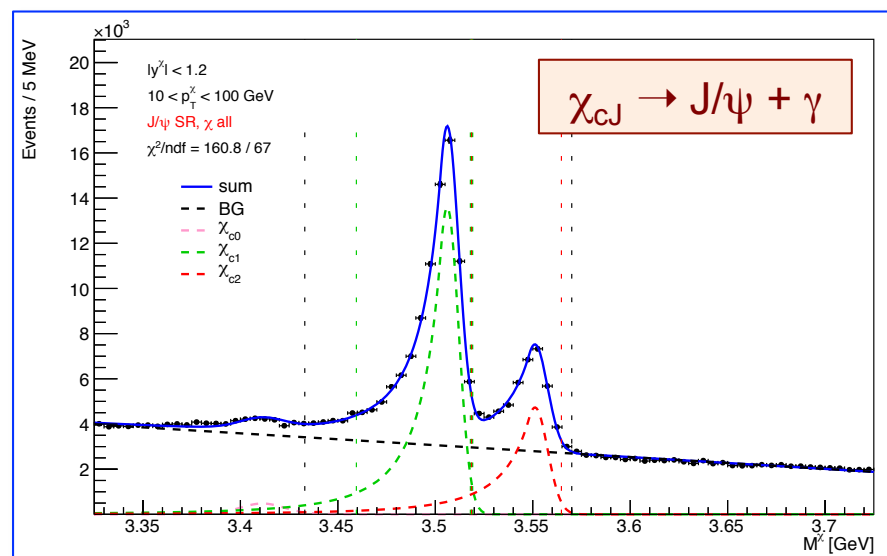
Essential measurements to study charmonium LDMEs

- J/ψ and $\psi(2S)$ cross sections up to very high p_T : ≈ 100 GeV reached by CMS, to be published soon
- χ_{c1} and χ_{c2} polarizations to be expected from CMS in 2015, first measurement of this kind
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*The 2012 CMS dataset has not been exploited yet for quarkonium studies \rightarrow many results to be expected
LHC RunII will significantly increase the p_T reach, but more complicated analyses*



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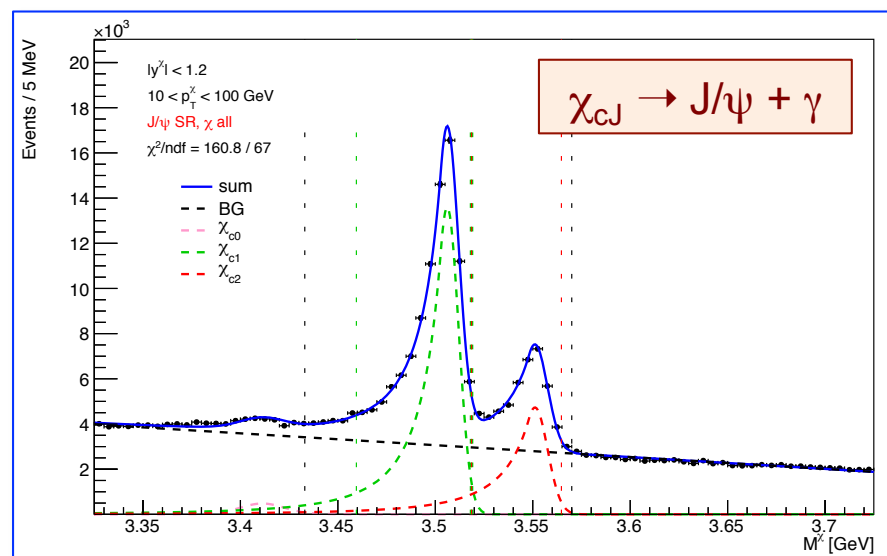
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Quarkonia at 13 TeV and beyond

Associated production: Quarkonia + VB / γ / Jets

„Orthogonal“ information to polarization and production data

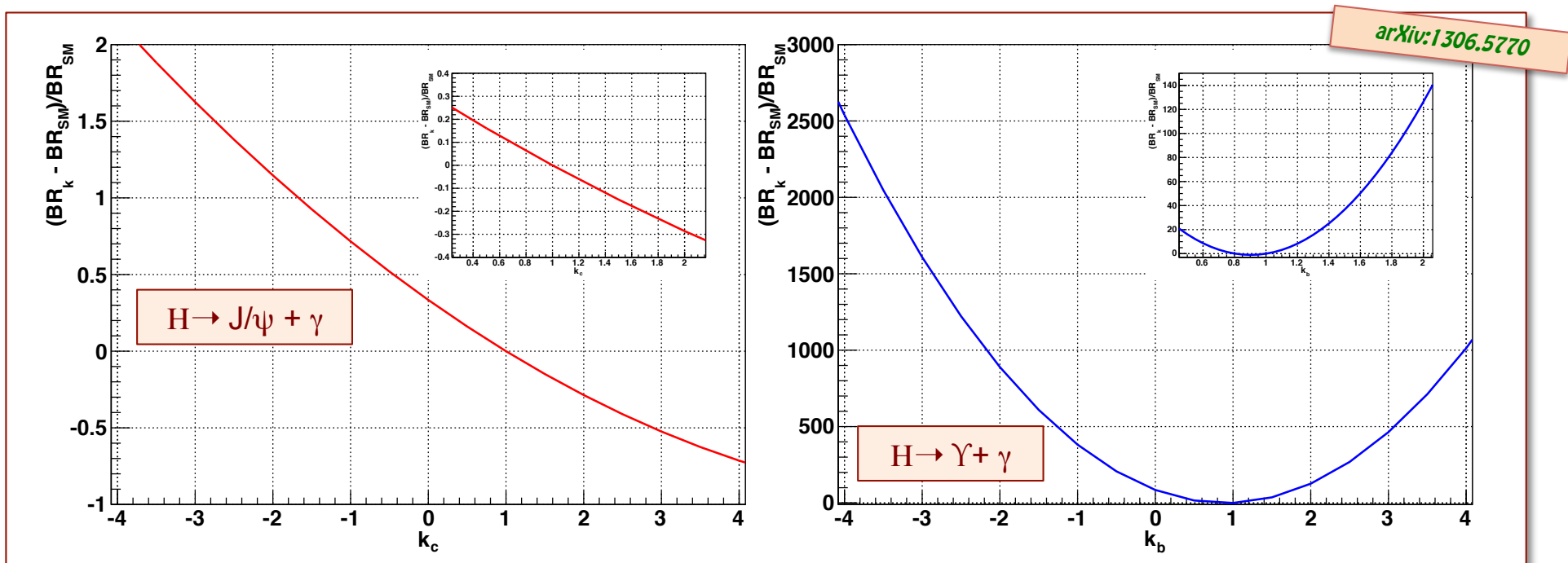
Higgs- $q\bar{q}$ couplings: The decay Higgs to quarkonium + γ is the only means to measure $Hc\bar{c}$ and $Hb\bar{b}$ couplings directly at the LHC

→ Understanding quarkonium production is essential to interpret these measurements

→ Seeing $H \rightarrow Y + \gamma$ would imply new physics: large deviations of the $Hq\bar{q}$ coupling from its SM value

LDME universality: Crucial prediction of NRQCD, cannot be tested in pp collisions

Same behavior for any $q\bar{q}$ pair, produced in pp, ee, pA, AA collisions or through Higgs decays...



Conclusion

Combination of

High quality measurements of quarkonium production at the LHC

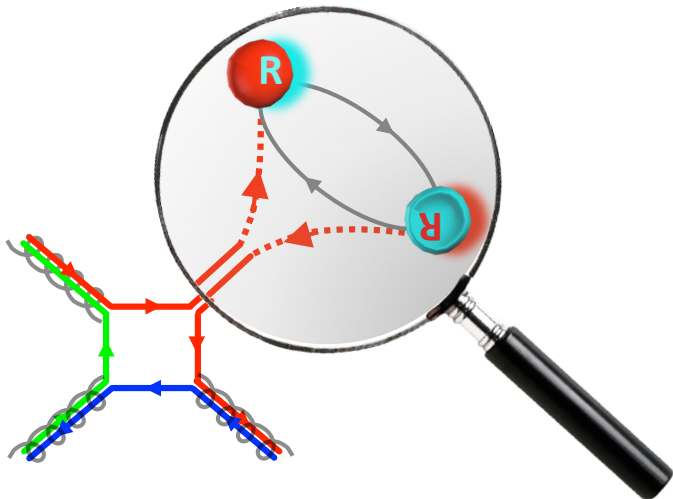
Data-driven physics interpretations of the LHC measurements

→ turn quarkonium data into **high-precision studies of (non-perturbative) QCD**

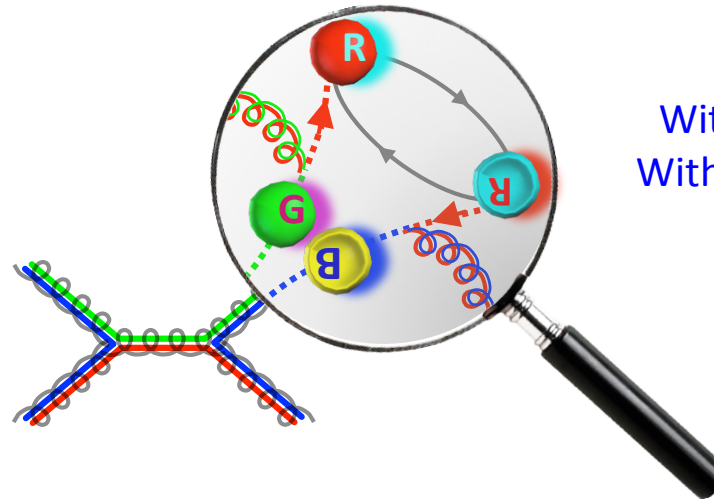
→ open new paths to *finally* address **the interesting questions about hadron formation**

We have **learned a lot**, but even more **remains to be understood!**

Is the QQbar immediately produced colour neutral?



Or does it undergo transitions changing colour (and angular-momentum)?



Which ones?
With what *hierarchies*?
With what dependences
on *quark mass*,
binding energy,
angular momentum
and *spin*?