Gaps between jets at the LHC

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Thomas Binoth Symposium
University of Edinburgh
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Outline

- Introduction: soft gluons in QCD
- Jet vetoing
  - Global and non-global logarithms
  - Discovery of super-leading logarithms
- Some phenomenological studies
  - Global logarithms
  - Super-leading logarithms
- Conclusions and Outlook

Resummation in QCD

• Given a particular hard scattering process we can study how it is modified by (perturbatively calculable) additional radiation.

• The most important emissions are those involving either *collinear* quarks and gluons or *soft* gluons.

• They are important because the usual suppression in the strong coupling is compensated by a large logarithm

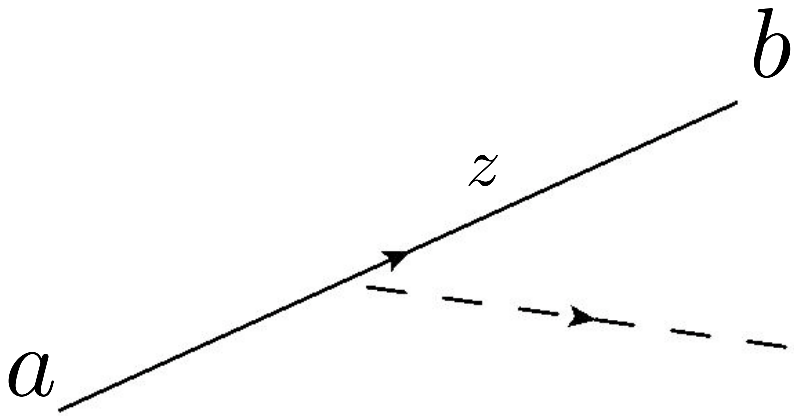
$$\sigma = \alpha_s a_0 \ln \omega + \alpha_s^2 \left(a_1 \ln^2 \omega + b_0 \ln \omega\right) + \dots$$

• It is important to reorganise (i.e. resum) the perturbative series in order to obtain meaningful predictions

$$\sigma = \sum_k (\alpha_s \ln \omega)^{k+1} a_k + \alpha_s \sum_k (\alpha_s \ln \omega)^{k+1} b_k + \dots$$

Collinear emissions

- It is as if emission is off the parton to which it is collinear $\sim$ "classical branching"
- Colour structure is easy



$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2} dz \; P_{ba}(z)$$

- DGLAP evolution enables us to resum collinear logarithms

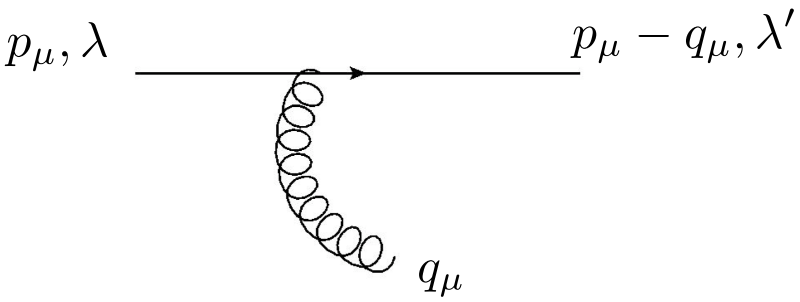
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} f(x,\mu^2) = \int dz P(x/z) f(z,\mu^2)$$

- Splitting functions known at NNLO

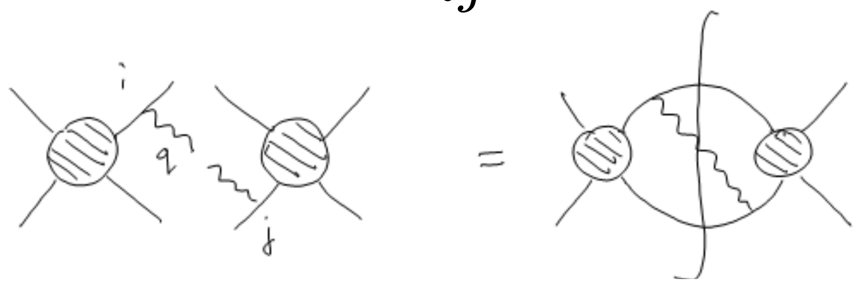
$$P(x) = \alpha_s P^{(0)}(x) + \alpha_s^2 P^{(1)}(x) + \alpha_s^3 P^{(2)}(x)$$

Soft emissions

The eikonal approximation



$$= 2 g p_\mu \delta_{\lambda\lambda'} T^a_{ij}$$



$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi} \frac{dE}{E} \frac{d\Omega}{2\pi} \sum_{ij} C_{ij} E^2 \, \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q}$$

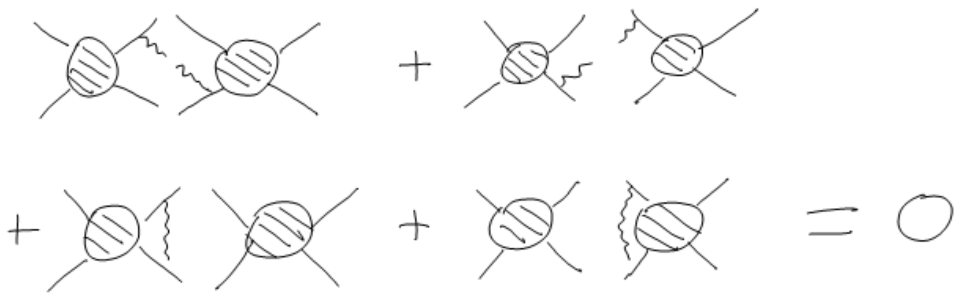
- Colour factor is the “problem”

In the Monte Carlo parton showers soft and/or collinear radiation is handled simultaneously using “angular ordered parton evolution”

Is this good enough ?

Soft gluons in QCD

- What happens if we dress a hard scattering with soft gluons?
- Sufficiently inclusive observables are not affected: real and virtual cancel via Bloch-Nordsieck theorem



- Soft gluon corrections play a role if the real radiation is constrained into a region of phase-space
- In such cases BN fails and miscancellation between real and virtual induces potentially large logarithms

$$-\alpha_s \int_0^{Q_0} \frac{dE}{E}\Big|_{\mathrm{real}} + \alpha_s \int_0^{Q} \frac{dE}{E}\Big|_{\mathrm{virtual}} = \alpha_s \int_{Q_0}^{Q} \frac{dE}{E}\Big|_{\mathrm{virtual}} = \alpha_s \ln \frac{Q}{Q_0}$$

Soft gluon corrections are important for observables that insist on only *small deviations from lowest order kinematics*

In such cases real radiation is constrained to a small corner of phase space and the logarithms are large

Some examples …

If V measures the 'distance' from the lowest order kinematics:

- Event shapes a such as thrust $V = 1 - T$
- Production near threshold $V = 1 - \frac{M^2}{\hat{s}}$
- Drell- Yan at low transverse momentum $V = \frac{M^2}{q_T^2}$
- DIS at large x $V = 1 - x$

Recent developments

- Structure of soft singularities in QCD amplitudes
  - 2 loop soft anomalous dimension

    Sterman *et al.*
    hep-ph/0607309
    arXiv:0903.3241 [hep-ph]

  - Symmetry of the massless case: all order structure of IR singularities

    (Dixon), Gardi and Magnea
    arXiv:0901.1091 [hep-ph]
    (arXiv:0910.3653 [hep-ph])

  - Soft Collinear Effective Theory

    Becher and Neubert
    arXiv:0904.1021 [hep-ph]

- Beyond the eikonal approximation: next-to-eikonal via path integral

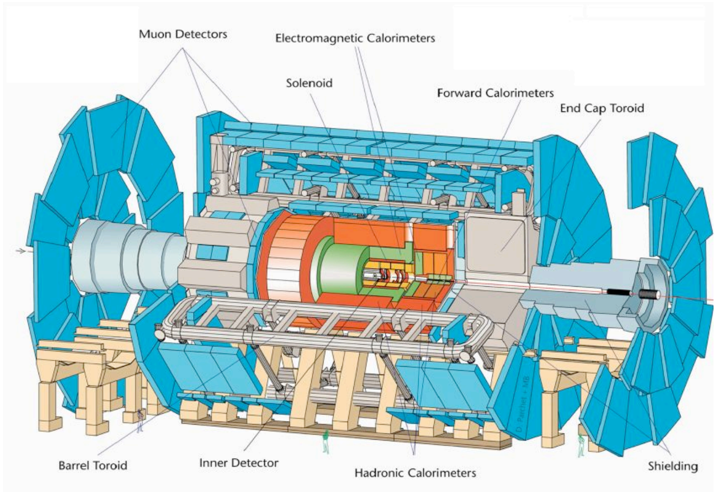
  Laenen, Stavenga, White
  arXiv:0811.2067 [hep-ph]

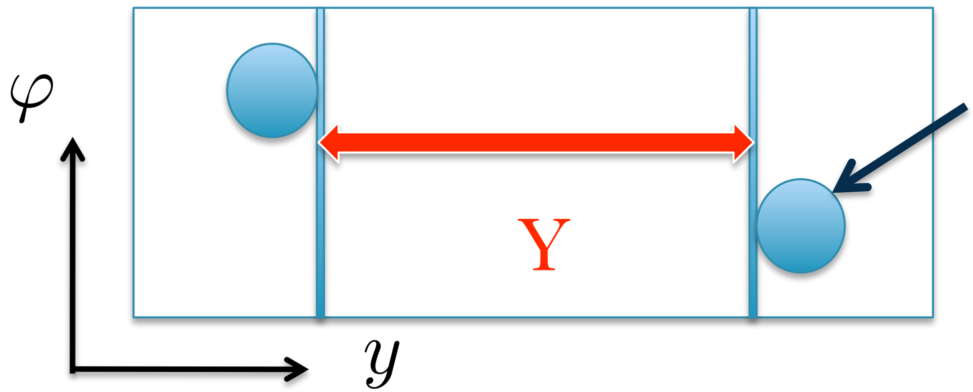
Jet vetoing:
Gaps between jets

The observable

Production of two jets with

- transverse momentum Q
- rapidity separation Y



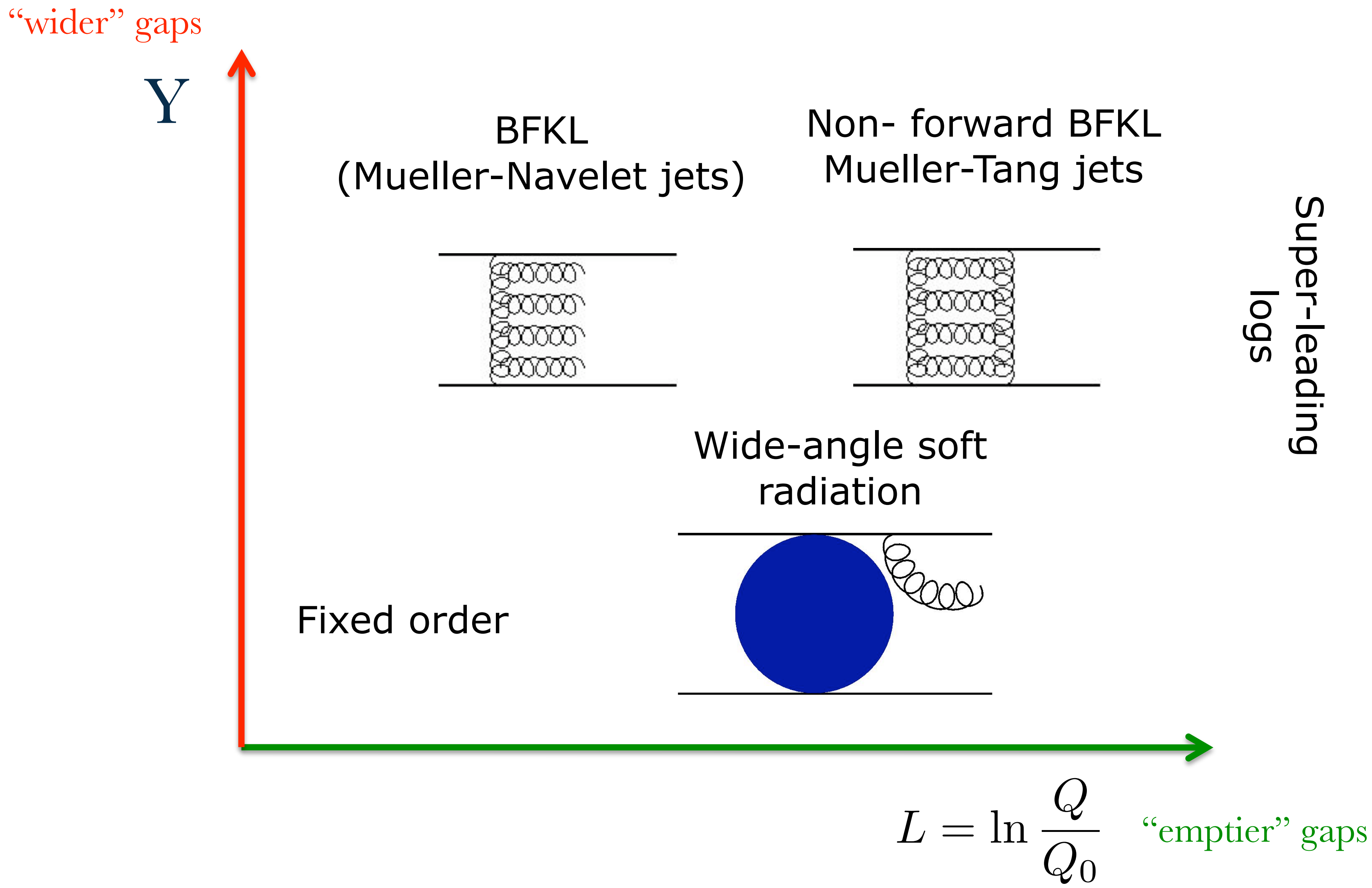


jet radius R

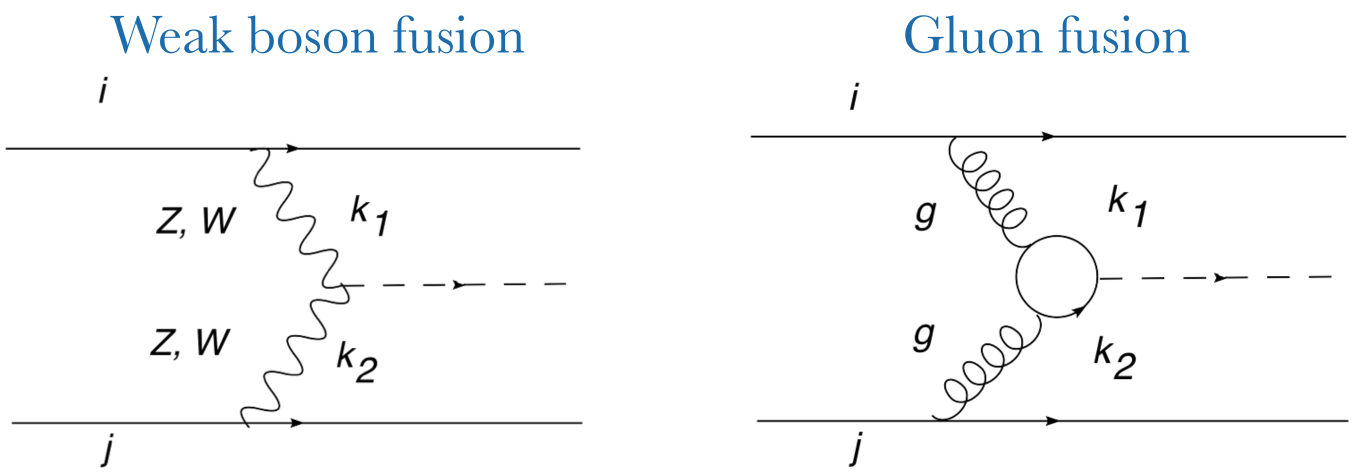
- Emission with $k_T > Q_0$ forbidden in the inter-jet region

Q_0 can be rather large: the gap is a region of limited hadronic activity

Plenty of QCD effects



Higgs +2 jets



- Different QCD radiation in the inter-jet region
- To enhance the WBF channel, one can make a veto Q_0 on additional radiation between the tagged jets
- QCD radiation as in dijet production

Forshaw and Sjödahl
arXiv:0705.1504 [hep-ph]

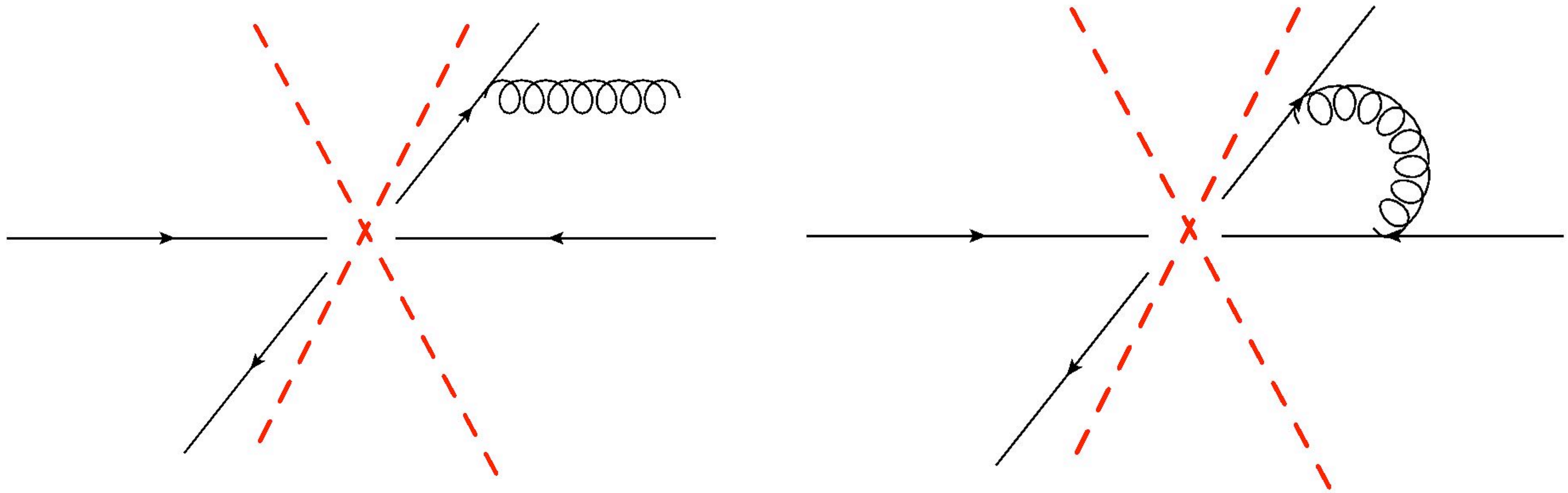
- Important in order to extract the VVH coupling

Soft gluons in gaps between jets: the old way

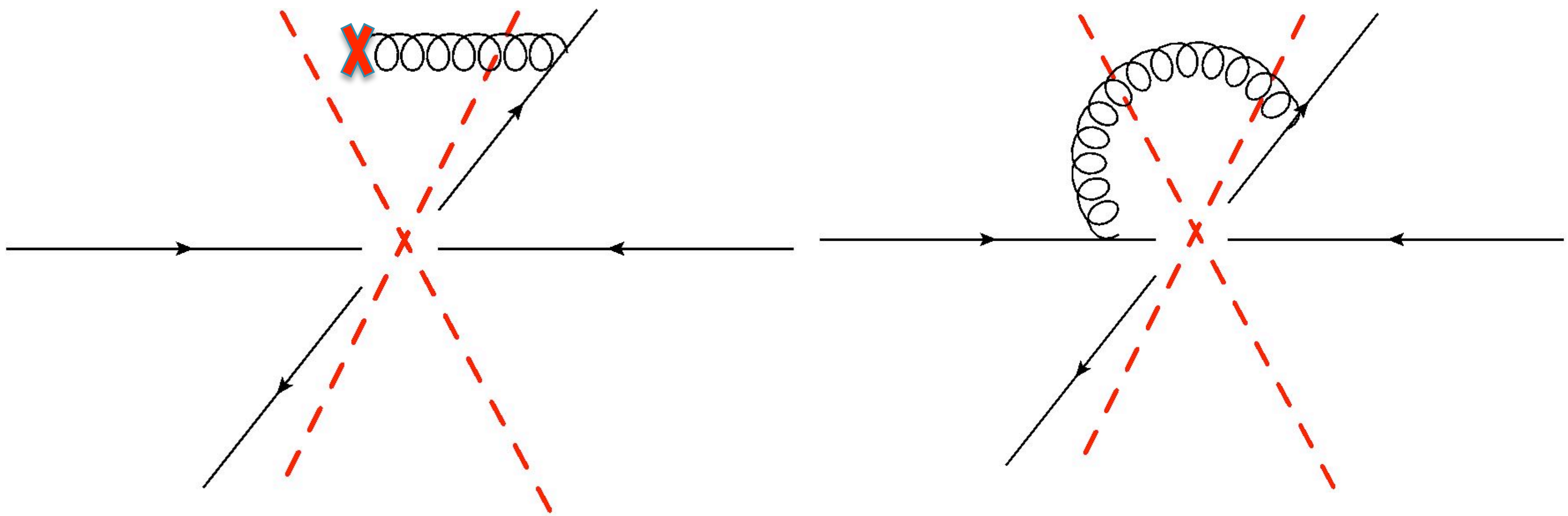
Oderda and Sterman
hep-ph/9806530

- Naive application of BN:

real and virtual contributions cancel outside the gap for every k_t:



The cancellation fails for in-gap gluons with $k_T > Q_0$



$$-\alpha_s \int_0^{Q_0} \frac{dk_T}{k_T}\Big|_{\rm real} + \alpha_s \int_0^{Q} \frac{dk_T}{k_T}\Big|_{\rm virtual} = \alpha_s \int_{Q_0}^{Q} \frac{dk_T}{k_T}\Big|_{\rm virtual} = \alpha_s L$$

One only needs to consider virtual corrections with

$$Q_0 < k_T < Q$$

Soft gluons exponentiation

- Leading logs (LL) are resummed by iterating the one-loop result
- This contribution exponentiates

$$\mathcal{M} = e^{-\alpha_s L \Gamma} \mathcal{M}_0$$

soft anomalous dimension

Born

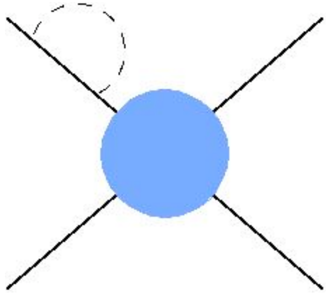
- The colour structure is not trivial
- The soft anomalous dimension is an operator in colour space
- Once we have fixed a basis it is represented by a matrix

Colour evolution

- The anomalous dimension can be written as

$$\Gamma = \frac{1}{2} Y T_t^2 + i\pi T_1 \cdot T_2 + \frac{1}{4}\rho(T_3^2 + T_4^2)$$

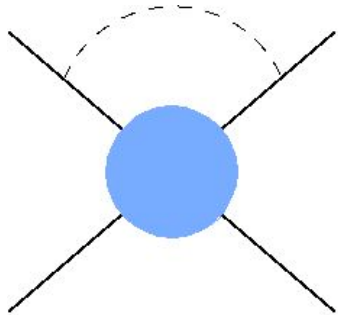
- T_i is the colour charge of parton *I*



- T_i^2 is a Casimir

- $T_t^2 = (T_1^2 + T_3^2 + 2T_1 \cdot T_3)$

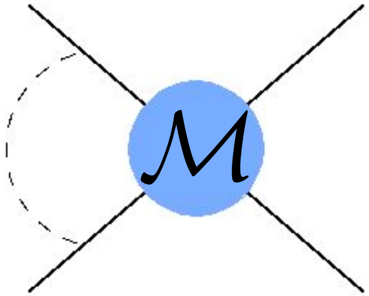
  is the colour exchange in the *t*-channel



Coulomb gluons

- The $i\pi$ term is due to Coulomb gluon exchange

$$i\pi\, T_1 \cdot T_2\, \mathcal{M} =$$



- Coulomb gluons put on-shell the parton propagators
- It doesn't play any role for processes with less than 4 coloured particles (e.g. DIS or DY)

$$T_1 + T_2 + T_3 = 0 \Rightarrow T_1 \cdot T_2 = \frac{1}{2}\left(T_3^2 - T_1^2 - T_2^2\right)$$

leading to an unimportant overall phase

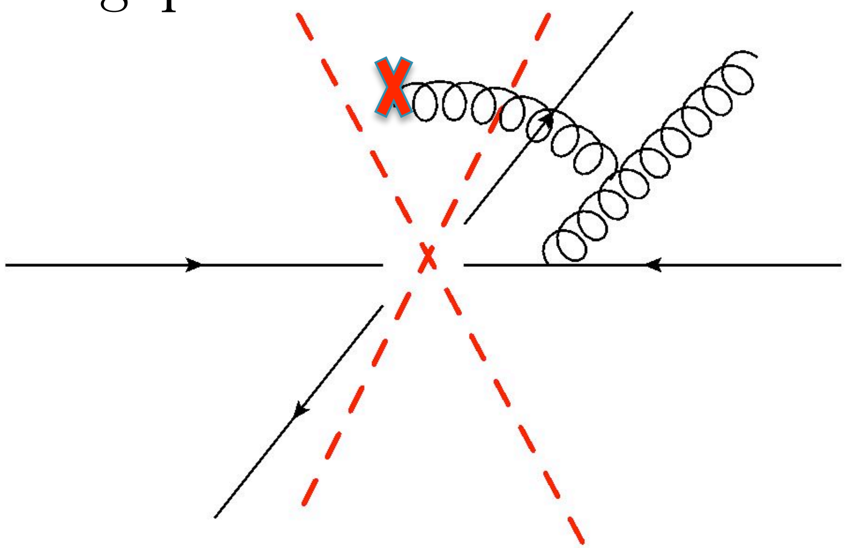
- Coulomb gluon contributions are *not* implemented in parton showers

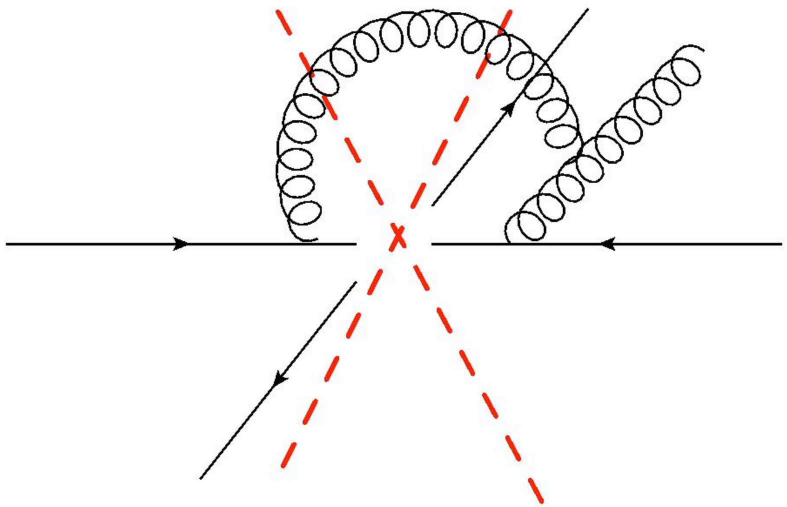
Non-global effects

Dasgupta and Salam
hep-ph/0104277

- However this naive approach completely ignores a whole tower of LL

- Virtual contributions are not the whole story because real emissions out of the gap are forbidden to remit back into the gap





Resummation of non-global logarithms

- The full LL result is obtained by dressing the $2 \to n$ (i.e. *n-2* out of gap gluons) scattering with virtual gluons (and not just $2 \to 2$)

- The colour structure soon becomes intractable

- Resummation can be done (so far) only in the large N_c limit

Dasgupta and Salam
hep-ph/0104277

Banfi, Marchesini and Smye
hep-ph/0206076

Back to gaps between jets

- One would like to resum the non-global logs but keeping the full N_c dependence

- We can start by considering only one out-of-gap gluon

Forshaw Kyrieleis Seymour
hep-ph/0604094

- We need to consider $2 \to 3$ processes dressed with virtual in-gap gluons

- The one gluon outside the gap cross section is

$$\sigma^{(1)} = -\frac{2\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk_T}{k_T} \int_{\mathrm{out}} (\Omega_R + \Omega_V)$$

The real contribution

- Real emission vertex D^{μ}
- 5 - parton anomalous dimension Λ
- Computed now for all partonic processes

Sjödahl
arXiv:0807.0555 [hep-ph]

$$\Omega_R = \mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}^\dagger\right)\mathbf{D}_\mu^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Lambda}^\dagger\right)\mathbf{S}_R$$
$$\exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Lambda}\right)\mathbf{D}^\mu \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)\mathbf{M}_0$$

The virtual contribution

- Virtual eikonal emission γ
- 4-parton anomalous dimension Γ

$$\Omega_V = \left[\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk'_T}{k'_T} \mathbf{\Gamma}^\dagger \right) \right. \\ \left. \exp\left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{k_T} \frac{dk'_T}{k'_T} \mathbf{\Gamma} \right) \gamma \ \exp\left(-\frac{2\alpha_s}{\pi} \int_{k_T}^{Q} \frac{dk'_T}{k'_T} \mathbf{\Gamma} \right) \mathbf{M}_0 + \text{c.c.} \right]$$

A big surprise

Conventional wisdom ("plus prescription" of DGLAP) when the out-of-gap gluon becomes collinear with one of the external partons the real and virtual contributions should cancel

- It works when the out-of-gap gluon is collinear to one of the outgoing partons ✓
- But it fails for initial state collinear emissions ✗
- Cancellation *does* occur for up to 3rd order relative to the Born, but fails at 4th order
- The problem is entirely due to the exchange of Coulomb gluons, i.e. cancellation is restored if one artificially considers only the real part of the anomalous dimension

The collinear limit

- Let's look more in details at what is happening
- The virtual contribution has the expected form:

$$\Omega_V^{\text{coll}} = \frac{1}{V_c} \langle m_0 | \mathbf{t}_i^2 e^{-\xi \mathbf{\Gamma}^\dagger} e^{-\xi \mathbf{\Gamma}} | m_0 \rangle$$

- The real contribution is more complicated and cancellation fails at 4$^{\text{th}}$ order and beyond

$$\Omega_R^{\text{coll}} = -\frac{1}{V_c} \langle m_0 | e^{-\xi(k_T,Q)\mathbf{\Gamma}^\dagger} \mathbf{t}_i^{a\,\dagger} e^{-\xi(Q_0,k_T)\mathbf{\Lambda}^\dagger} e^{-\xi(Q_0,k_T)\mathbf{\Lambda}} \mathbf{t}_i^a e^{-\xi(k_T,Q)\mathbf{\Gamma}} | m_0 \rangle$$

- As result we are left with super-leading logarithms (SLL):

$$\sigma^{(1)} \sim -\alpha_s^4 L^5 \pi^2 + \dots$$

Effects on other observables

- Resummation of soft gluons is based on the idea of coherence: large-angle radiation only sees the sum of the colour charges of a bunch of collinear partons
- The appearance of SLL challenges this picture
- Does this effect other observables, i.e. event shapes ?
- Coherence Violating Logarithms are worst in forward suppressed observables

$$V \sim k_t^a e^{-by} \Rightarrow \alpha_s^n L^{2n-3}$$

Banfi, Salam, Zanderighi
arXiv:1001.4082 [hep-ph]

- They are NLL in the exponent but NNNLL in the expansion

Some LHC phenomenology

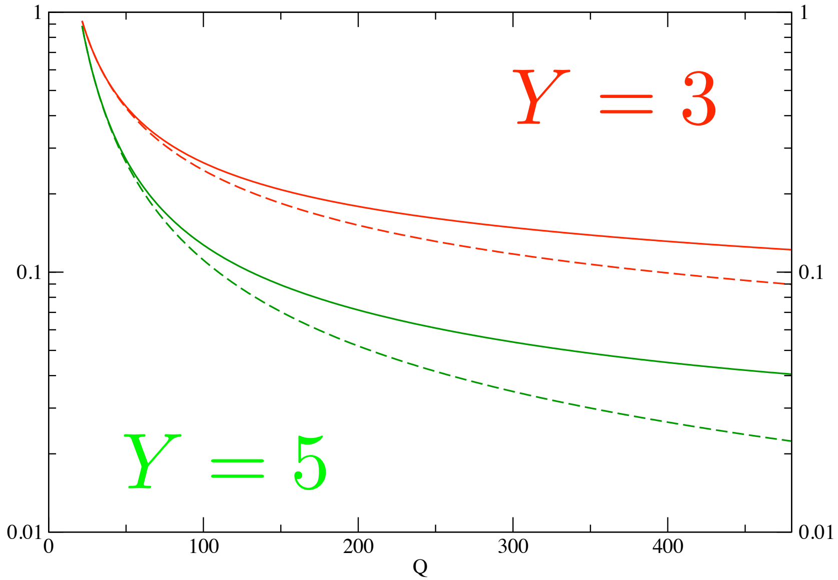
Forshaw, Keates, SM
arXiv:0905.1350

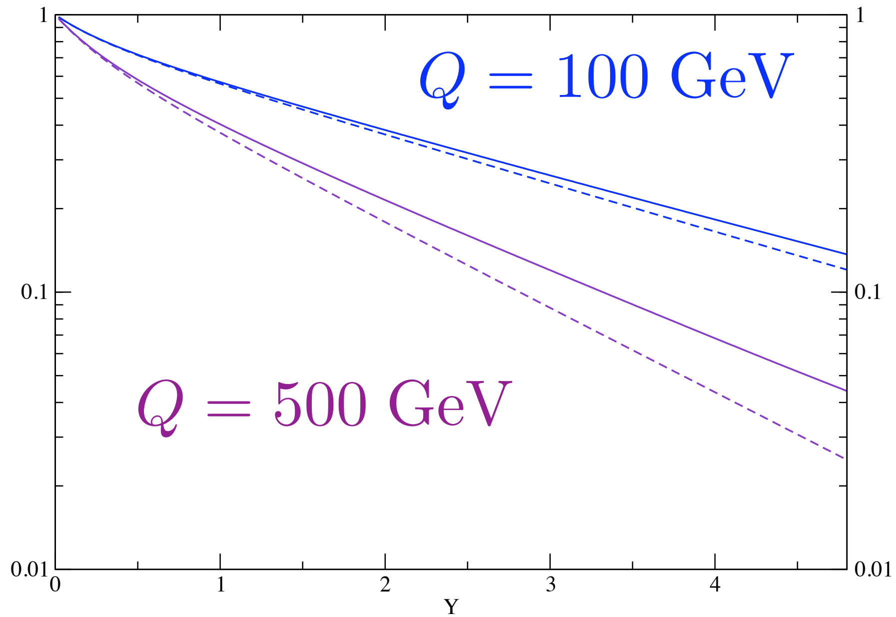
And work in progress with Duran, Forshaw and Seymour

Global logs and Coulomb gluons (no gluon outside the gap)

$$f^{(0)} = \sigma^{(0)}/\sigma^{\text{born}}$$

$\sqrt{S}$ = 14 TeV
Q_0 = 20 GeV
R = 0.4
η_{cut} = 4.5





- solid lines: full resummation
- dashed lines: ignoring $i\,\pi$'s

Large Coulomb gluon contributions !

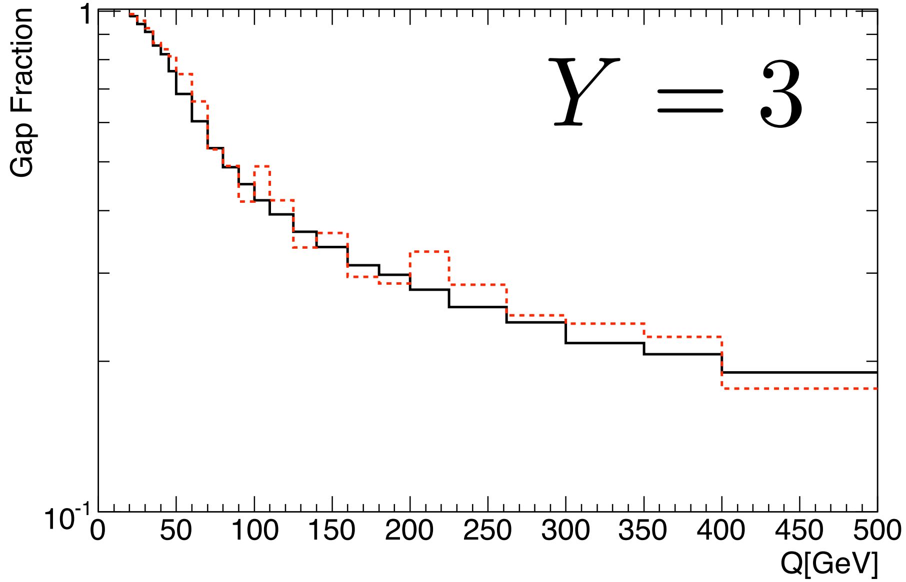
Angular-ordered parton showers

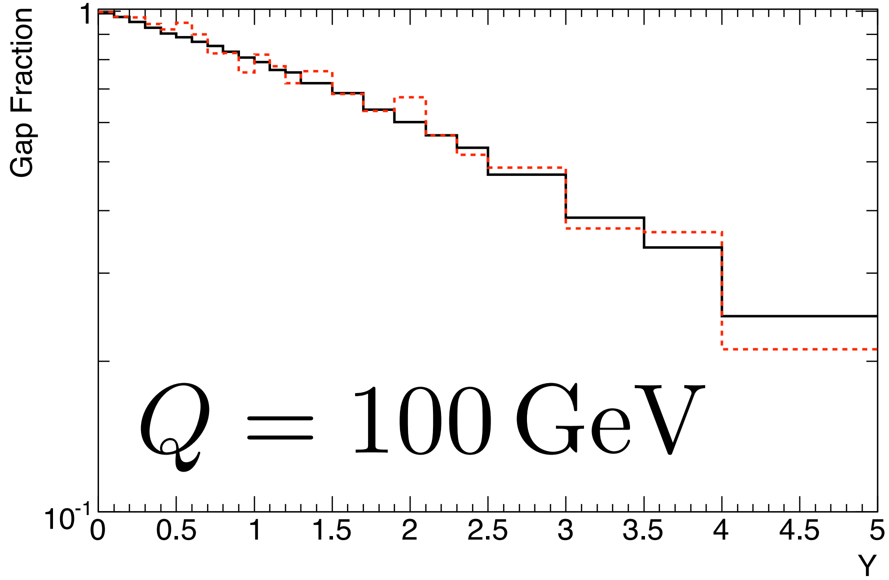
- We want to compare our resummed calculation to a standard event generator
- Thanks to coherent branching soft gluon contributions can computed to all orders in parton showers

  (but Coulomb gluons)
- Coherent branching is obtained by considering the opening angle as the evolution variable
- HERWIG has an angular-ordered parton shower

Hadronisation effects

- Hadronisation is "gentle"
- It does not spoil the gap fraction

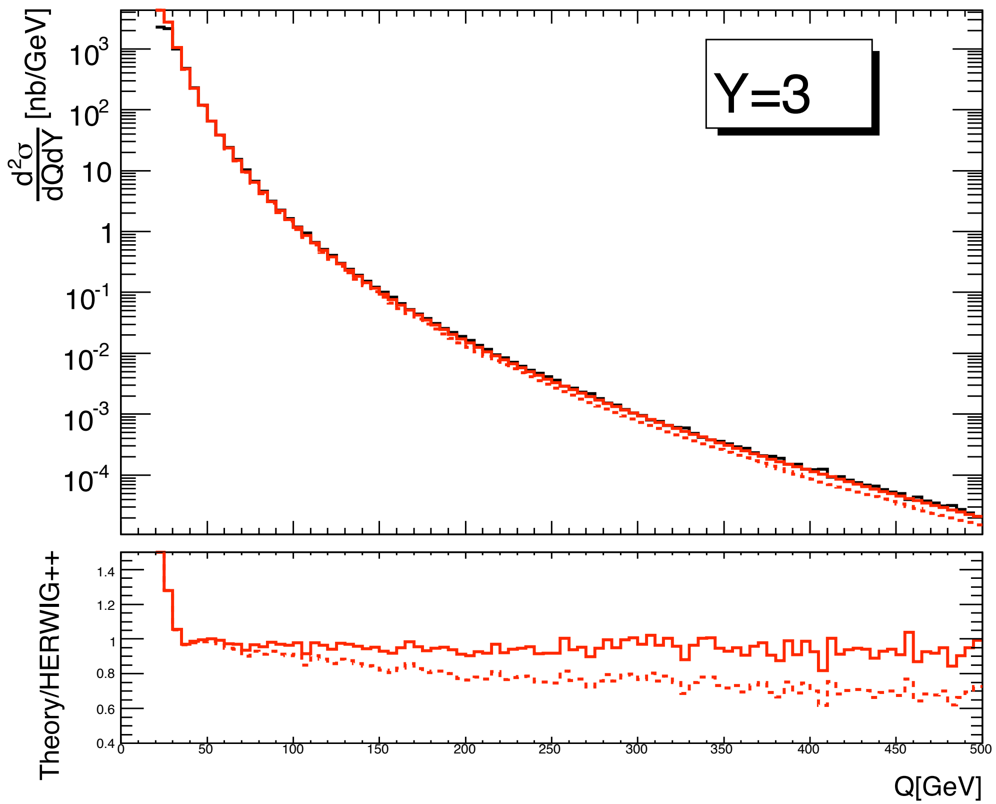




black line: after parton shower

red line: after hadronisation

Comparison to HERWIG++ (gap cross-section)



- We compare our results to HERWIG++
- LO scattering + parton shower (no hadronisation)
- Q is the mean p_T of the leading jets
- Jet algorithm SIScone

- The overall agreement is encouraging
- One should compare the histogram to the dotted curve (no Coulomb gluons)

Parton shower VS resummation (i)

HERWIG parton shower

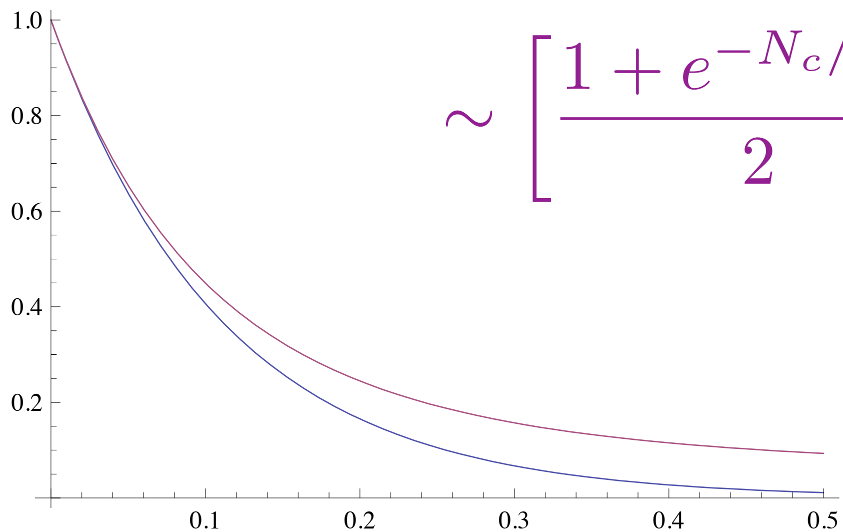
- enforces energy-momentum conservation
- misses Coulomb gluons
- contains *some* non-global contributions
- does not have the full colour structure: ~ large N_c
    - the colour partner of a parton is chosen in each event with equal probability
    - this is not the same as taking the large N_c limit

Correct large N_c limit

$$\sim e^{-N_c Y \xi}$$

HERWIG

$$\sim \left[\frac{1+e^{-N_c/2Y\xi}}{2}\right]^4$$



$$\xi = \frac{2}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\alpha_s(k_T)$$

Thanks to Mike Seymour

Parton shower VS resummation (ii)

Soft gluon resummation

- has the full colour structure

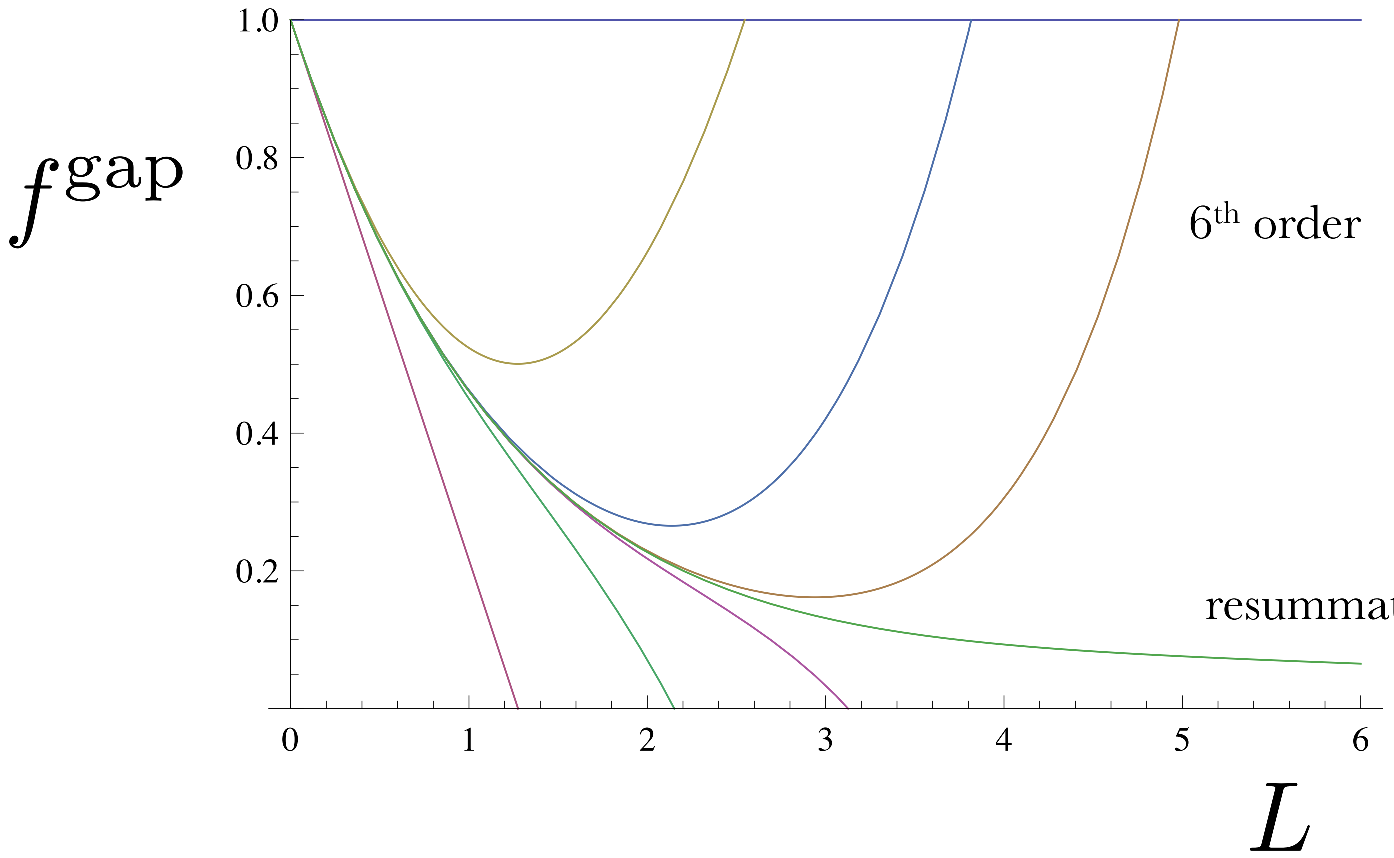
- doesn't have non-global logs (yet)

- does not conserve energy and momentum (eikonal approximation)

- Because of the fairly large value of Q_0 the region considered is not asymptotic and fixed-order effects are not negligible

- Thus we need matching to fixed order

Order by order contributions



$Y = 3$

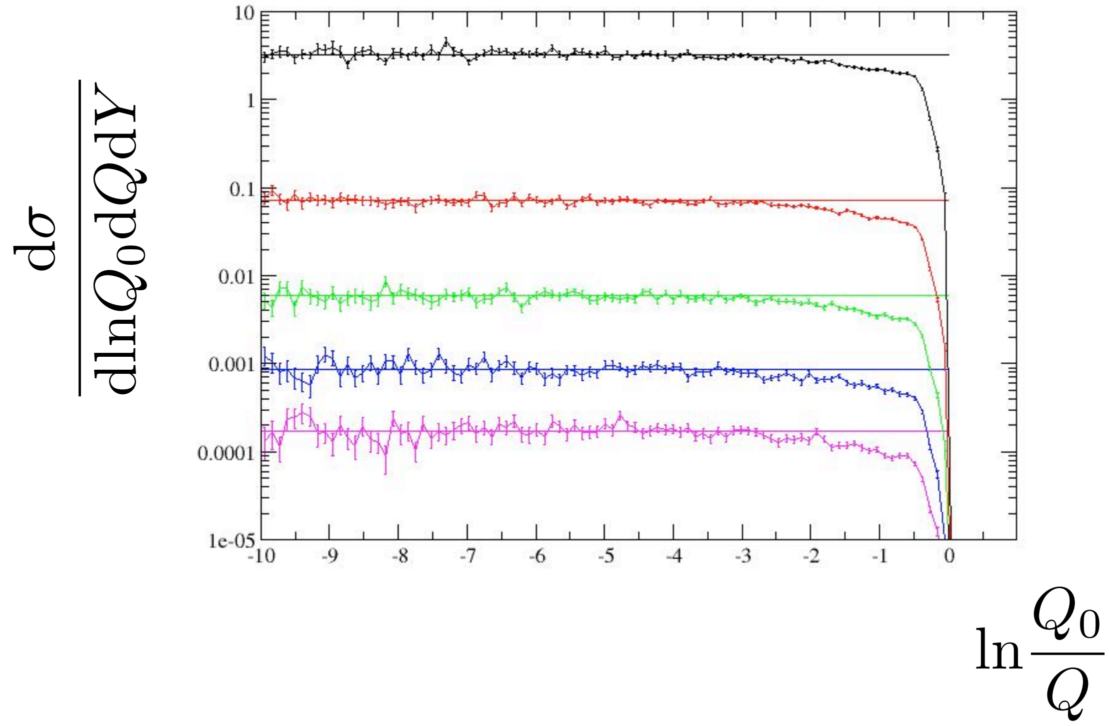
Toy: qg channel only, no PDFs

Matching to fixed order

$$f = 1 + \alpha_s c_1 + \alpha_s^2 c_2 + \ldots$$

- Fixed order computed with NLOJET++
- Check of the logs using the distribution

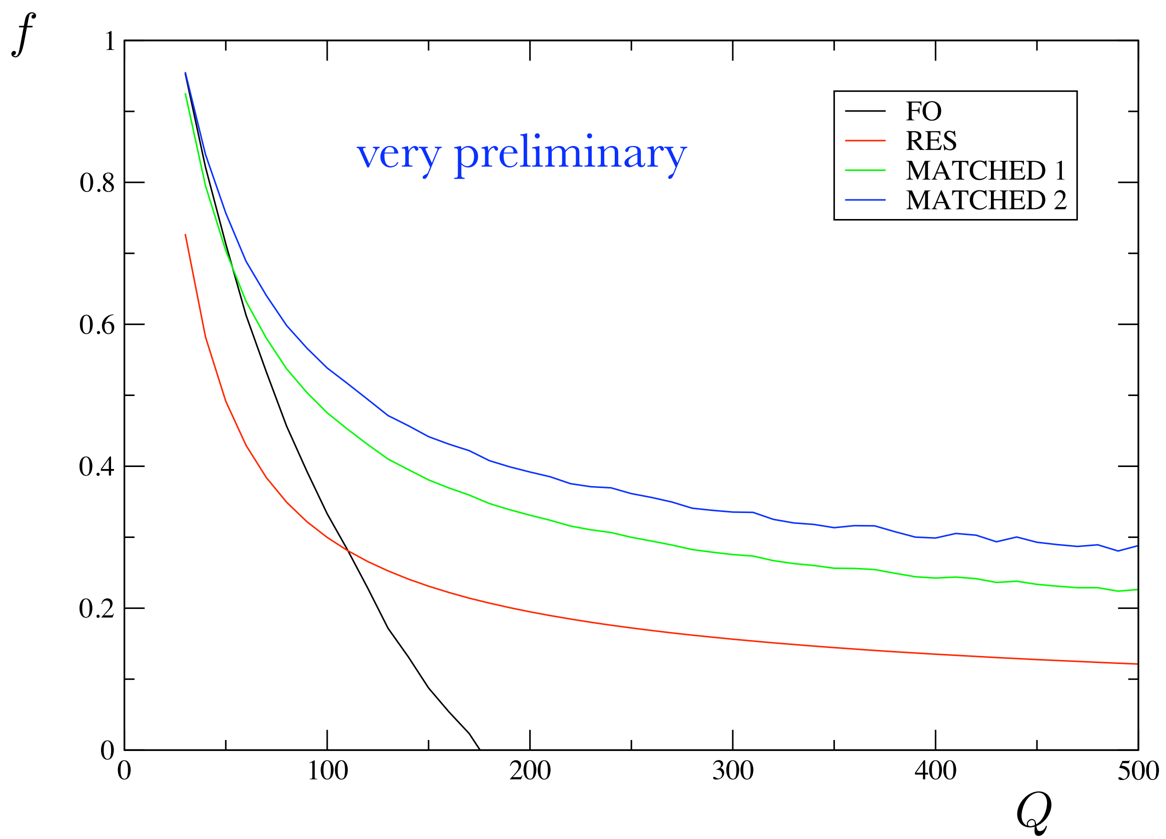
$$\begin{aligned} Y &= 3 \\ Q &= 100 - 500\,\mathrm{GeV} \end{aligned}$$



- The LO matching can be done with just tree-level matrix elements: studies with Madgraph as well

The matched gap fraction

$$f = f_{\rm res}(1 + \alpha_s c_0)e^{\alpha_s d_0}$$



$$b_0 = c_0 + d_0$$

Obtained from fixed order calculation with the logarithm subtracted

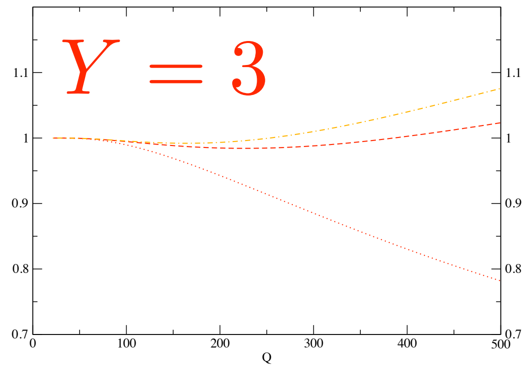
To do:

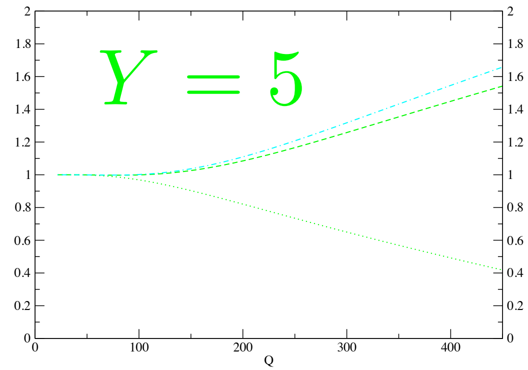
- check these results
- include non-global logs
- include scale uncertainties
- matching to the next order

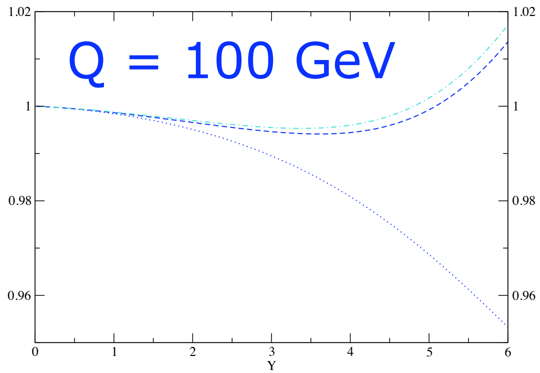
Back to
super-leading logarithms

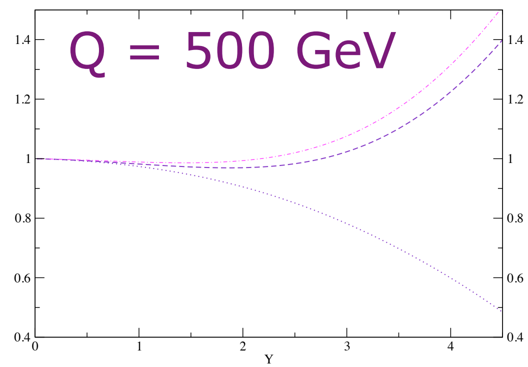
Super-leading logarithms (fixed order)

$$(\sigma^{(0)} + \sigma^{(1)} + \sigma^{(2)})/\sigma^{(0)}$$







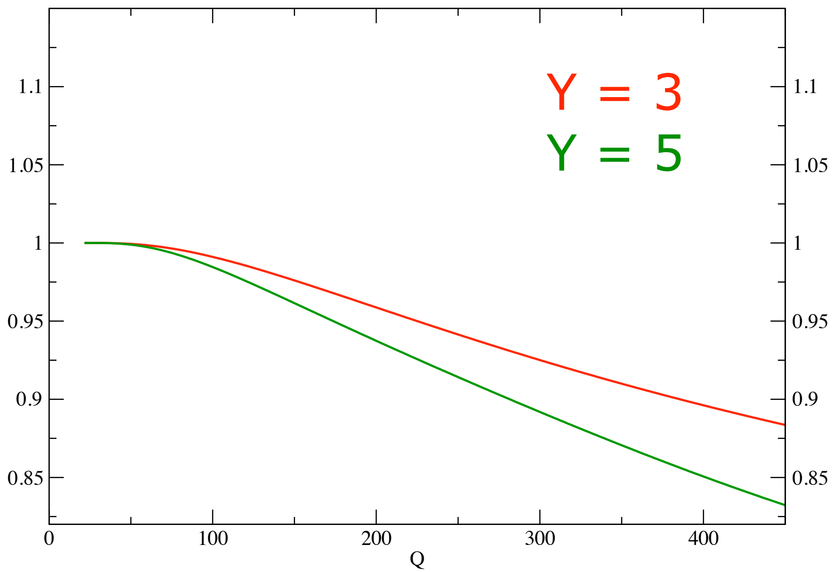


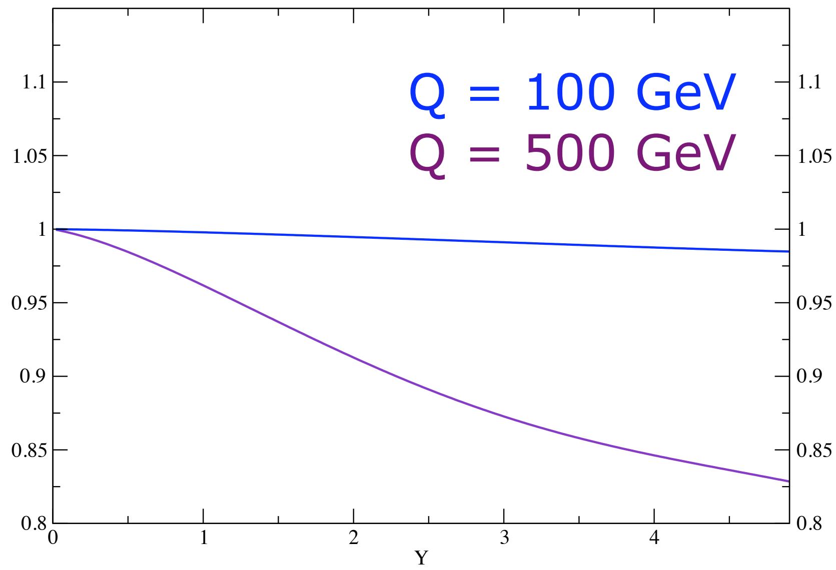
- dotted, one gluon, $\alpha_s^{\,4}$
- dashed: one gluon, up to $\alpha_s^{\,5}$
- dash-dotted: one+two gluons, up to $\alpha_s^{\,5}$

instability:
need of resummation

Resummation of SLL

Resummed results (one out-of-gap gluon) $\sigma_{\text{res}}^{(1)}/\sigma^{(0)}$





- Y = 3, ~ 5 - 10 %
- Y = 5, ~10 – 15 %

- Q = 100 GeV, ~ 2 %
- Q = 500 GeV, ~10 -15 %

- SLL could have an effect as big as 10-15 % in quite extreme dijet configurations
- There are no SLL effect on Higgs+ jj, unless Q_0 < 10 GeV

Conclusions

- There is plenty of interesting QCD physics in gaps between jets
- Soft logs may be relevant for extracting the Higgs coupling to the weak bosons

- Coulomb gluons play an important role: at large enough transverse momenta the PS approach is not valid

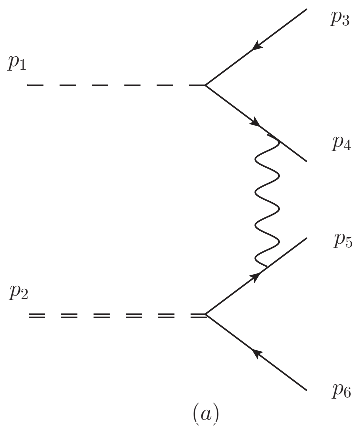
- Dijet cross-section could be sensitive to SLL at large Y and L (e.g. 300 GeV and Y = 5, ~15%)

Outlook (phenomenology)

- Compute the best theory prediction for gaps between jets at the LHC:
  - matching with NLO
  - complete one gluon outside the gap
  - jet algorithm dependence
  - BFKL resummation

Outlook (theory)

- What is the origin of super-leading logs?
- We need to test the hypothesis of the calculation:
  - k_t ordering ?
  - interaction with the remnants ?



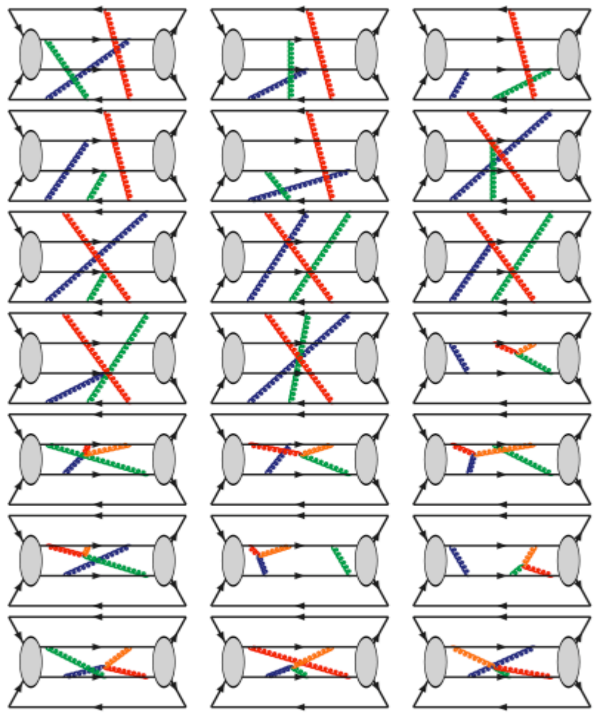
on-going projects in Manchester

Thank you !!!

BACKUP SLIDES

Fixed order calculation

- Gluons are added in all possible ways to trace diagrams and colour factors calculated using COLOUR
- Diagrams are then cut in all ways consistent with strong ordering
- At fourth order there are 10,529 diagrams and 1,746,272 after cutting.
- SLL terms are confirmed at fourth order and computed for the first time at 5^{th} order



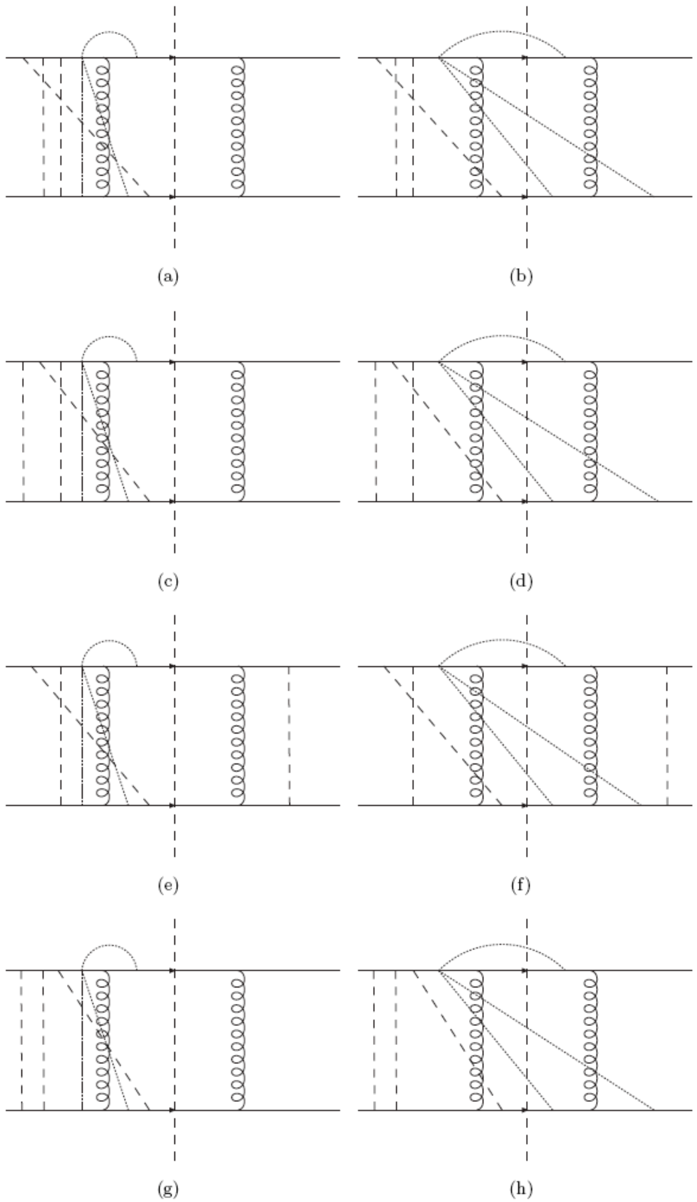
Keates and Seymour
arXiv:0902.0477 [hep-ph]

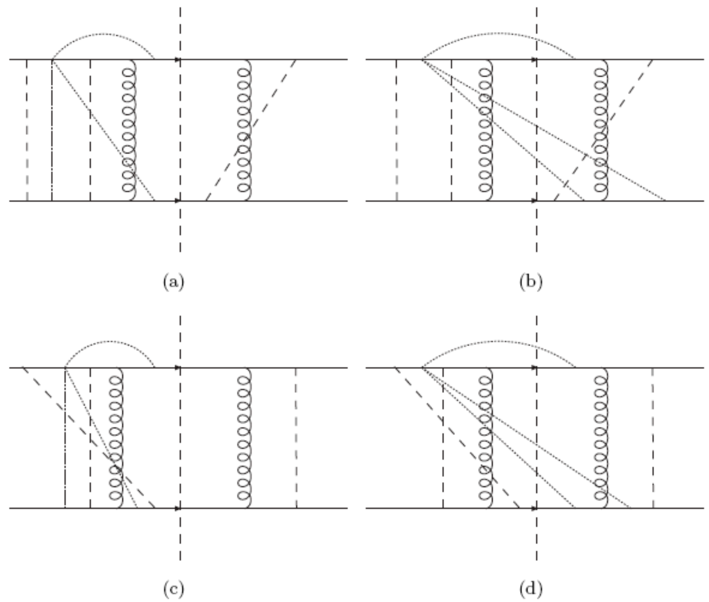
The non-cancelling diagrams (Feynman gauge)

Dotted line is the out-of-gap gluon.

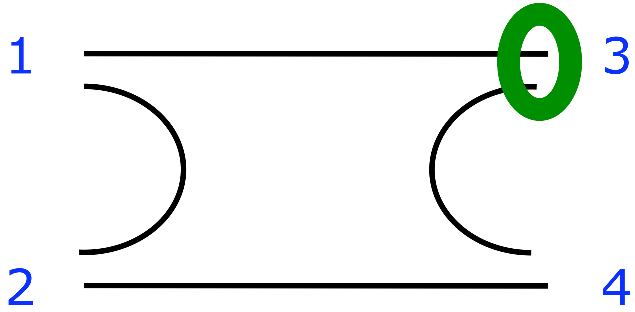
Dashed lines are in-gap & Coulomb gluons.

Springs are hard scatter gluons.

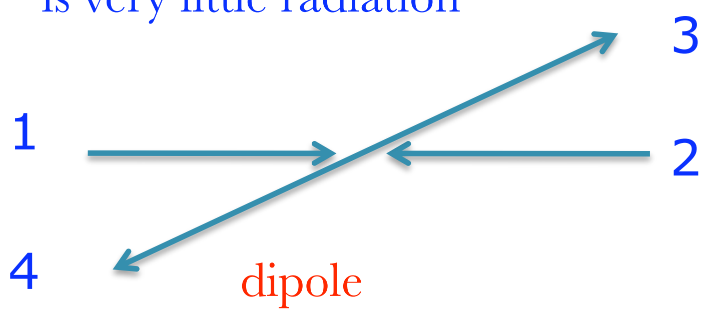




The large N_c limit in HERWIG



- The colour partner of gluon 3 is chosen in each event between 1 and 4 with equal probability
- If the partner is on the same side of the gap there is very little radiation



dipole

- Inclusive interjet radiation

$$4 \times \frac{1}{2} \times \frac{1}{2} \times N_c = N_c$$

jets

Partner across the gap

- No radiation probability

$$\left[\frac{1}{2} + \frac{1}{2} e^{-N_c/2Y\xi}\right]^4$$

Thanks to Mike Seymour

An interesting link to small-*x*

- The non-linear evolution equation which resums non-global logs resembles the BFKL/BK equations (in the dipole picture)

$$\frac{d^2\Omega_c}{4\pi}\frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ac})(1-\cos\theta_{cb})} \to \frac{d^2\mathbf{x}_c}{2\pi}\frac{\mathbf{x}_{ab}^2}{\mathbf{x}_{ac}^2\mathbf{x}_{cb}^2}$$

- The two kernels can be mapped via a stereographic projection

$$\Omega = (\theta, \phi) \to \mathbf{x} = (x^1, x^2)$$

Avsar, Hatta and Matsuo
arXiv:0903.4285 [hep-ph]

- Is there a fundamental connection between non-global (soft) evolution and small-*x* ?