Calculation of multi-particle processes at the one-loop level: precise predictions for the LHC

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in collaboration with
T. Binoth, N. Kauer, R. Rückl [Phys. Rev. D **74** (2006) 113008]
and T. Binoth, T. Gleisberg, N. Kauer, G. Sanguinetti [Phys. Lett. B **683** (2010) 154]

Thomas Binoth symposium, Edinburgh

Outline

- Motivation and short overview of the tensor reduction with GOLEM
- 2 applications: Results for multi Higgs production and
- Z-boson pair +1-jet production at NLO QCD
- Summary

Motivation of NLO calculations

- LO predictions usually have large theoretical uncertainties
- Large impact of higher order corrections due to new channels and experimental cuts possible
- no sensible way to estimate NLO corrections without doing the work!
- discrimination of (small) signals from (large) backgrounds: maximal control of theory predictions for signals and backgrounds needed

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$\Longrightarrow$ NLO necessary for precision comparisons of data to theory

Difficulties in NLO multi-leg calculations

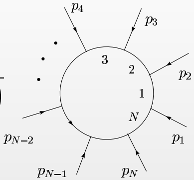
- large number of Feynman diagrams due to many final particles and the higher order in the perturbative expansion
- huge amount of algebra, long expressions
  $\rightarrow$ Computeralgebra (Maple, Mathematica, FORM,...), automation
- complicated structure of singularities: real and virtual corrections

$$\sigma_n^{\text{NLO}} = \int_n \sigma_n^{\text{tree}} + \int_n \sigma_n^{\text{virt}} + \int_{n+1} \sigma_{n+1}^{\text{real}}$$

  only the sum is finite
  cancellations of singularities between different phase spaces
  $\rightarrow$ various subtraction methods exist
- numerically stable evaluation of one-loop tensor-integrals when integrating over the multi-dimensional phasespace

Reduction method: (based on Feynman diagrams)

$$I_N^{n,\mu_1\ldots\mu_r} = \int \frac{d^n k}{i\,\pi^{n/2}} \, \frac{q_1^{\mu_1}\ldots q_r^{\mu_r}}{(q_1^2-m_1^2+i\delta)\ldots(q_N^2-m_N^2+i\delta)}$$



- main problem: numerically fast and stable evaluation needed
- standard approach: lorentz-inv. decomp.: $I_2^{\mu\nu} = A g^{\mu\nu} + B p^\mu p^\nu$
  $A, B \propto 1/\det G * (\sum \text{scalar integrals } I_N^n)$
- our method (GOLEM-coll.): reduce tensor int. to scalar int. in shifted dimensions (Davydychev 91)
  avoids inverse Gram determinants, algebraic separation of IR poles
  (T. Binoth, et al. hep-ph/0504267)

$$I_N^{n,\mu_1\ldots\mu_r} = \sum \tau^{\mu_1\ldots\mu_r}(r_{j_1},\ldots,r_{j_r},g^{\times m})\, I_N^{n+2m}(j_1,\ldots,j_R)$$

$$I_N^D(j_1,\ldots,j_R) = (-1)^N\,\Gamma(N-D/2)\int_0^\infty d^N z\,\delta(1-\sum_{l=1}^N z_l)\frac{z_{j_1}\ldots z_{j_r}}{(z\cdot S\cdot z/2)^{N-D/2}}$$

$$S_{ij} = (r_i-r_j)^2 - m_i^2 - m_j^2,\; r_j = p_1+\ldots+p_j$$

- two alternatives for evaluation:
  - further algebraic reduction: occuring basis integrals: I_4^{n+2}, I_3^n, I_2^n
    But $1/\det G$ unavoidable ($G_{ij} = 2\, r_i \cdot r_j$)
  - direct numerical evaluation in critical regions of phasespace feasible
    for ZZ+jet and 3H: fully algebraic reduction
  - see Thomas talk for more information about the GOLEM method, golem95 and GOLEM2.0!
  - $N = 5$ rank 2 example: $I_5^{\mu_1\mu_2}$

$$I_5^{\mu_1\mu_2} = T_{00}^{5,2} g^{\mu_1\mu_2} + \sum T_{i,j}^{5,2} r_i^{\mu_1} r_j^{\mu_2}$$

$$T_{00}^{5,2} = -\frac{1}{2} \sum_l b_l I_{4,l}^{n+2} + \mathcal{O}(\varepsilon)$$

$$T_{ij}^{5,2} \rightarrow \sum_l I_{4,j}^{n+2}, \sum_{l,m} I_{3,l,m}^{n}$$

The $PP \rightarrow HH, HHH$ amplitude

(T. Binoth, N. Kauer, R. Rückl)

- mechanism of mass generation: Higgs mechanism
- Higgs potential:

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda_{3H} H^3 + \frac{1}{4} \lambda_{4H} H^4$$

  - SM: $\lambda_{3H} = \lambda_{4H} v$, where $v = 246\,\mathrm{GeV}/c^2$
  - MSSM: $\lambda_{3h}/\lambda_{4h} = v \sin(\beta + \alpha)/\cos(2\alpha)$ (for the CP-even scalar h)

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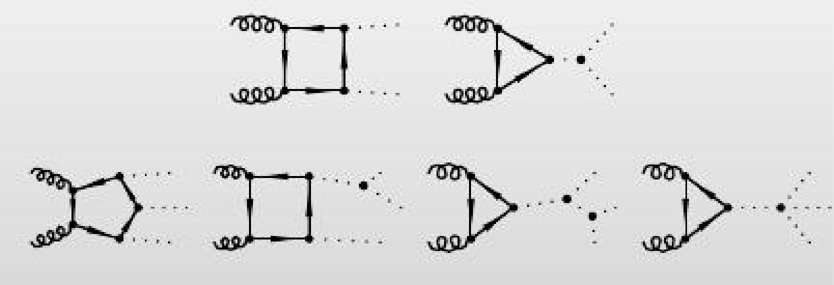
- $gg \to H$: most important channel for Higgs production at the LHC, measures the Higgs mass m_H
- both λ_3 and λ_4 are needed to fully determine the Higgs potential
  $\to$ measurement of $gg \to HH, HHH$ needed to fully confirm the Higgs mechanism
- test of the Standard Model

multi-Higgs production at high energy at hadron colliders:

- $\lambda_{Hf\bar{f}} \propto m_f \rightarrow$ gluon fusion: $gg \rightarrow 2H, 3H \quad m_t = 178\text{GeV}$



- pentagon, box and triangle contributions with different coupling constants dependence (Plehn,Rauch;Phys.Rev.D72:053008,2005, comparison → agreement)
- determine trilinear and quartic Higgs self-couplings
- probe Higgs potential

Amplitude representation:

$$\Gamma^{gg\to 3H} = \epsilon_{1,\mu}^{\lambda_1}\epsilon_{2,\nu}^{\lambda_2} M^{\mu\nu}, \quad \lambda_{1,2} \in [+,-]$$

with the Lorentz-invariant decomposition

$$M^{\mu\nu} = A g^{\mu\nu} + \sum_{i,j=1}^{4} B_{ij} p_i^\mu p_j^\nu \quad (17 \text{ coefficients})$$

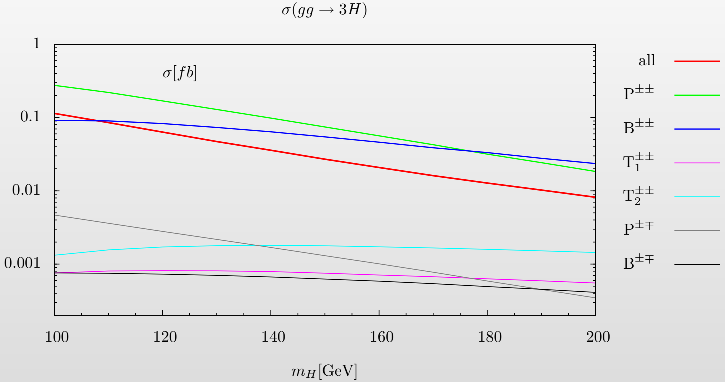
- due to transversality ($\epsilon_j \cdot p_j = 0$), 4d-kinematics, gauge- and bose-symmetry:
  $\to$ only 2 independent coefficients: B_{33}, B_{34} $\to$ used to check coefficients
- helicity amplitudes:

$$\Gamma^{++} = M^{\mu\nu}\epsilon_{1,\mu}^{+}\epsilon_{2,\nu}^{+} \quad (\text{spin } 0) \text{ with}$$

$$\epsilon_{1,\mu}^{+}\epsilon_{2,\nu}^{+} = \frac{-e^{i\phi}}{2p_1\cdot p_2}\left(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - p_1\cdot p_2\, g^{\mu\nu} - \varepsilon^{\mu\nu\rho\sigma} p_{1,\rho} p_{2,\sigma}\right)$$

similar for Γ^{+-} (spin 2)

Results for 3H production:



- $T_1 \propto \lambda_{4H}, \quad T_2 \propto \lambda_{3H}^2$
- $\frac{\sigma(\lambda_4 = \lambda_{SM})}{\sigma(\lambda_4 = 0)} = 0.93$ ($m_H = 120$ GeV)
- $\frac{\sigma(\lambda_4 = \lambda_{SM})}{\sigma(\lambda_4 = 0)} = 0.64$ ($m_H = 200$ GeV)
  $\rightarrow$ significant impact of mod. self-coupling on total cross section!

Comparison with eff. theory: Glover, van der Bij (1988)

- expansion parameter: $\hat{s}/4m_t^2$, $\quad m_t \to \infty$ ($\sqrt{\hat{s}}$ = partonic CM energy)
- $\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{12\pi}\, G^a_{\mu\nu} G^{\mu\nu}_a \log(1 + H/v)$ $\quad\to$ vertices with 2 gluons and n Higgs

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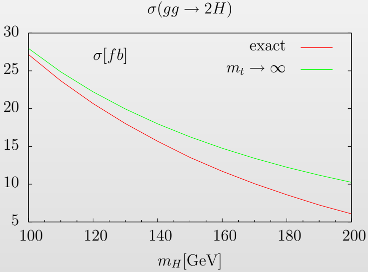

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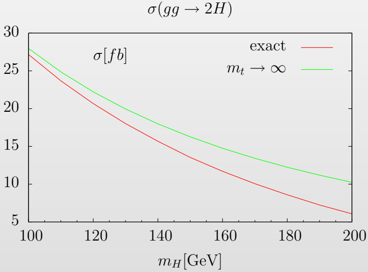

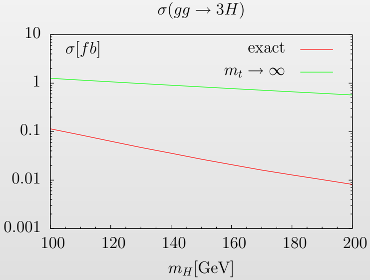

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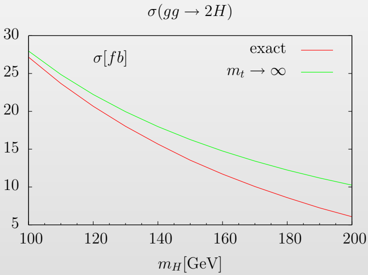

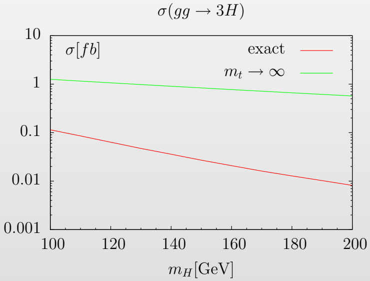

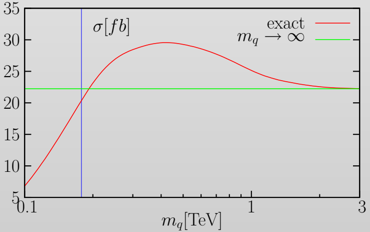

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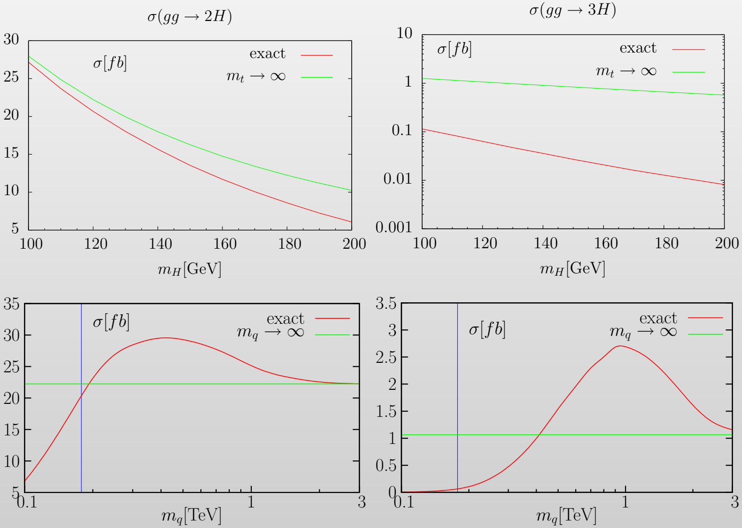


The $\mathrm{PP} \to \mathrm{VV} + jet$ amplitude

(T. Binoth, T. Gleisberg, SK, N. Kauer, G. Sanguinetti)

- Importance for LHC physics: background process to $H \to VV + \text{jet}$, anomalous gauge boson couplings, part of $PP \to VV$ at NNLO
- Virtual corrections: $\sim$ 100 Feynman diagrams: (tensor reduction with GOLEM methods)

- tuned comparison of WWjet with [Dittmaier, Kallweit, Uwer 07], [Campbell, Ellis, Zanderighi 07] and ZZjet [DKU 07]→ [NLM Les Houches report 08,10]
- 6 scales: $s_{12}, s_{23}, s_{34}, s_{45}, s_{15}, M_Z^2$
- regularisation scheme: 'tHooft/Veltman (anti-commuting γ_5), $\overline{\text{MS}}$
- 36 helicity amplitudes, related by bose symmetry, charge conjugation and parity transformation
  $\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} = \epsilon^{\lambda_3}_{3,\mu_3}\epsilon^{\lambda_4}_{4,\mu_4}\epsilon^{\lambda_5}_{5,\mu_5}\ \langle 2^{\lambda_2}|\Gamma^{\mu_3\mu_4\mu_5}|1^{\lambda_1}\rangle$
- real emissions: Sherpa dipoles (Gleisberg, Krauss), cross checked with MadDipole (Frederix,Gehrmann, Greiner), Helac dipoles (Czakon, Papadopoulos, Worek) and partial in house implementation

Helicity projection for $q\bar{q}VVg \to 0$

- replace momenta of the massive vectorbosons ($p_{3,4}$) with light-like momenta ($k_{3,4}$) to apply spinor formalism

$$k_{3,4} = \frac{1}{2\beta}\left[(1+\beta)p_{3,4} - (1-\beta)p_{4,3}\right] \quad \text{with } k_{3,4}^2 = 0$$

$$\epsilon^{+}_{3,\mu} = \frac{1}{\sqrt{2}}\frac{\langle 4^-|\mu|3^-\rangle}{\langle 43\rangle}, \quad \epsilon^{0}_{3,\mu} = \frac{1}{\sqrt{2}}\frac{(1+\beta)k_{3,\mu} - (1-\beta)k_{4,\mu}}{2M_V}$$

  Use to define projectors on helicity amplitudes, schematically:

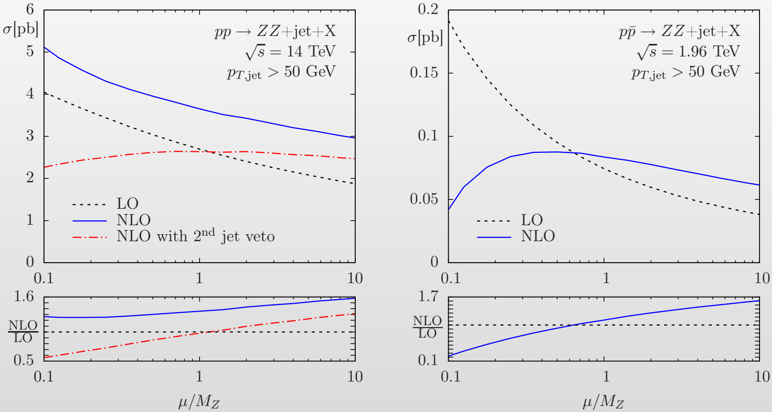
$$\begin{aligned} \mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} &= \mathcal{P}^{\lambda_3\lambda_4\lambda_5}_{\mu_3\mu_4\mu_5}\langle 2^{\lambda_2}|\Gamma^{\mu_3\mu_4\mu_5}|1^{\lambda_1}\rangle \\ &= (\text{global spinorial factor}) \times (\text{contracted tensor integrals}) \end{aligned}$$

- Lorentz indices saturated, at most rank 1 pentagons (+ rank 3 boxes)
- spinor products can be treated as global factors
- further simplifications in analytical expressions possible and performed

Results for $ZZ + \text{jet}$

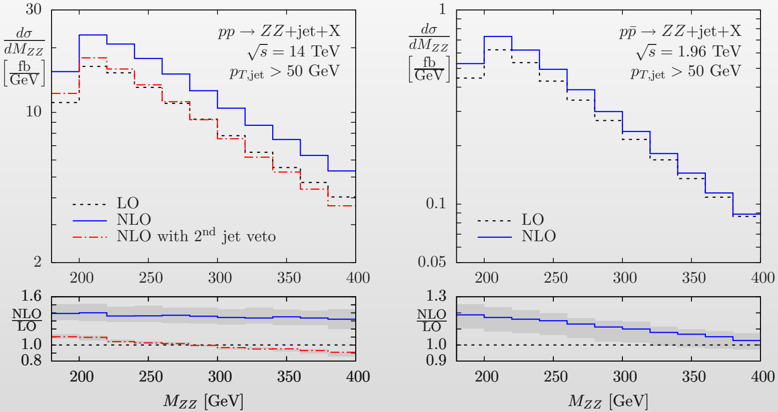
- differences to $WW + \text{jet}$: additional Bose symmetry, also right-handed couplings to fermions, no box-type diagrams from WWZ, WWA vertex
- Input parameters/settings:
  $N_F = 5$, $m_q = 0$, $M_Z = 91.188$ GeV, $\alpha(M_Z) = 0.00755391226$,
  $\sin^2\theta_W = 0.222247$
  PDFs: CTEQ6L1(LO), CTEQ6M(NLO)
  Cuts: $p_{Tjet} > 50$ GeV
  central scale choice: $\mu_F = \mu_R = M_Z$

Scale variations:



	$\Delta\sigma/\sigma(pp \to ZZ + \text{jet})$, $\sqrt{s} = 1.96(14)$ TeV		
	$\mu/M_Z \in [\frac{1}{2}, 2]$	$\mu/M_Z \in [\frac{1}{4}, 4]$	$\mu/M_Z \in [\frac{1}{8}, 8]$
LO	23%	44%	62%
NLO	6%	11%	19%
LO	12%	23%	34%
NLO	7%	15%	23%
NLO with 2nd jet veto	0.5%	3%	6%

Distributions:



- new channels at NLO, (e.g. $gg \to ZZq\bar{q}$), formally of LO-type
- new color structures (ggg-vertex)
- turn out to be numerically important for the LHC, partially spoil the scale uncertainty reduction (Tevatron: dominated by $q\bar{q}$)
  $\to$ jet veto suppresses these contributions

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- larger 3 Higgs cross section of up to $\mathcal{O}(10)$ fb in BSM scenarios
- Report on the calculation of NLO QCD corrections to ZZ+jet for LHC and Tevatron, as an important background to Higgs and new physics searches
- Reduced scale uncertainties (at the LHC: only after a jet veto)
- Corrections for the M_{ZZ} distribution depend on the kinematical region (Tevatron, LHC NLO(excl))

	LHC	SLHC	VLHC
$\sqrt{s}$ [TeV]	14	14	200
$\mathcal{L}$ [fb^{-1}]	600	6000	1000

m_H = 120 GeV

	σ [fb]		# of events		
2H	21	2000	$1.3*10^4$	$1.3*10^5$	$2.0*10^6$
3H	0.06	9	36	360	9000

m_H = 160 GeV

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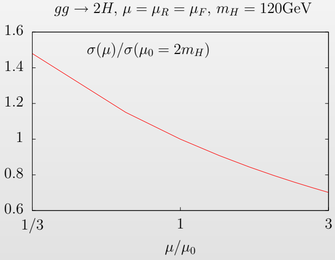
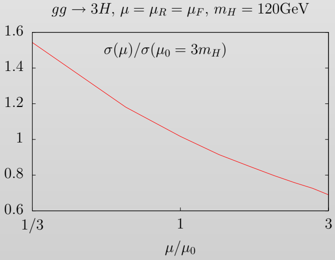
Factorization and renormalisation scale variation (pdfs: MRST2002)





MSSM: Minimal Supersymmetric Standard Model

- two complex Higgs doublets, five physical fields: $h, H, A, H^{\pm}$
- modified Higgs and Yukawa couplings, e.g.
  $\lambda_{ht\bar{t}} = \frac{m_t}{v}\frac{\cos\alpha}{\sin\beta} \propto \tan^{-1}\beta, \quad \lambda_{hb\bar{b}} = \frac{m_b}{v}\frac{\sin\alpha}{\cos\beta} \propto \tan\beta$
- two free parameters in the MSSM Higgs-potential: $M_A, \tan\beta = v_2/v_1$
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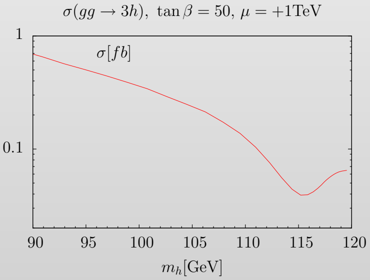

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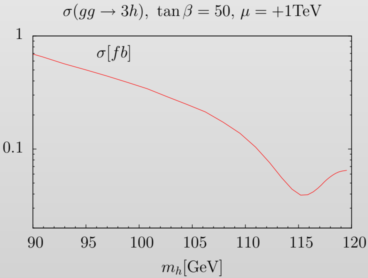

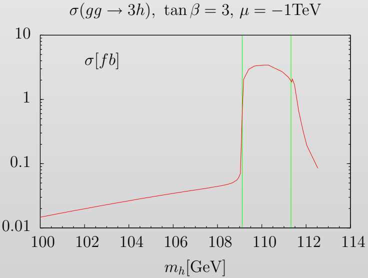

Tuned comparison of results

[Dittmaier, Kallweit, Uwer 07], [Campbell, Ellis, Zanderighi 07],
[Binoth, Guillet, SK, Kauer, Sanguinetti] → [NLM Les Houches report 08]

- Integrated LO results

$PP \to W^+W^-$+jet	$\sigma_{LO}[fb]$
DKU	10371.7(12)
CEZ	10372.26(97)
BGKKS	10371.7(11)

- Results for virtual corrections checked at one phase space point:

$$2\,\mathrm{Re}\{\mathcal{M}_V^* \cdot \mathcal{M}_{LO}\} = e^4 g_s^2\, \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_W^2}\right)^2 \left(\frac{1}{\epsilon^2} c_{-2} + \frac{1}{\epsilon} c_{-1} + c_0\right)$$

$u\bar{u} \to W^+W^- g$	c_{-2}	c_{-1}	c_0
DKU	$-1.0806993055087 \cdot 10^{-4}$	$7.8428619052630 \cdot 10^{-4}$	$-3.3829109154253 \cdot 10^{-3}$
CEZ	$-1.0806993055058 \cdot 10^{-4}$	$7.8428619052767 \cdot 10^{-4}$	$-3.3829109154640 \cdot 10^{-3}$
BGKKS	$-1.0806993055088 \cdot 10^{-4}$	$7.8428619052632 \cdot 10^{-4}$	$-3.3829109156162 \cdot 10^{-3}$

- three different methods, impressive agreement!