

Excited meson spectroscopy and radiative transitions from LQCD

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With Jo Dudek, Robert Edwards, Mike Peardon,
David Richards and the *Hadron Spectrum Collaboration*



Outline

- Introduction and motivation
- Excited spectra from LQCD – method outline
- Results – isovector and kaon spectra
- Photocouplings – charmonium
- Summary and outlook

PR D79 094504 (2009)
PRL 103 262001 (2009)
PR D82 034508 (2010)

Motivation

Renaissance in excited charmonium spectroscopy

BABAR, Belle, BES, CLEO-c

Upcoming experimental efforts, also in the light meson sector

GlueX (JLab), BESIII, PANDA

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Exotics ($J^{PC} = \mathbf{1}^{-+}, \mathbf{2}^{+-}, \dots$)? – can't just be a $q\bar{q}$ pair

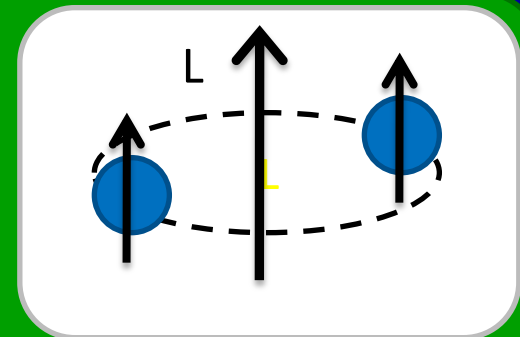
e.g. hybrids, multi-mesons

Two spin-half fermions: $2S+1L_J$

Parity: $P = (-1)^{(L+1)}$

Charge Conj Sym: $C = (-1)^{(L+S)}$

$J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}, 2^{--}, 2^{++}, 2^{-+}, \dots$



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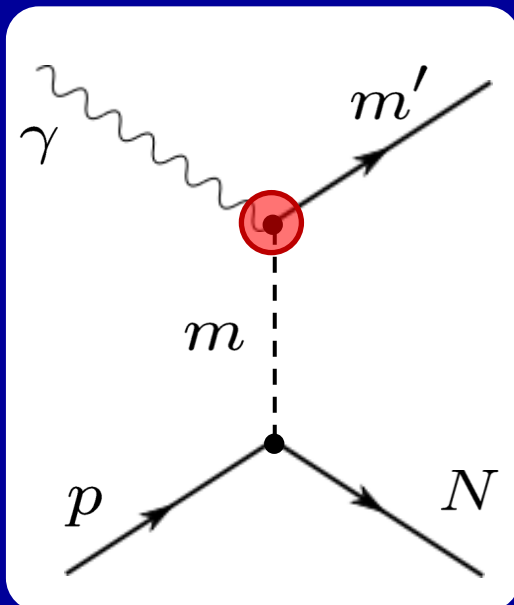
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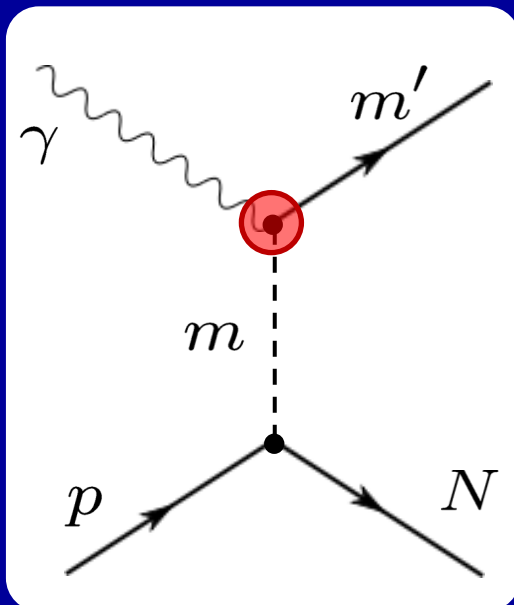
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Use Lattice QCD to extract excited spectrum...

... and photocouplings (tested in charmonium)

PR D77 034501 (2008), PR D79 094504 (2009)



Spectroscopy on the lattice

Calculate **energies** and **matrix elements** (“overlaps”, Z 's)
from correlation functions of meson interpolating fields

$$C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle$$

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(p = 0)

More about operators later...

‘Distillation’ technology for constructing
on lattice PR D80 054506 (2009)

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$$Z_i^{(n)} \equiv \langle 0 | O_i | n \rangle$$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2 E_n} \langle 0 | O_i(0) | n \rangle \langle n | O_j(0) | 0 \rangle$$

Variational Method

Large basis of operators \rightarrow matrix of correlators

$$C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle$$

Generalised eigenvector problem:

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

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Eigenvectors \rightarrow optimal linear combination of operators to overlap on to a state

$$\Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

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Var. method uses orthog of eigenvectors; don't just rely on separating energies

Spin on the lattice

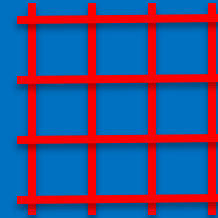
On a lattice, 3D rotation group is broken to Octahedral Group

2D Example

Eigenstates of angular momentum are $e^{iJ\phi}$

On a lattice, the allowed rotations are $\phi \rightarrow \phi + \pi/2$

Can't distinguish e.g. $J = 0$ and $J = 4$



Spin on the lattice

On a lattice, 3D rotation group is broken to Octahedral Group

In continuum:

Infinite number of *irreps*: $J = 0, 1, 2, 3, 4, \dots$

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In continuum:

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On lattice:

Finite number of *irreps*: A_1, A_2, T_1, T_2, E (and others for half-integer spin)

Irrep	A_1	A_2	T_1	T_2	E
Dim	1	1	3	3	2

Cont. Spin	0	1	2	3	4	...
Irrep(s)	A_1	T_1	$T_2 + E$	$T_1 + T_2 + A_2$	$A_1 + T_1 + T_2 + E$...

Spin and operator construction

Construct operators which only overlap on to one spin in the continuum limit

Circular basis for (spatial) Γ and $D \rightarrow$ transform as $J=1$

Couple using SU(2) Clebsch Gordans

$\Gamma \times D \times D \times \dots$ (up to 3 derivs)

$$\langle 0 | \mathcal{O}^{J,M} | J', M' \rangle = Z^{[J]} \delta_{J,J'} \delta_{M,M'}$$

definite J^{PC}

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'Subduce' operators on to lattice irreps ($J \rightarrow \Lambda$):

$$\mathcal{O}_{\Lambda,\lambda}^{[J]} = \sum_M \mathcal{S}_{\Lambda,\lambda}^{J,M} \mathcal{O}^{J,M}$$

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Up to 26 ops in Λ^{PC} channel

e.g. $\mathcal{O}^{[2]} \rightarrow T_2$ and $\mathcal{O}^{[2]} \rightarrow E$

Given continuum op \rightarrow
same Z in each Λ
(ignoring lattice mixing)

Spin and operator construction

Construct operators which only overlap on to one spin in the continuum limit

Circular basis

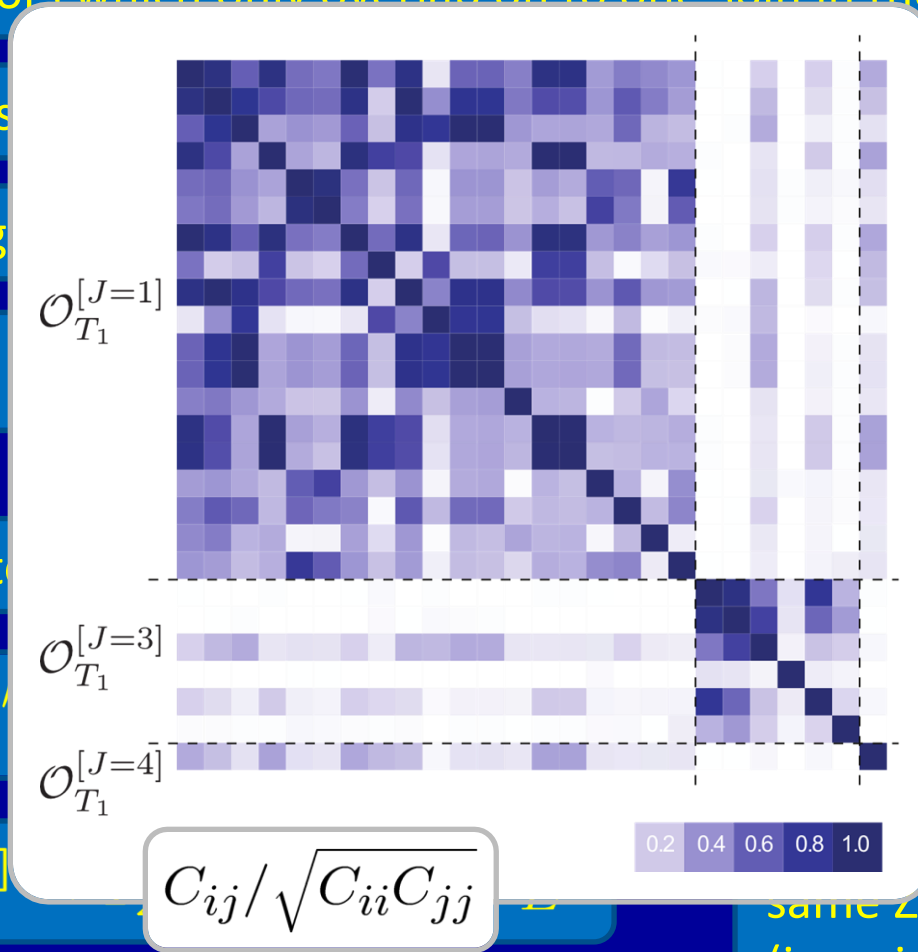
Couple using

$$\langle 0 | \mathcal{O}^{J,M} | J' \rangle$$

'Subduce' operator

$$\langle 0 | \mathcal{O}_{\Lambda,\lambda}^{[J]} | J' \rangle$$

e.g. $\mathcal{O}^{[2]}$



(up to 3 derivs)

$$= \sum_M \mathcal{S}_{\Lambda,\lambda}^{J,M} \mathcal{O}^{J,M}$$

ops in Λ^{PC} channel

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Calculation details

- Dynamical calculation. Clover fermions
- Anisotropic ($a_s/a_t = 3.5$), $a_s \sim 0.12$ fm, $a_t^{-1} \sim 5.6$ GeV
- Two volumes: 16^3 ($L_s \approx 2.0$ fm) and 20^3 ($L_s \approx 2.4$ fm)

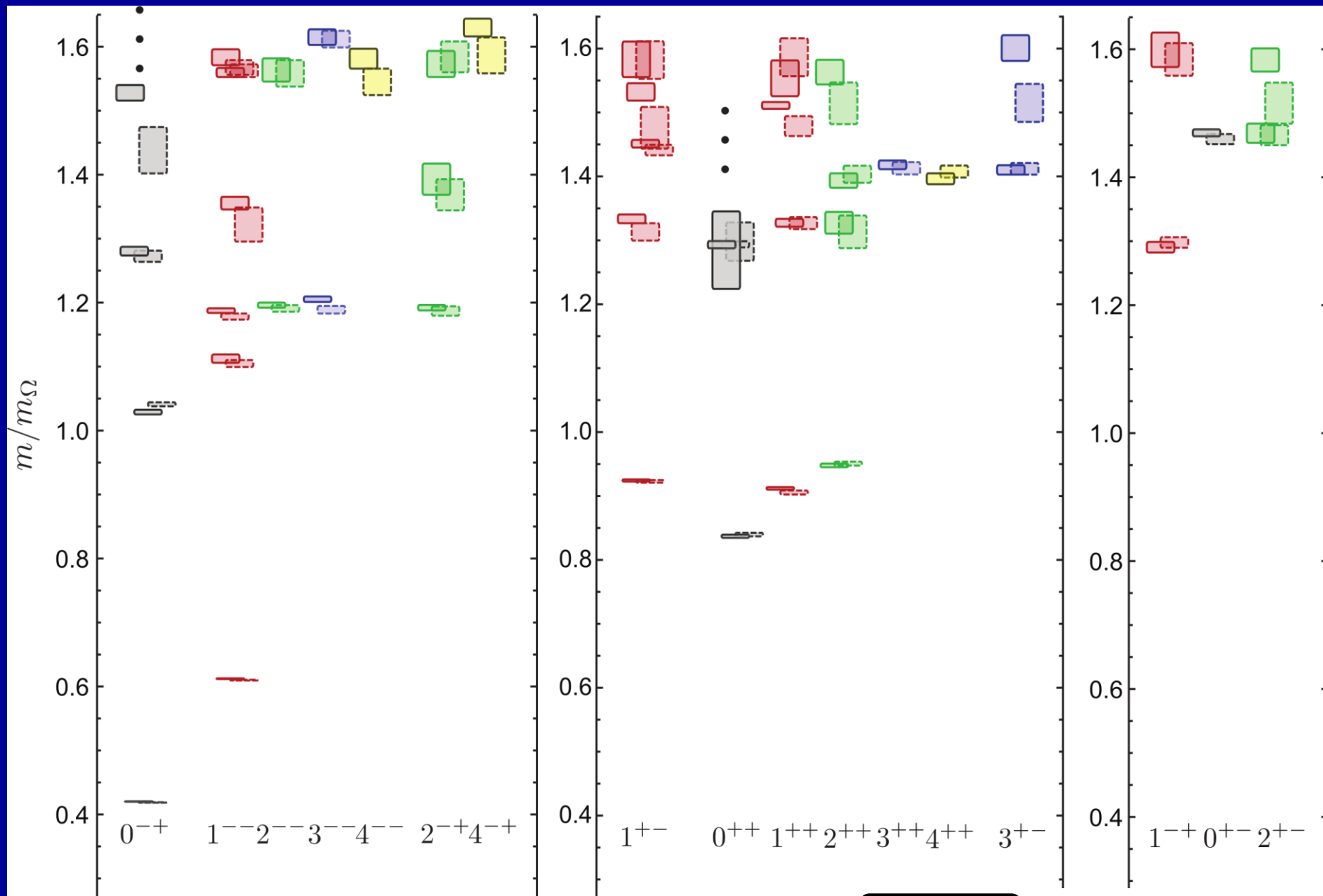
Lattice details in: PR D78 054501, PR D79 034502

- Only connected diagrams – Isovectors ($I=1$) and kaons
- As an example: three degenerate ‘light’ quarks ($N_f = 3$, $M_\pi \approx 700$ MeV)
- Also ($N_f = 2+1$) $M_\pi \approx 520, 440, 400$ MeV

SU(3) sym

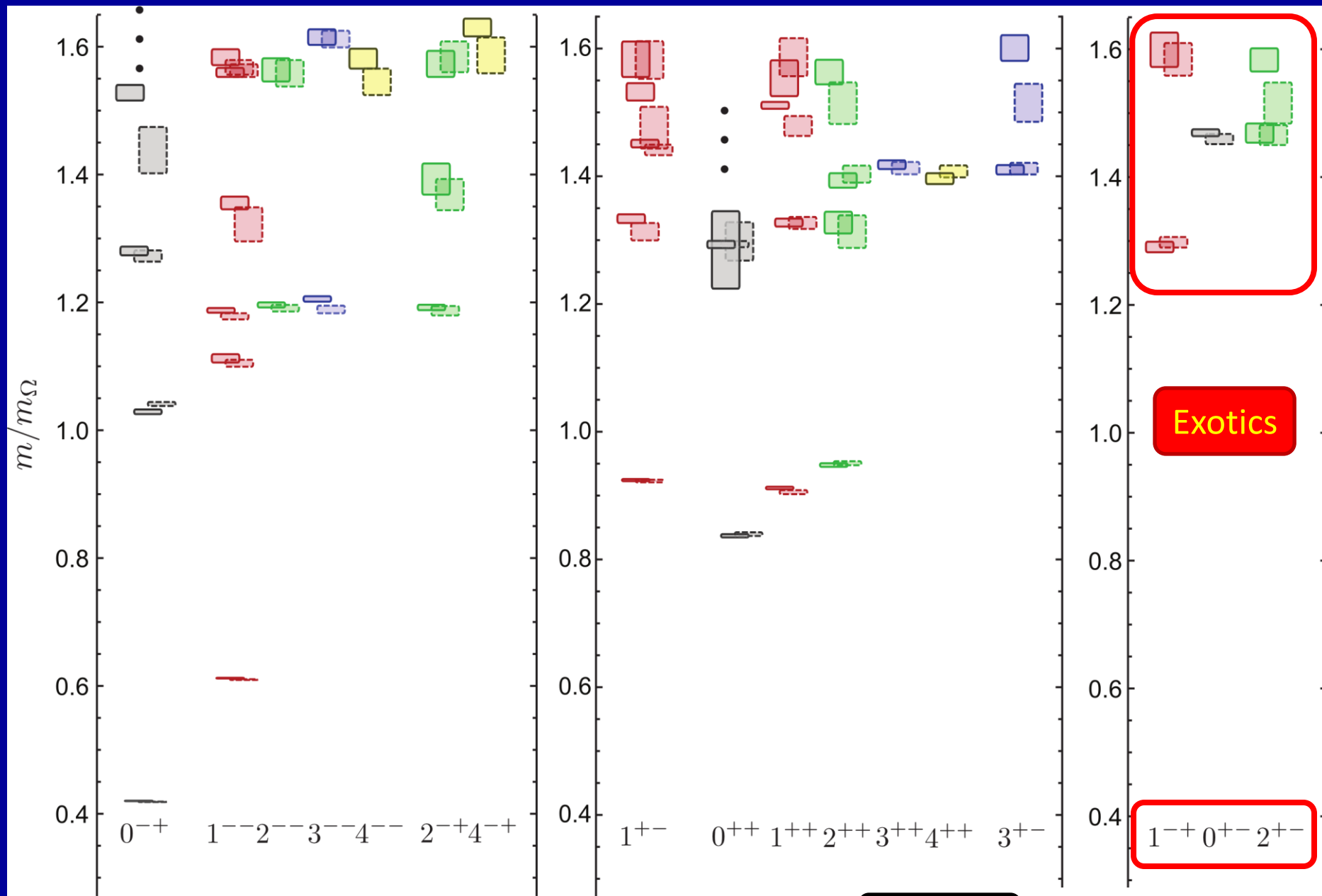
~ 500 cfgs x 9 t-sources

Method details and results: PRL 103 262001 (2009), PR D82 034508 (2010)



$N_f = 3$ isovectors

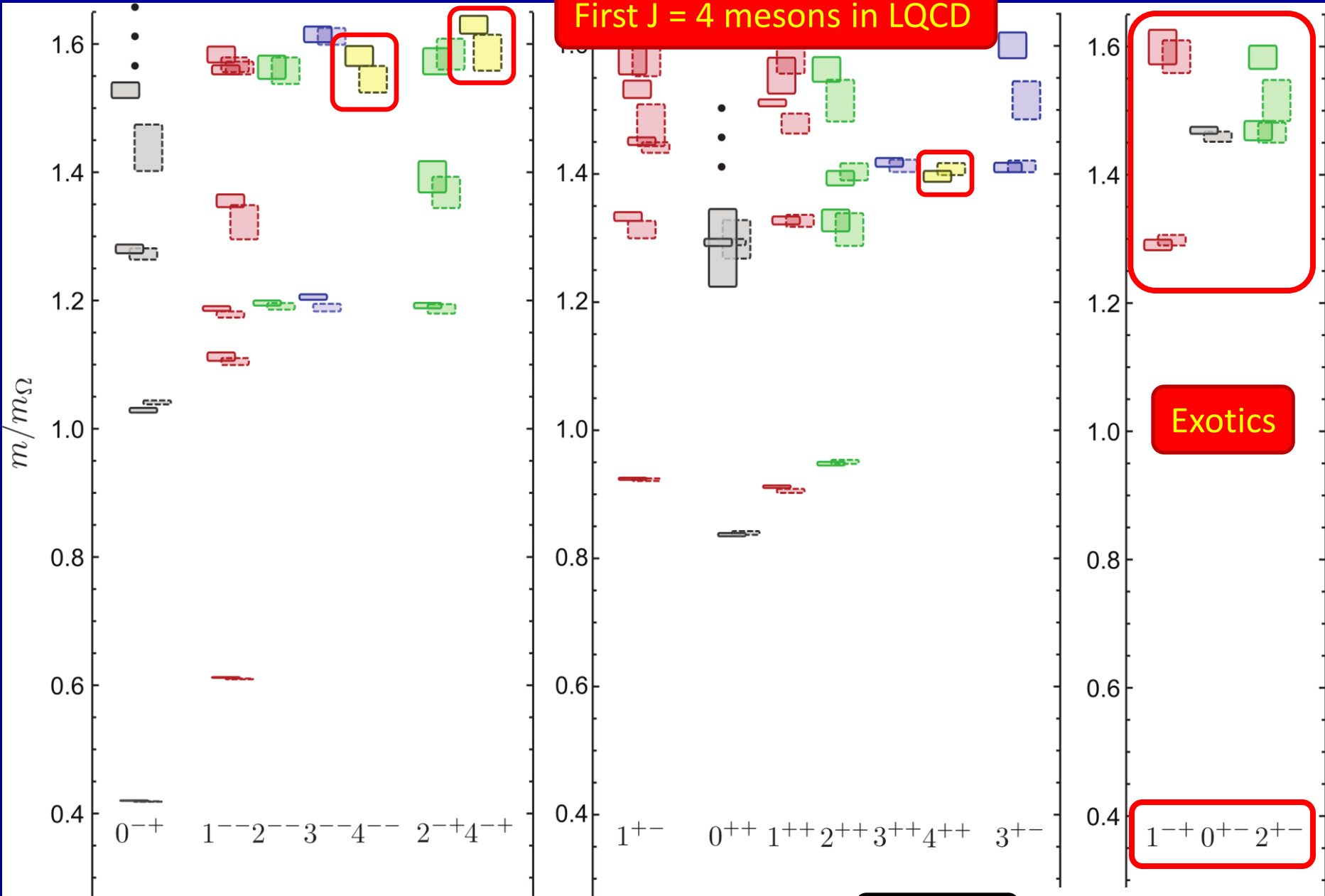
16^3 (~ 2 fm) and 20^3 (2.4 fm)



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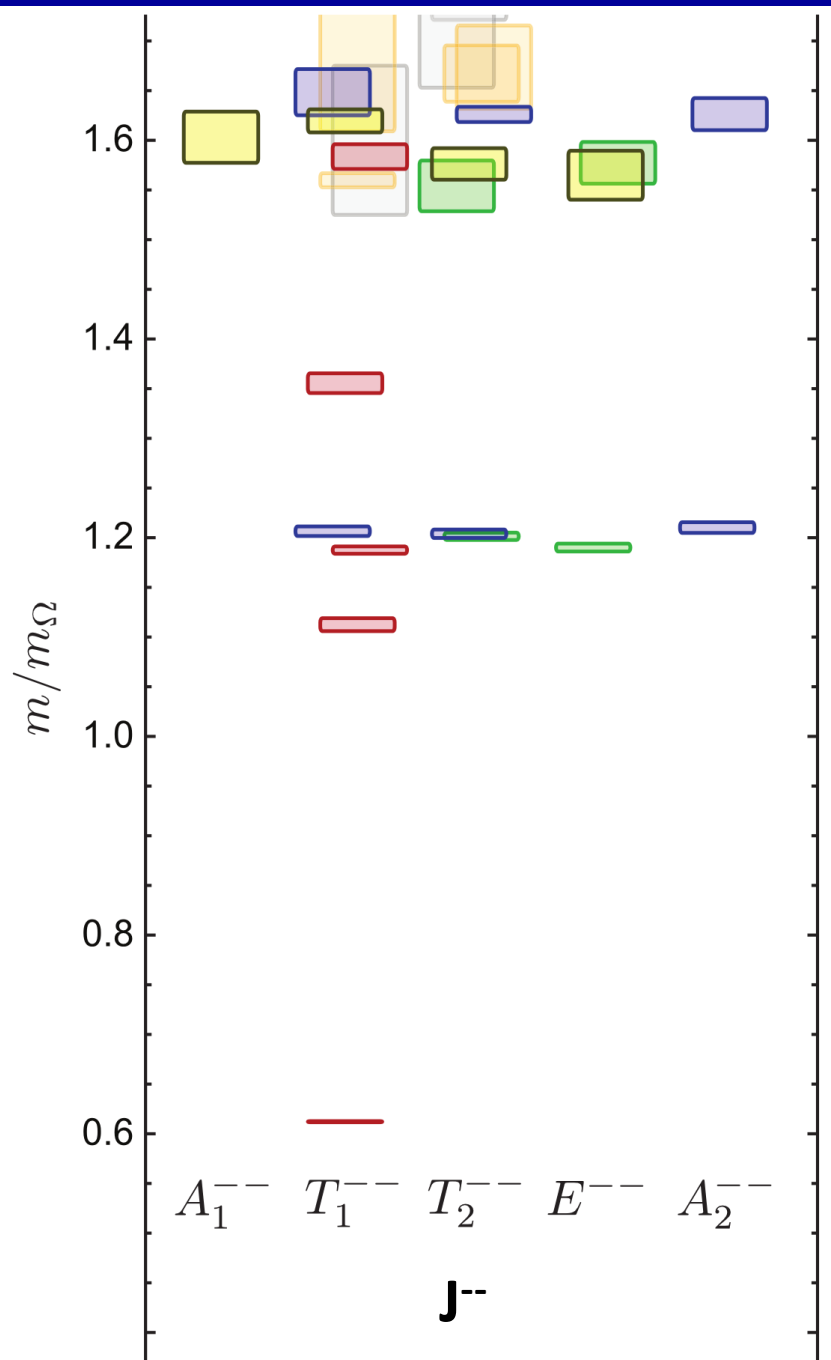
First $J = 4$ mesons in LQCD



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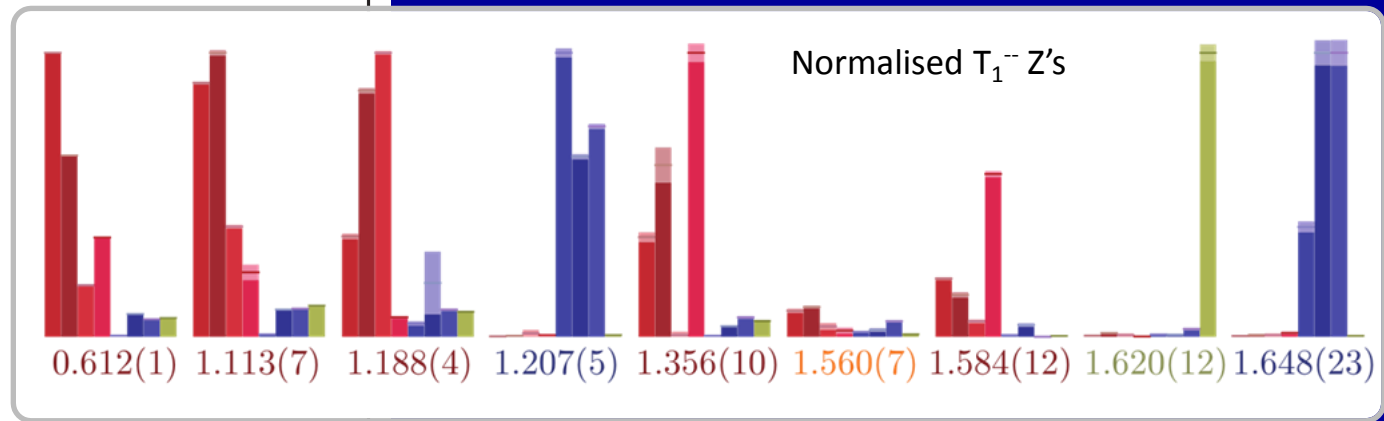
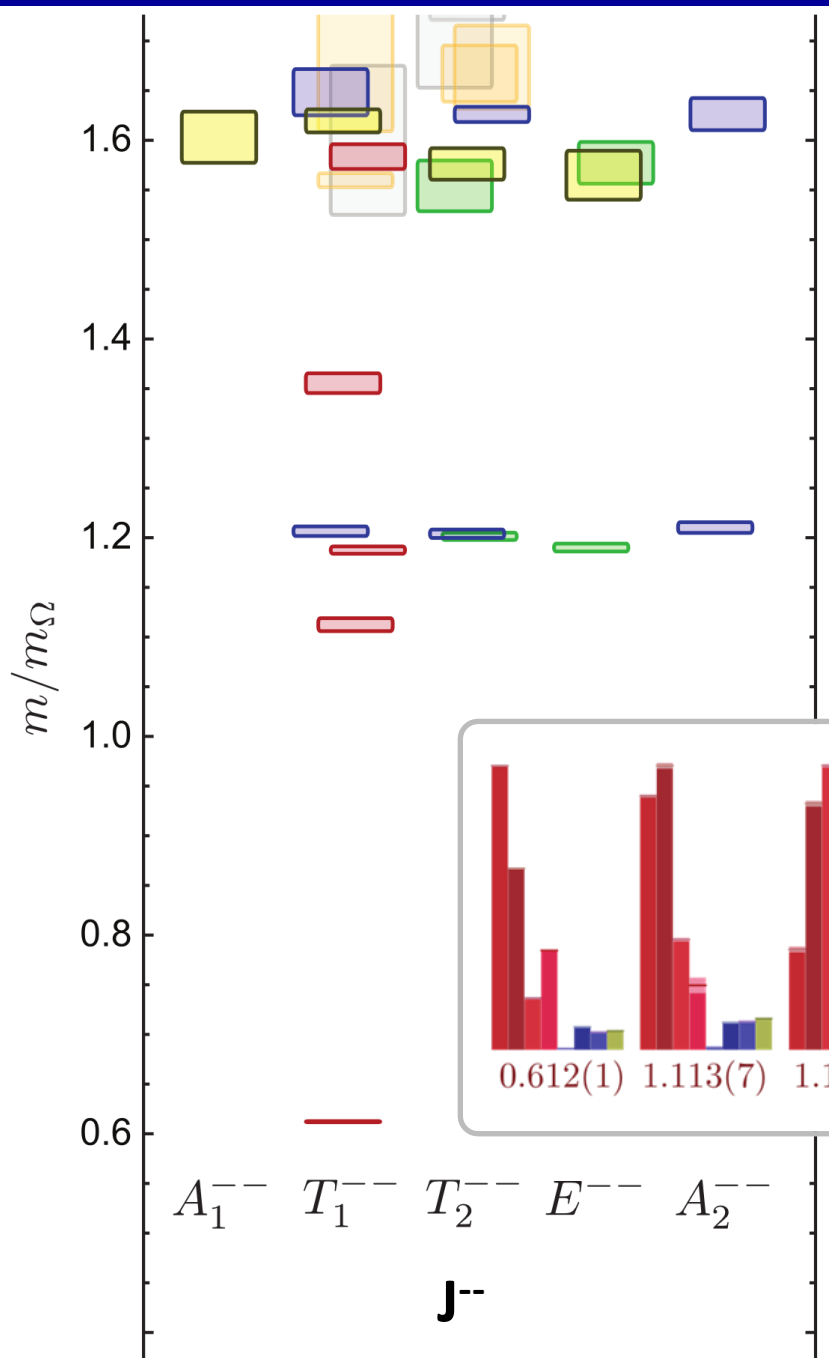
Z values



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$$\langle 0 | \mathcal{O}_{\Lambda, \lambda}^{[J]} | J', M \rangle = S_{\Lambda, \lambda}^{J, M} Z^{[J]} \delta_{J, J'}$$

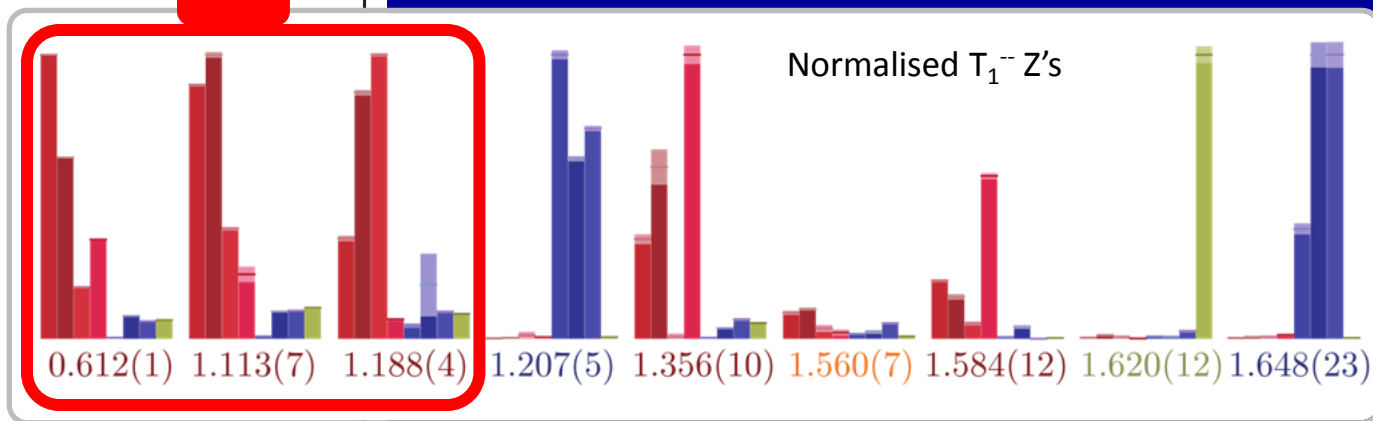
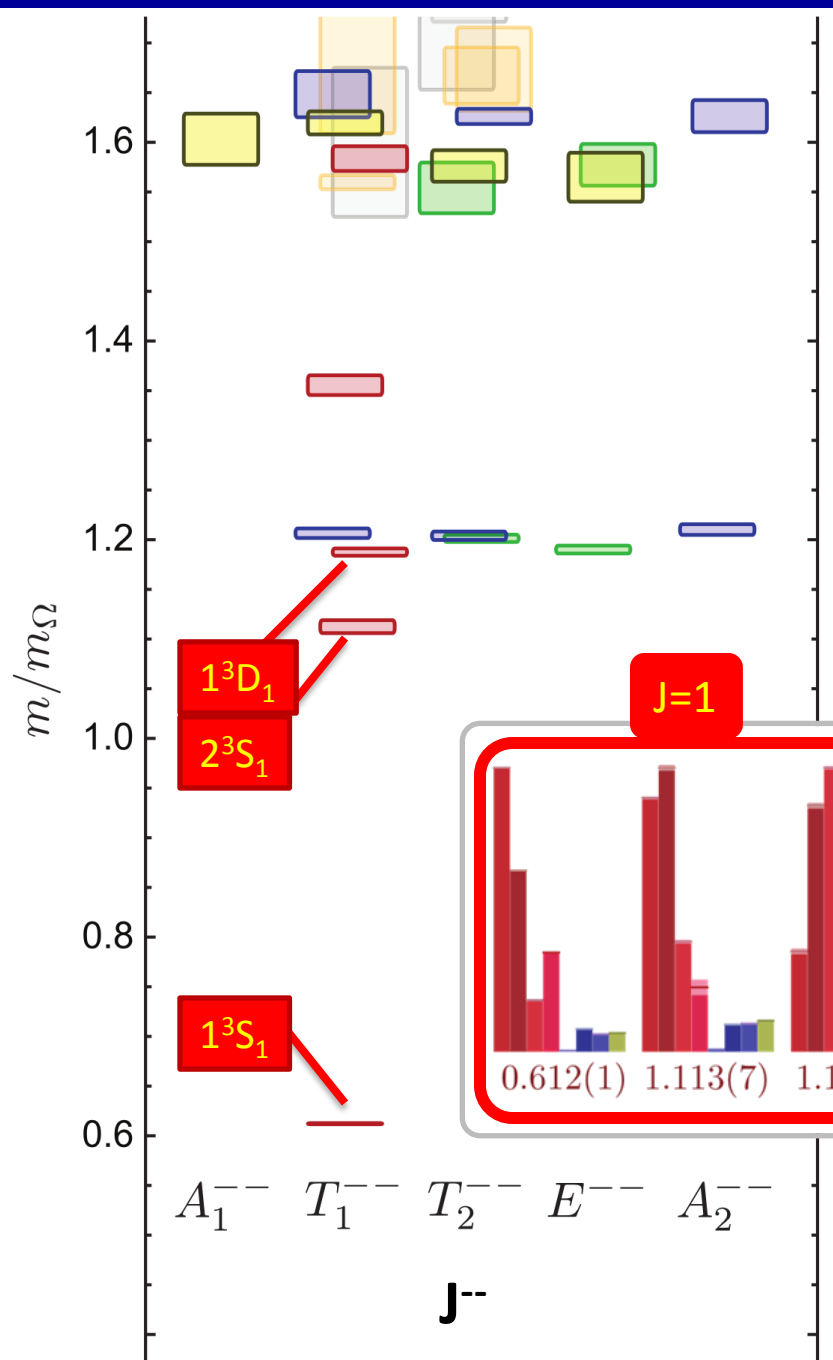
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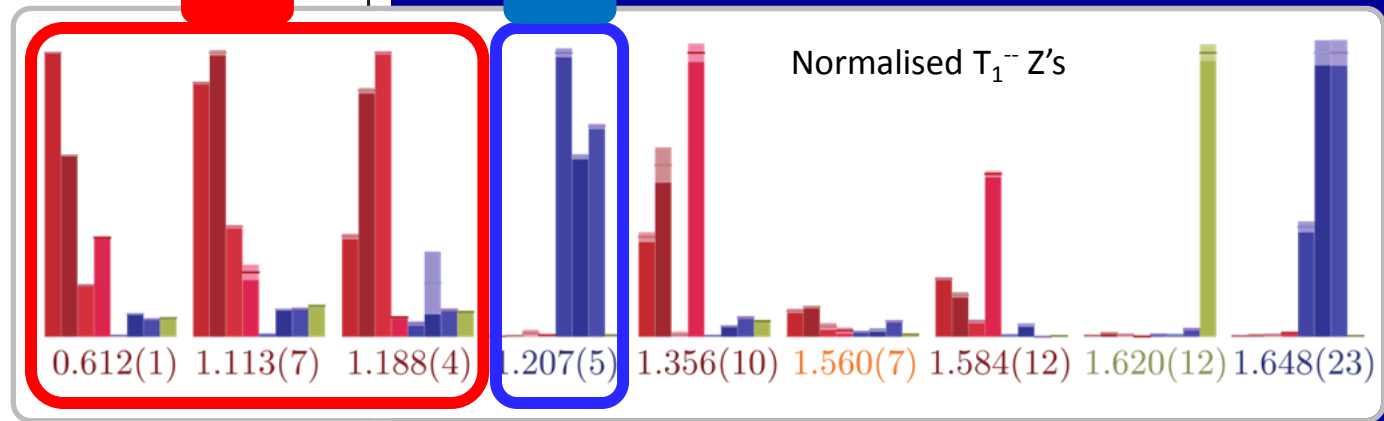
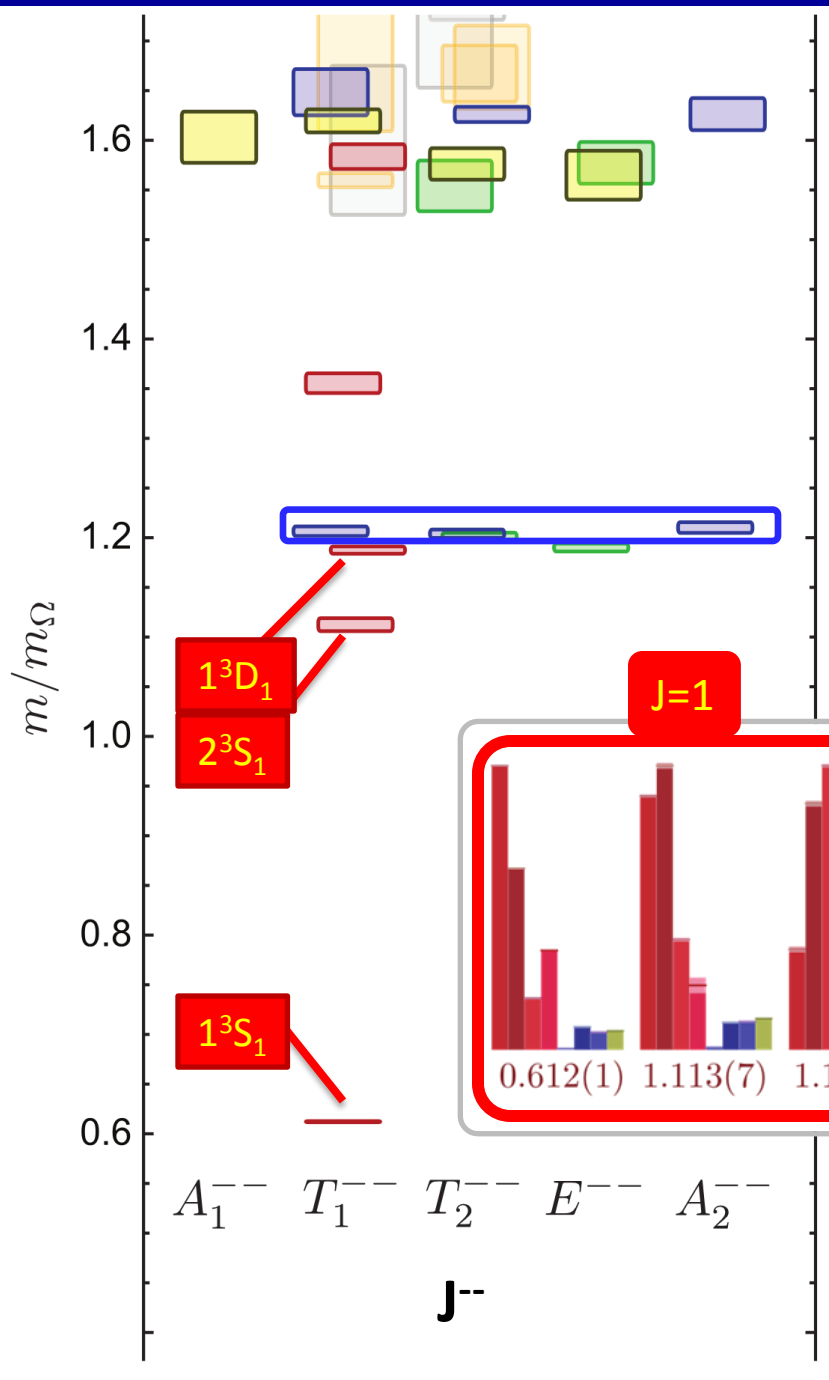
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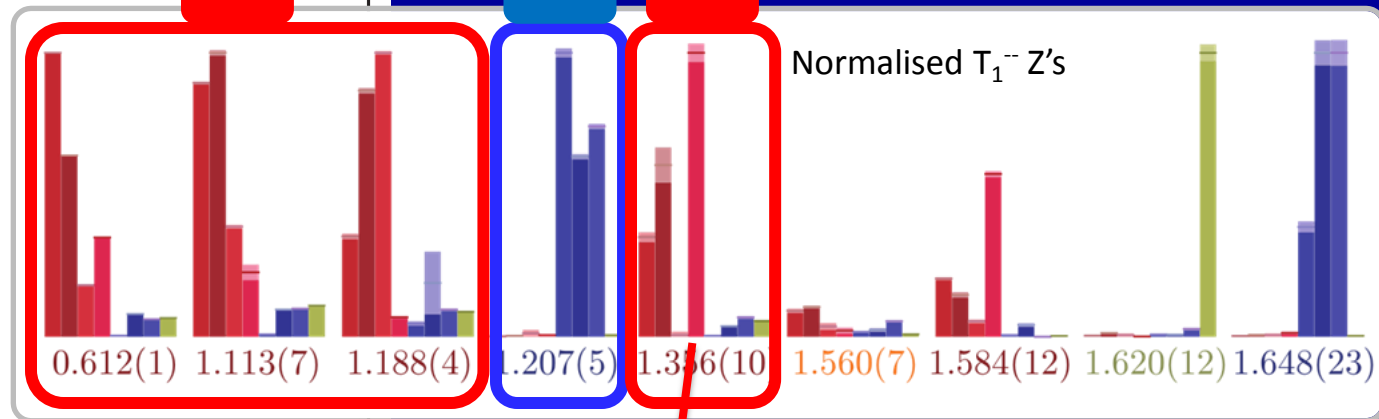
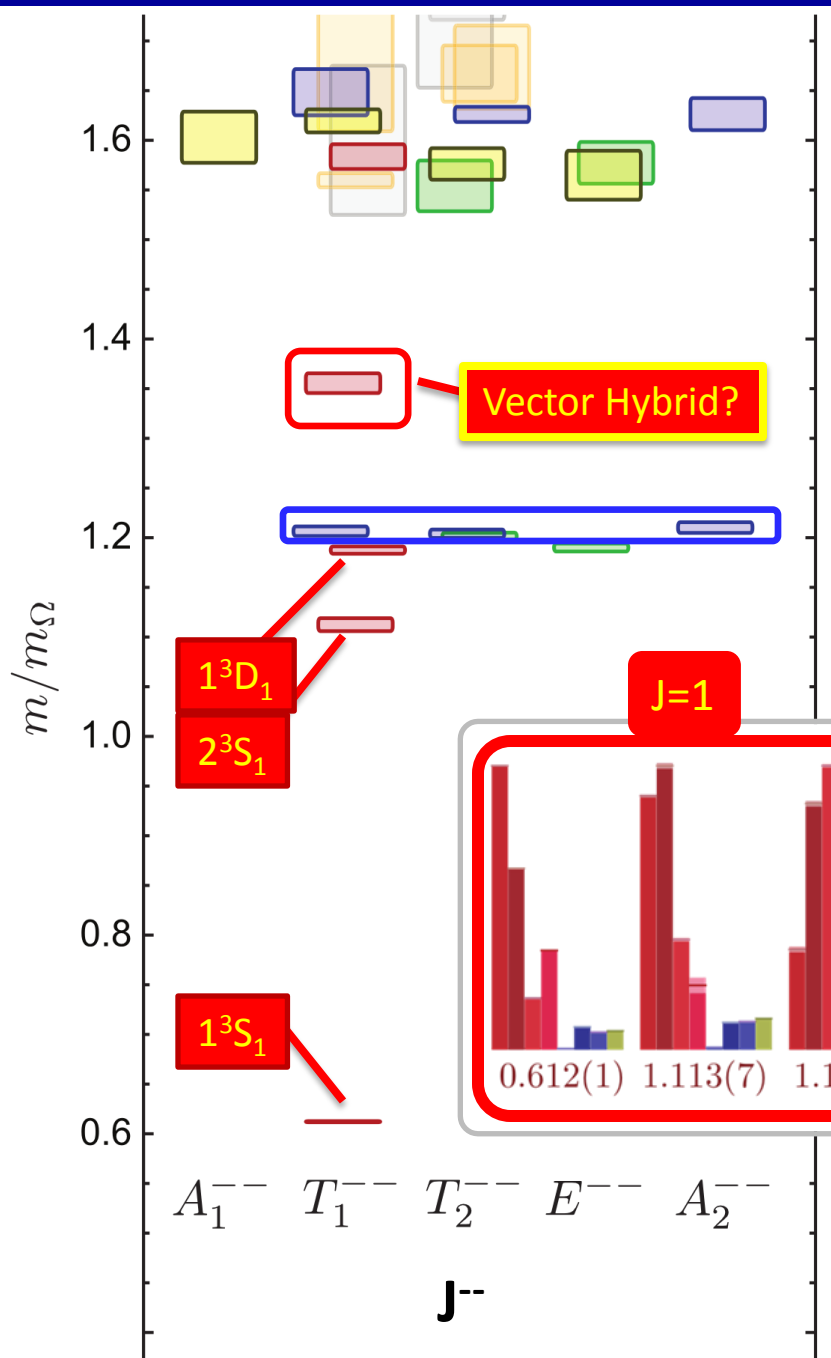
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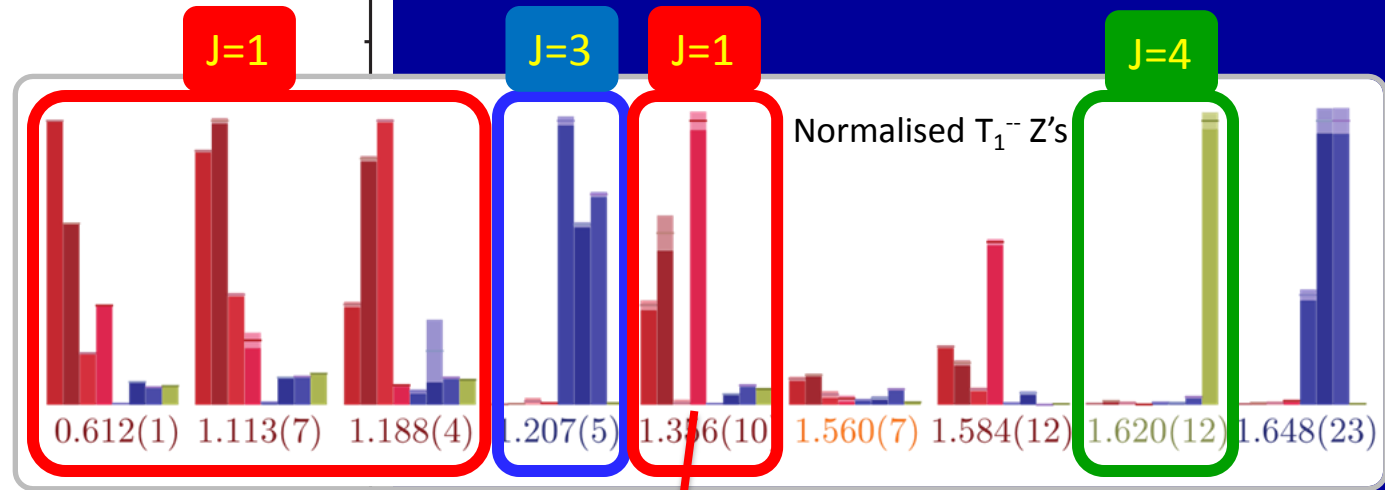
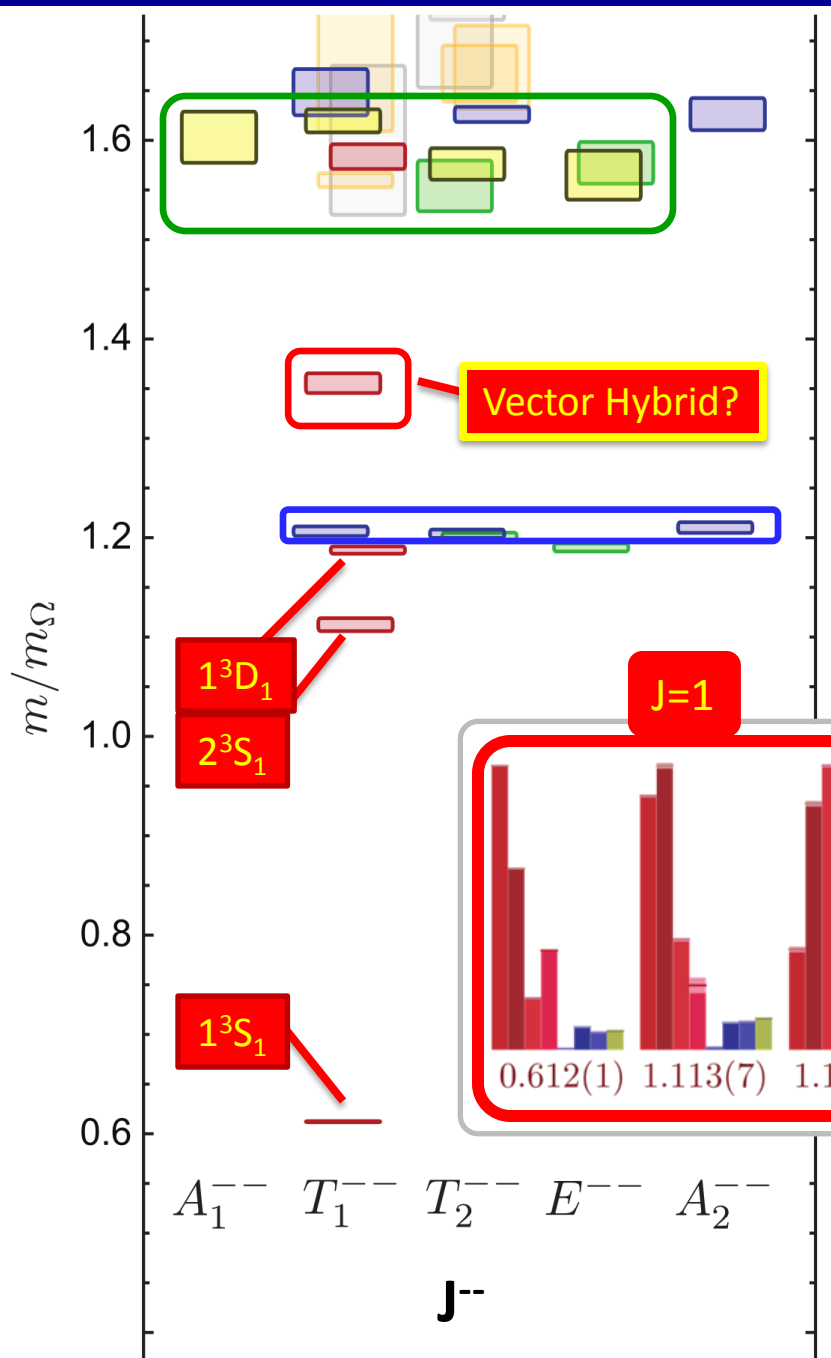


This operator $\sim [D_i, D_j]$

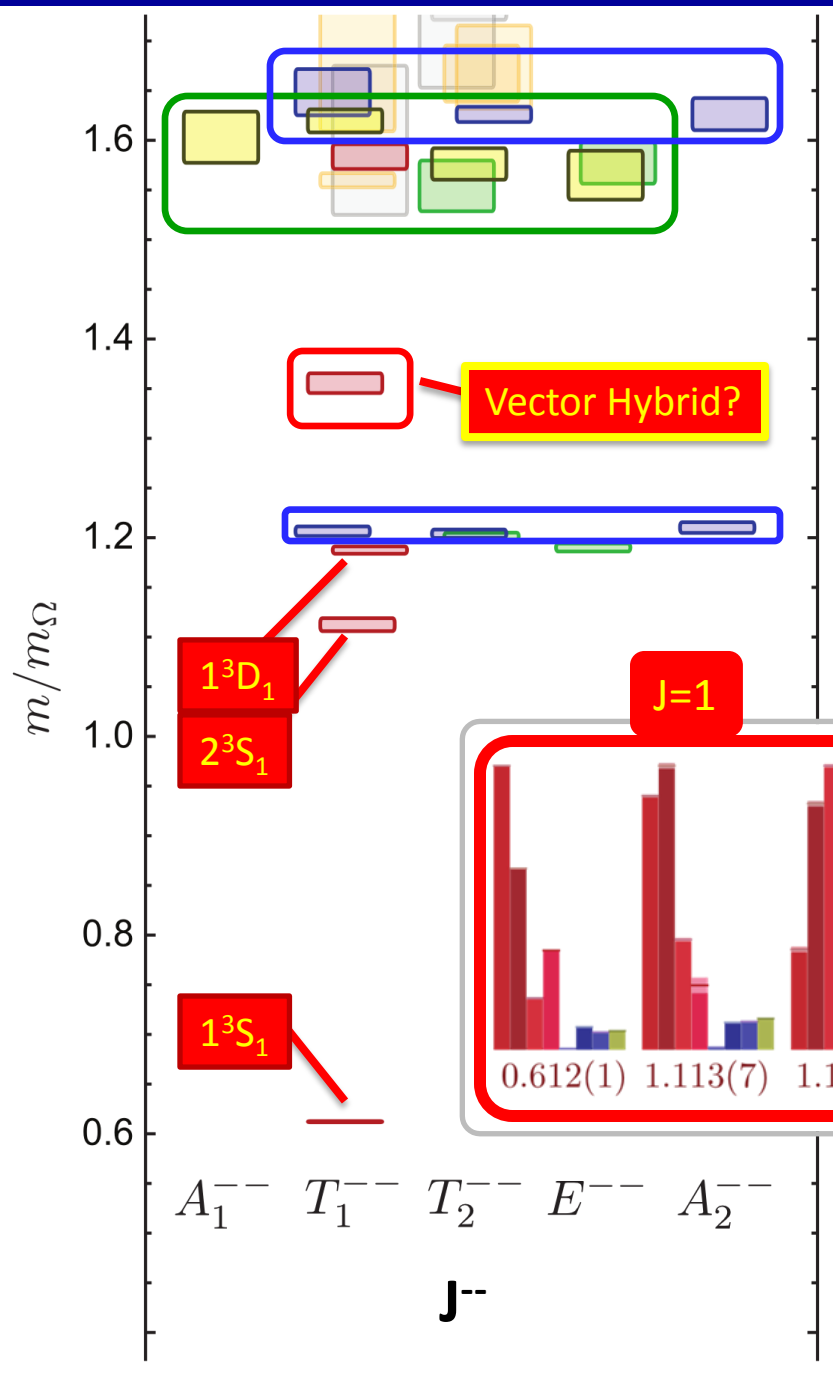
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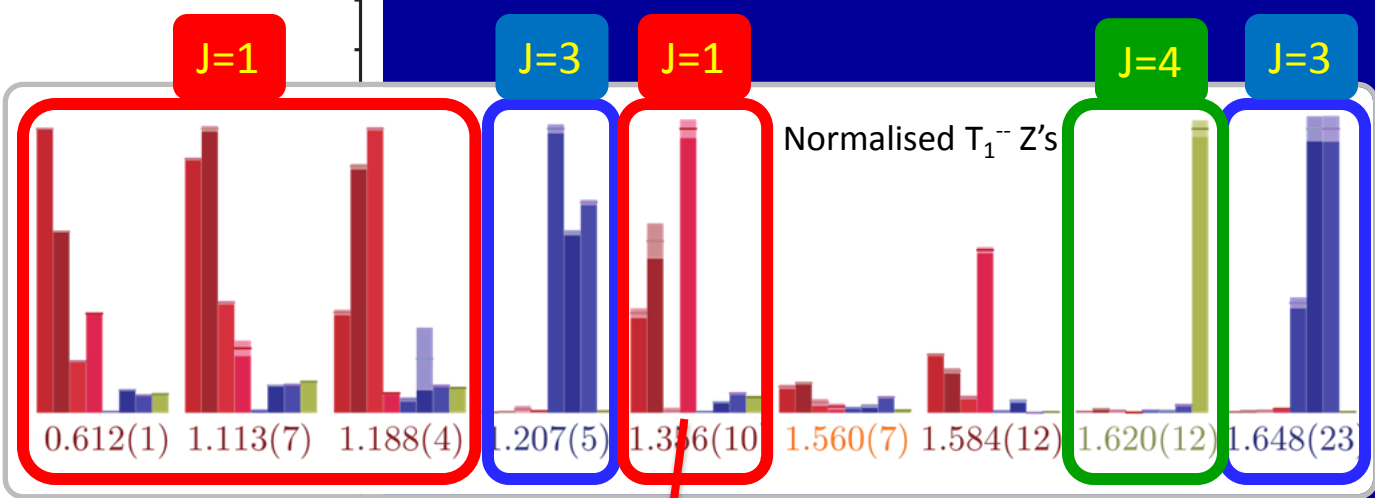
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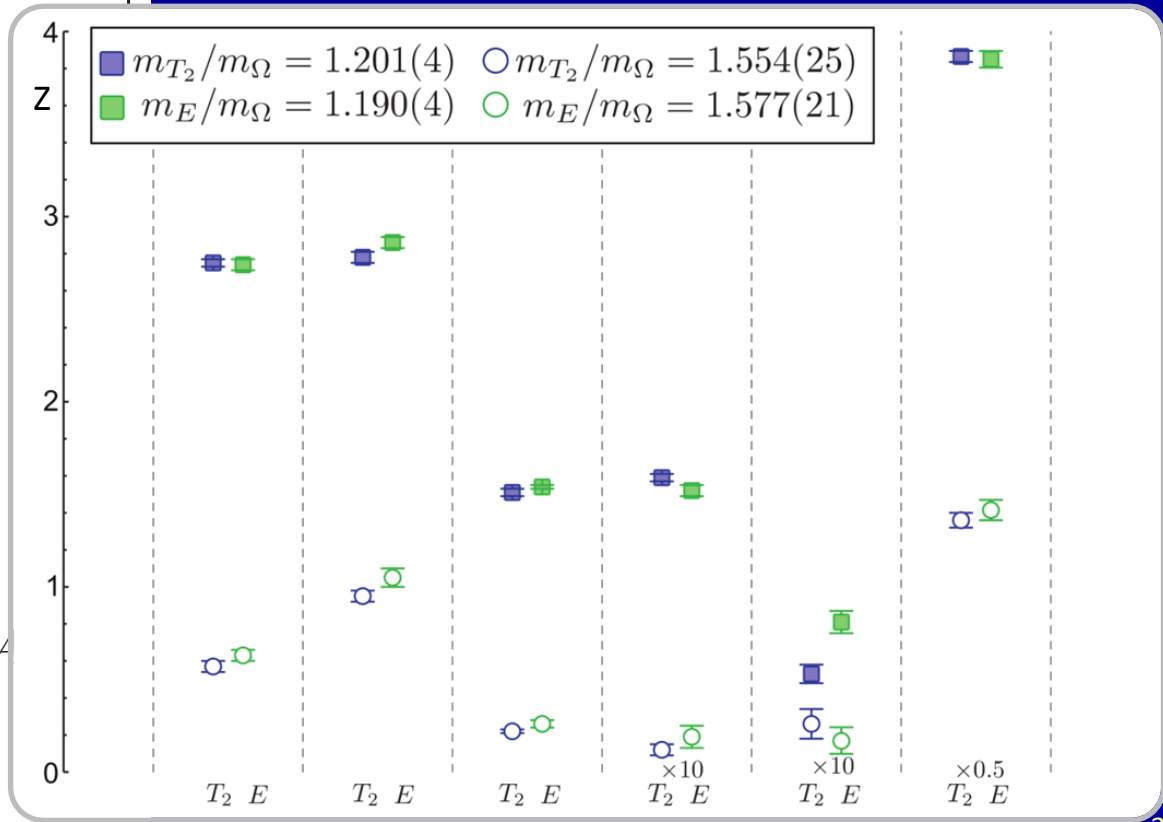
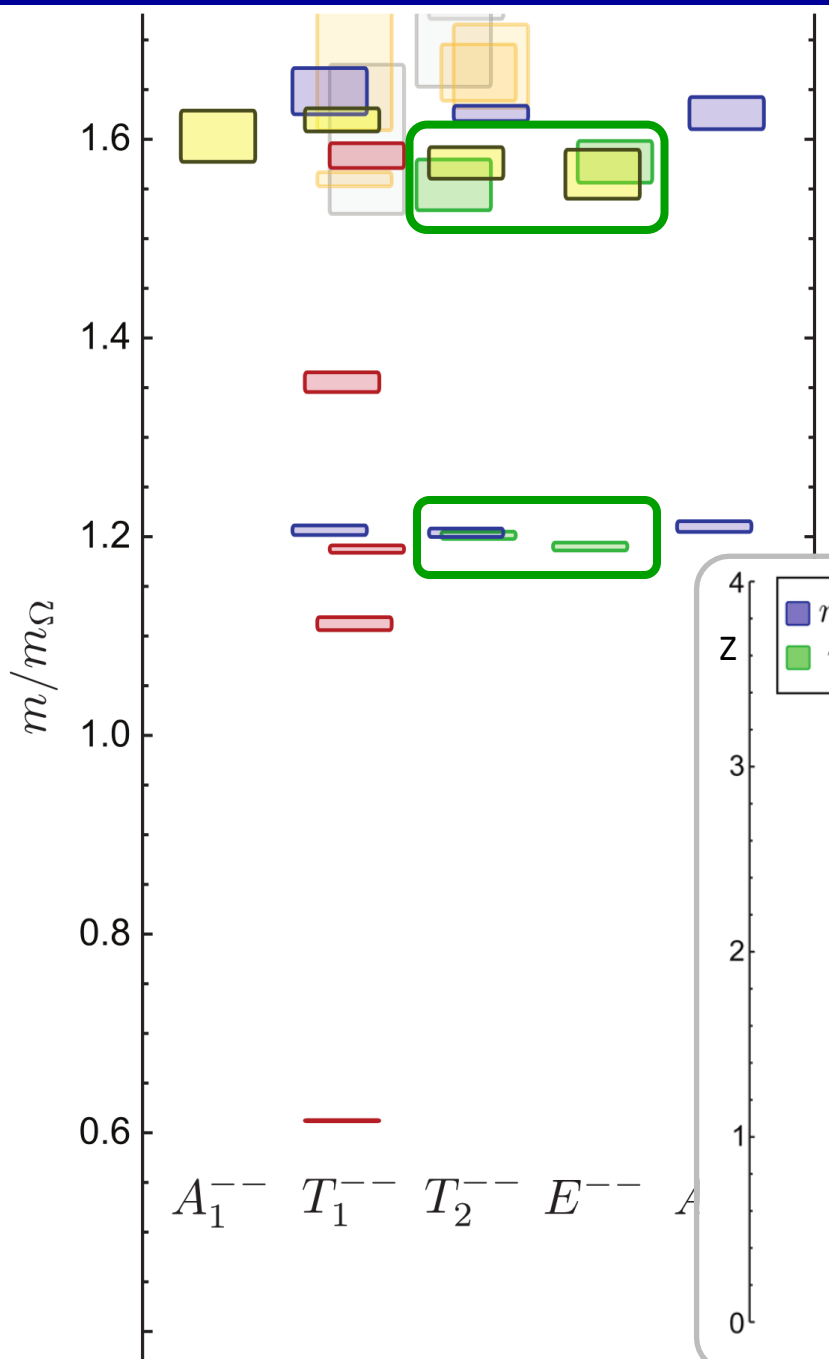


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Z values – spin 2

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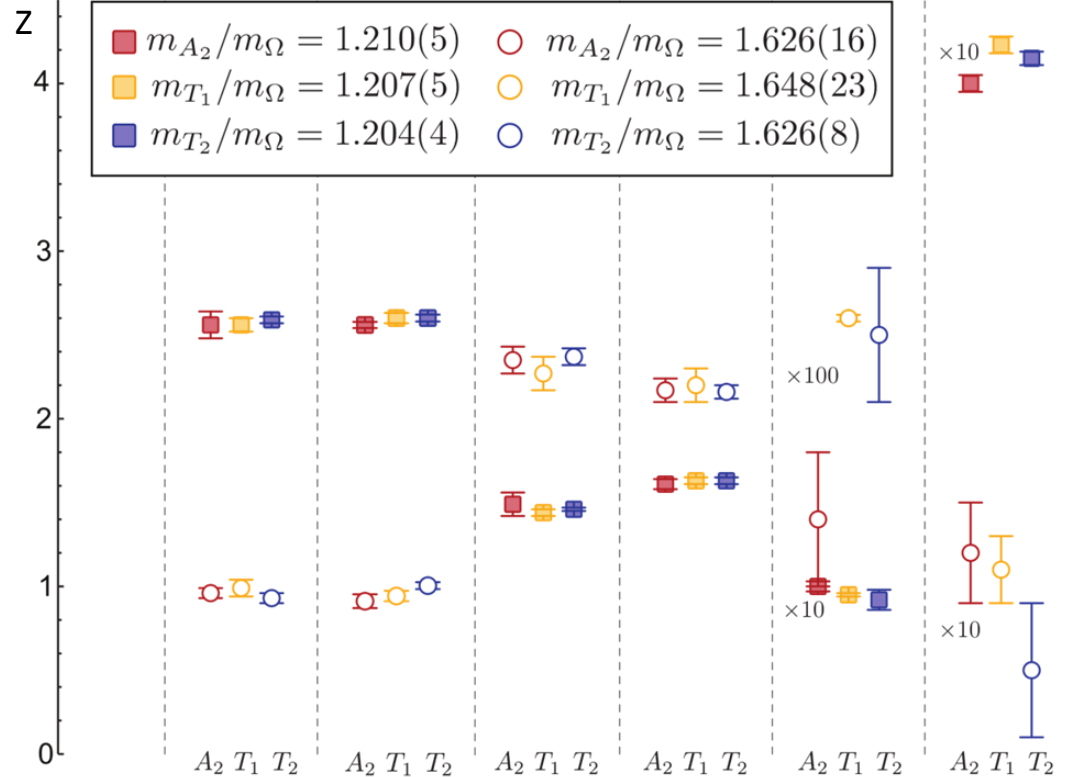
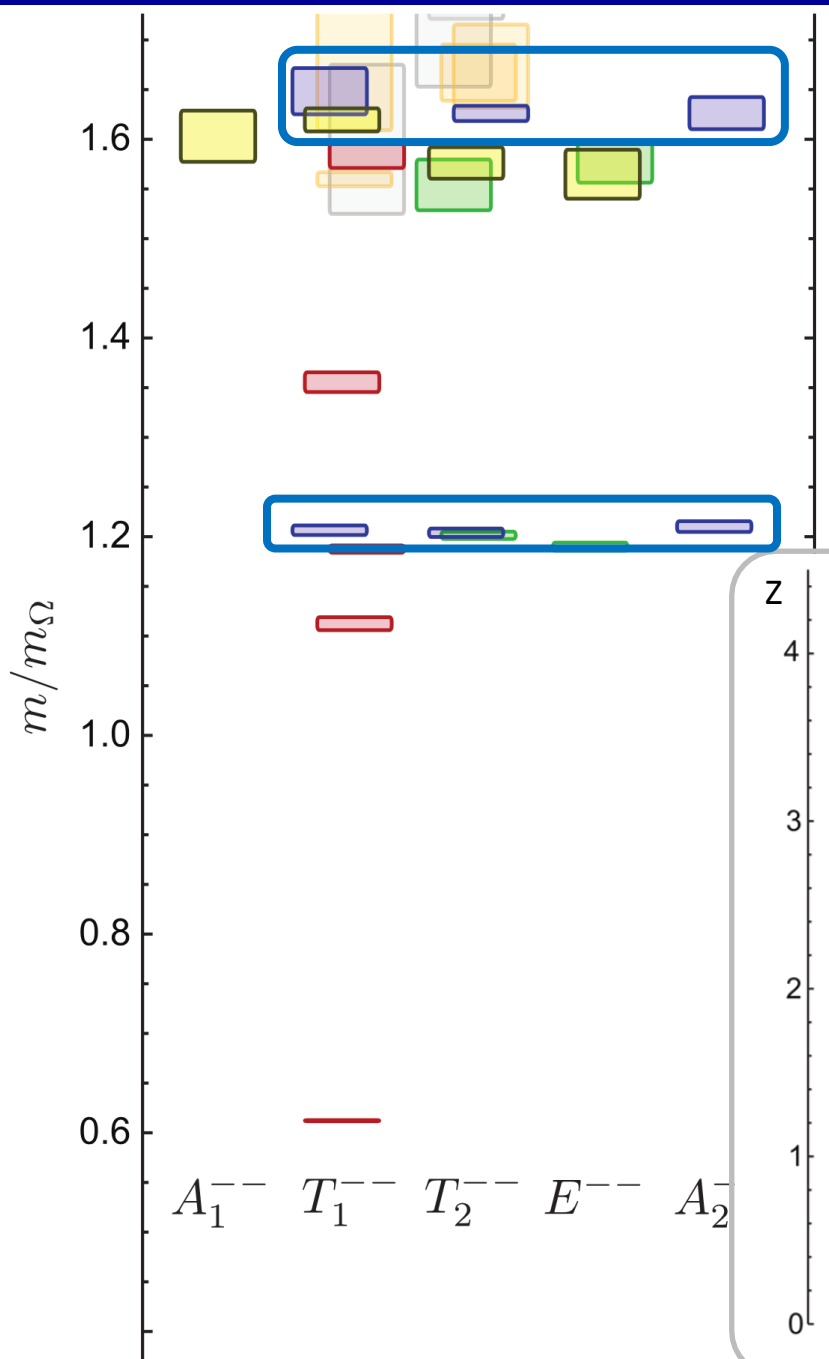
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same Z for each subduced irrep



Z values – spin 3

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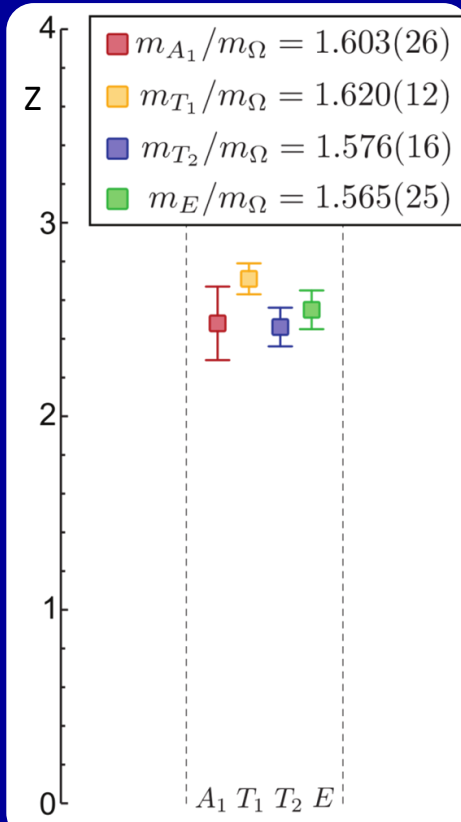
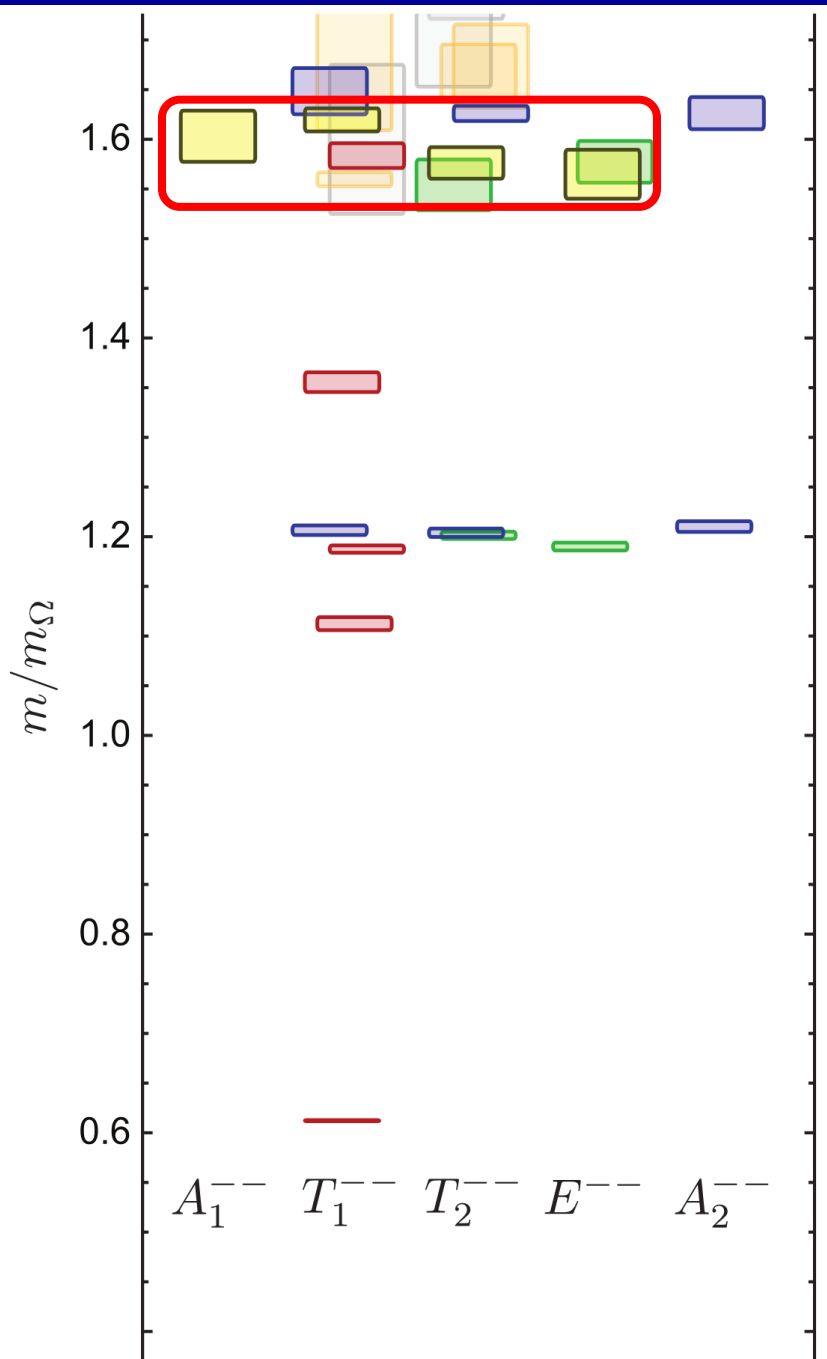
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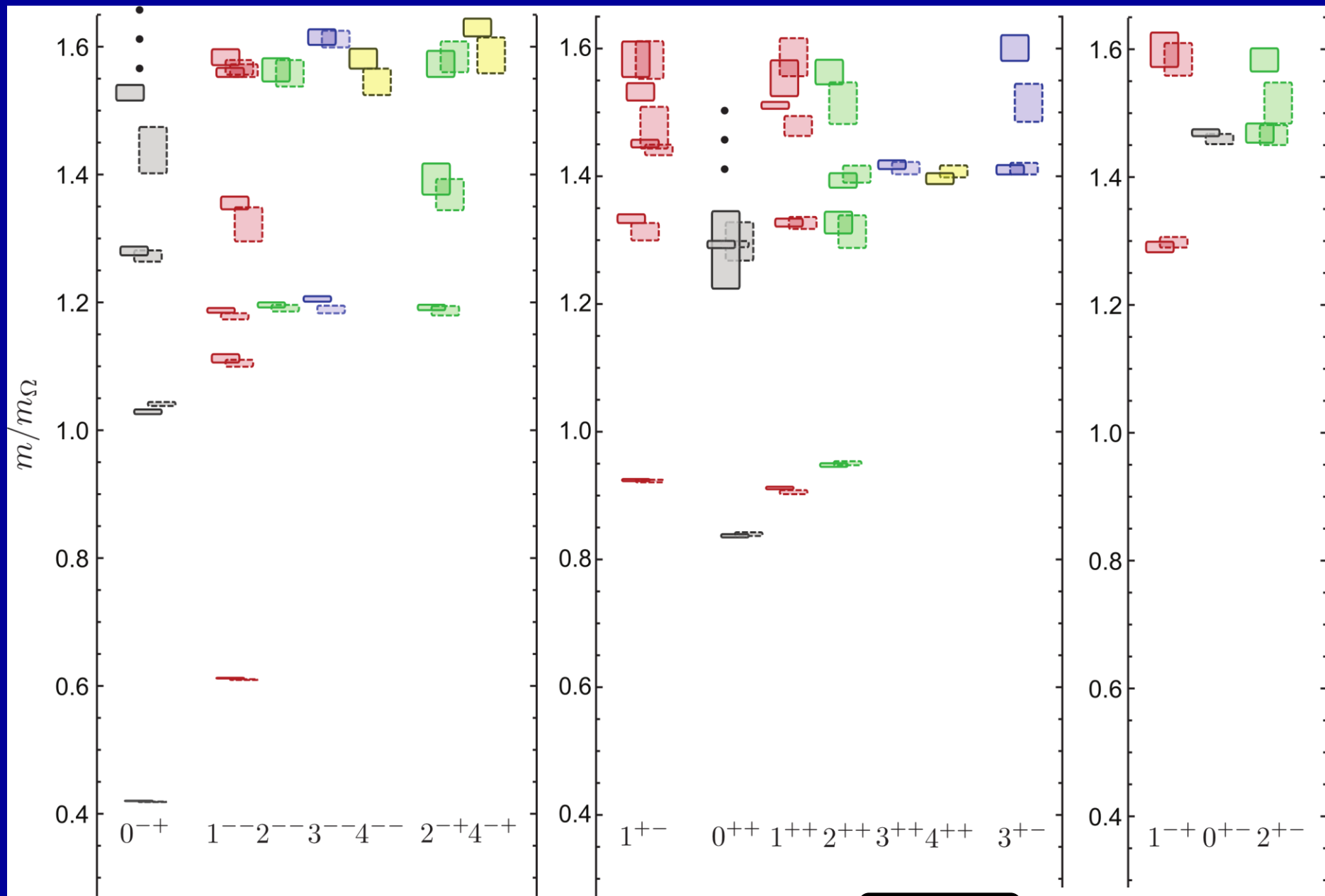


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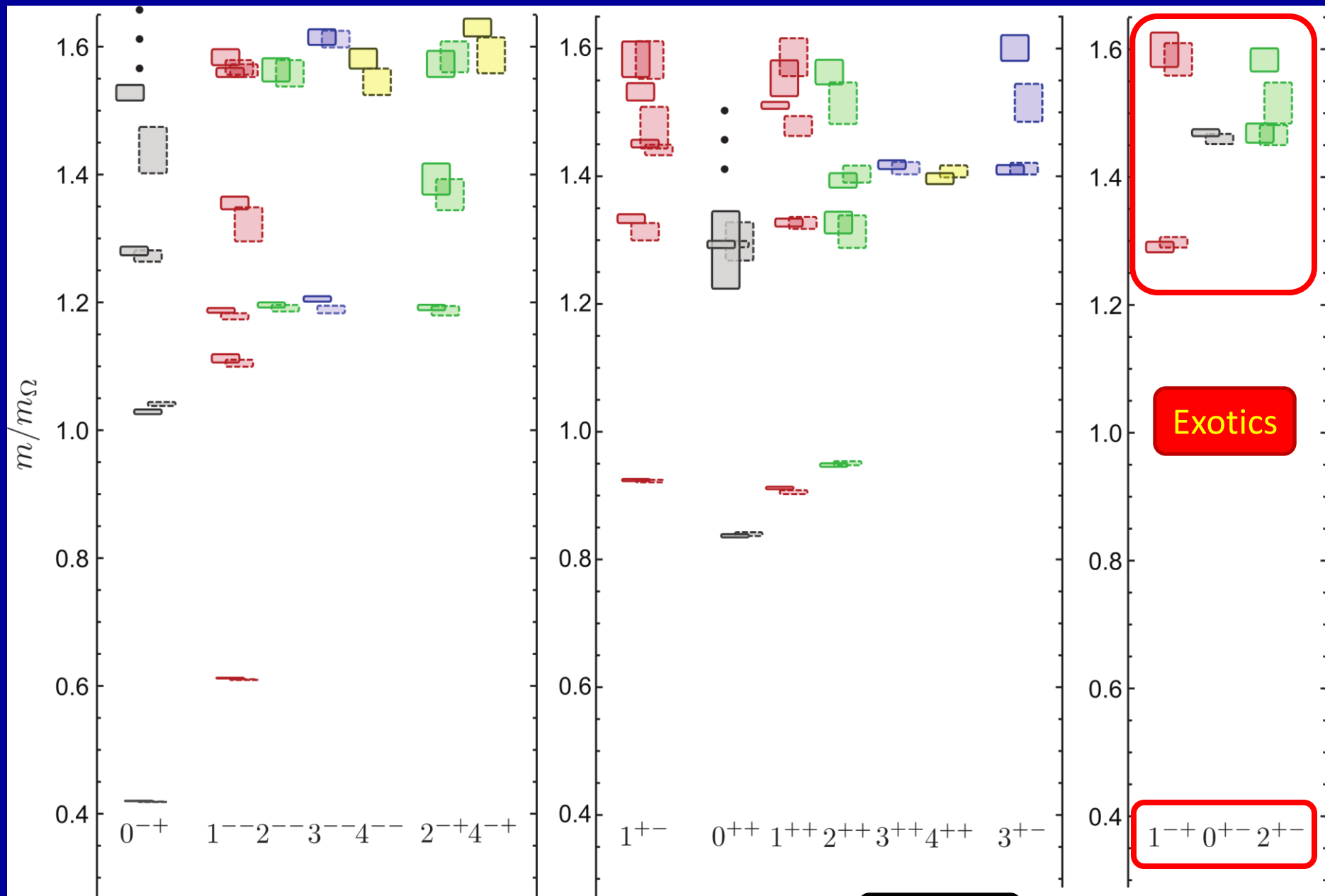
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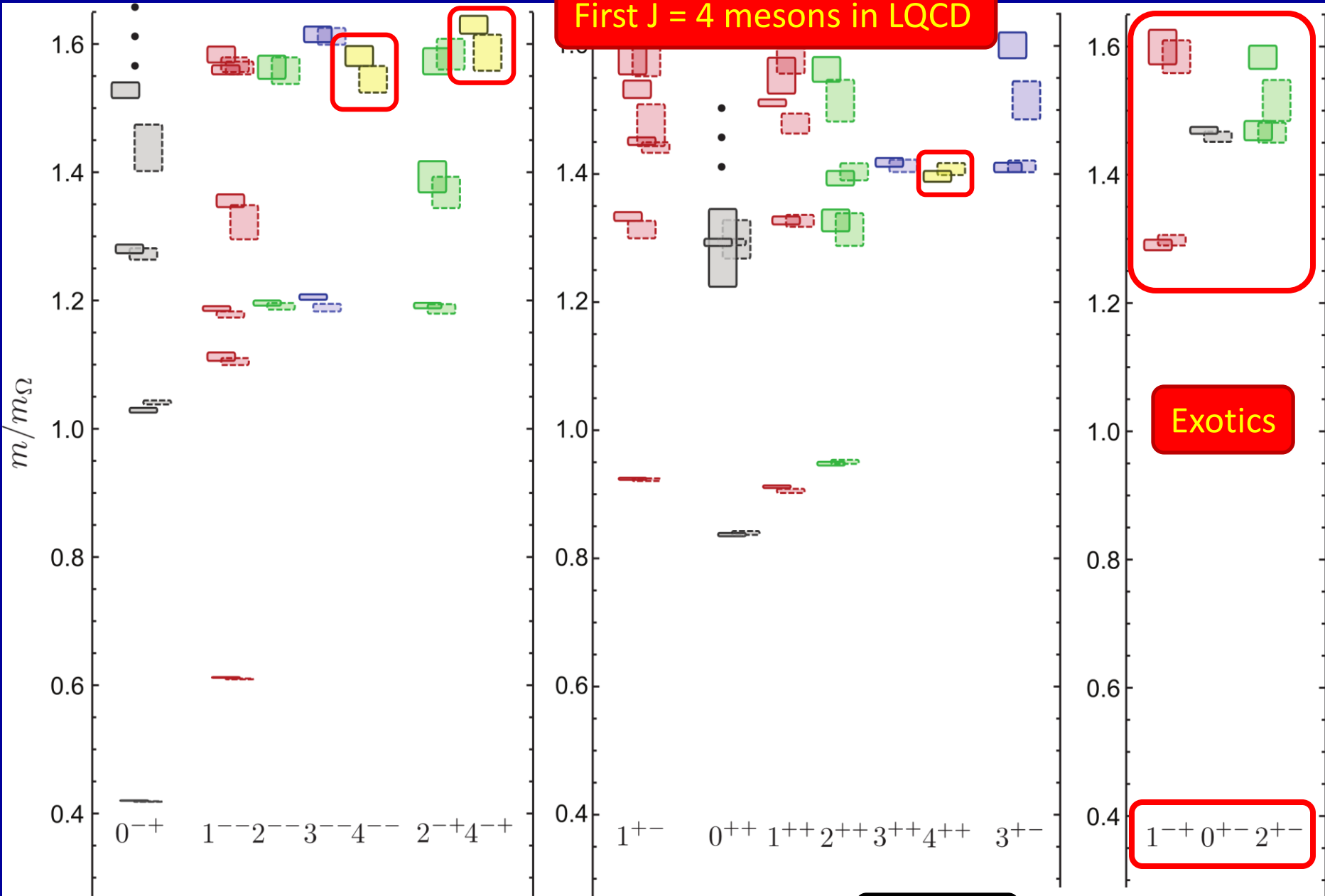
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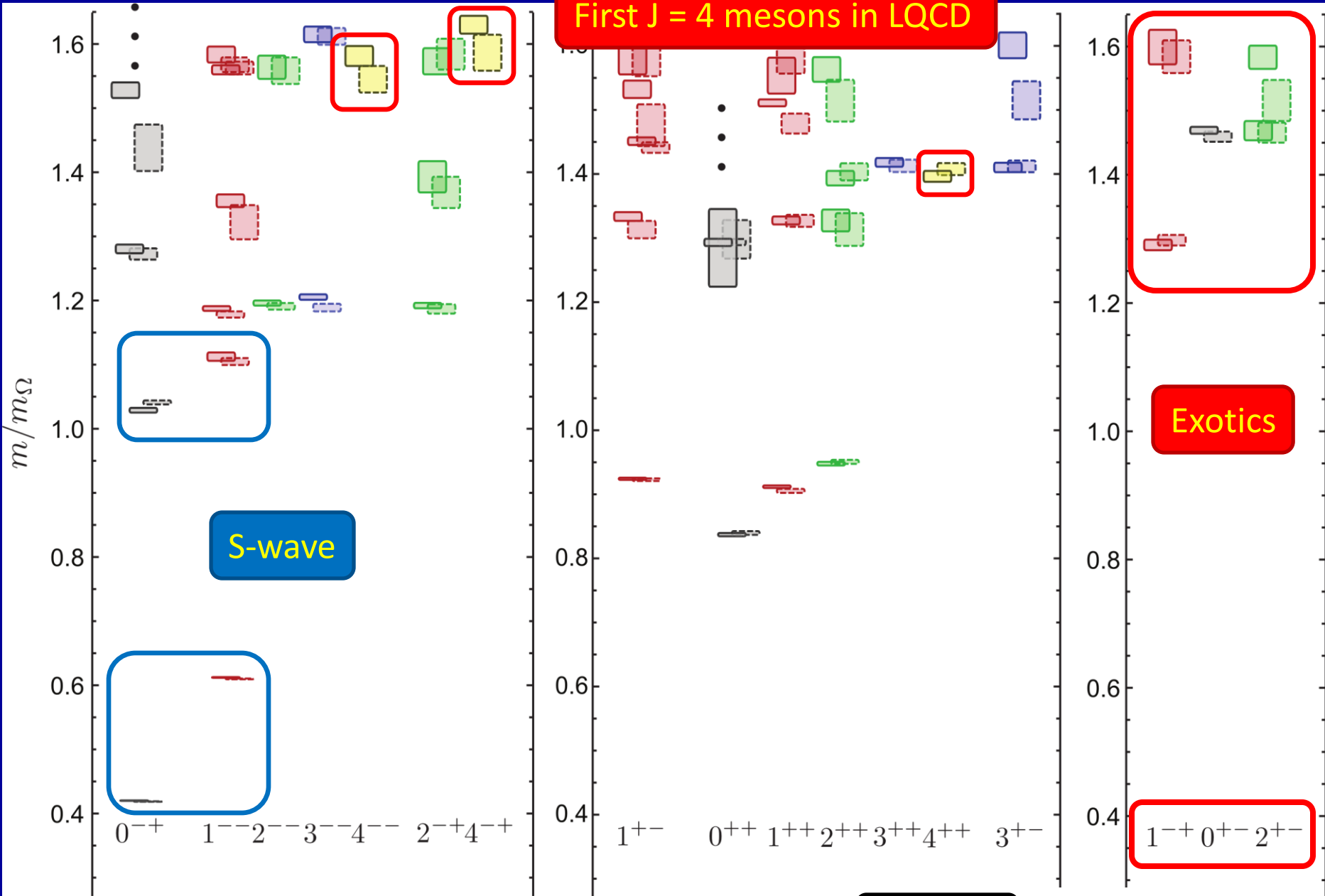
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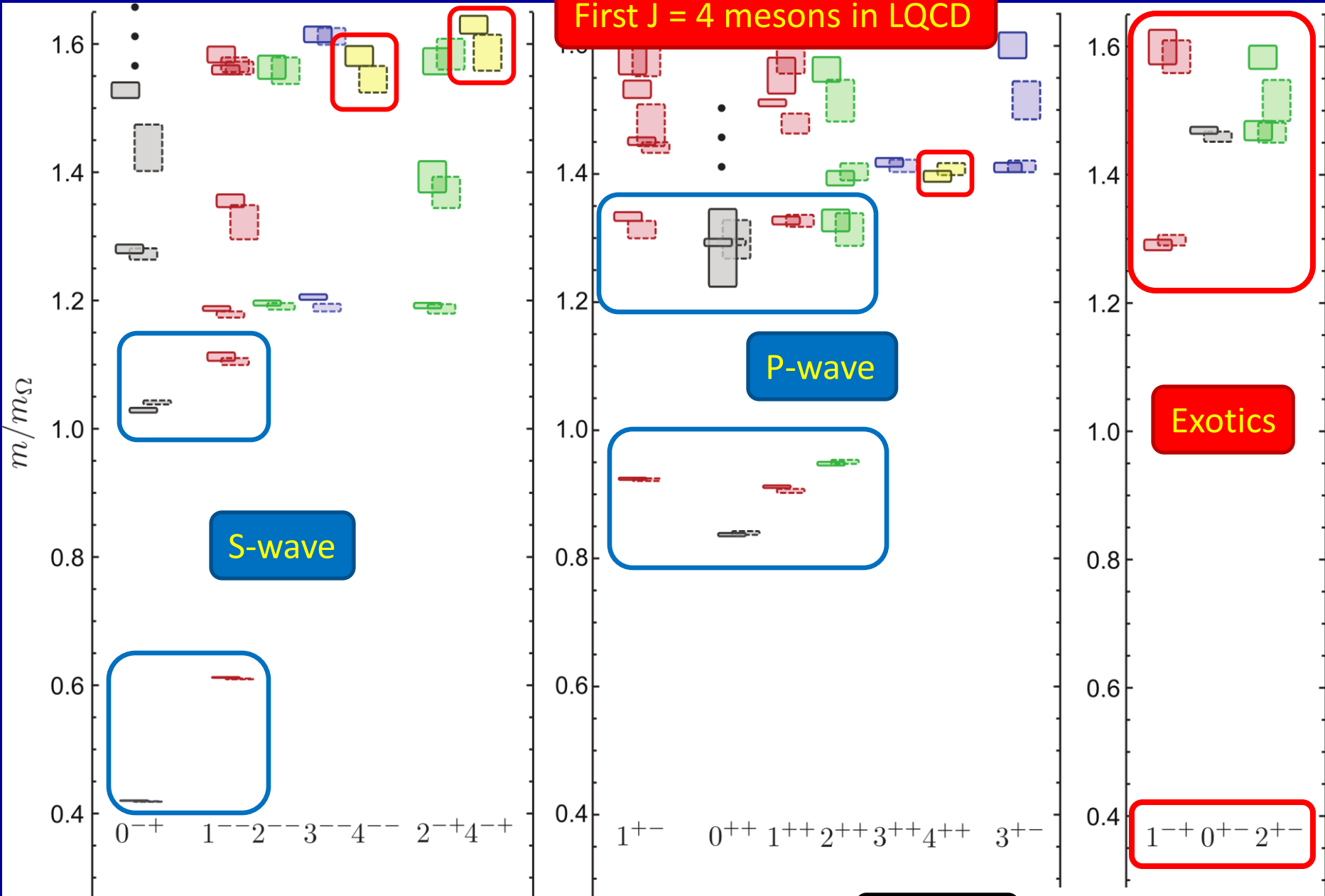
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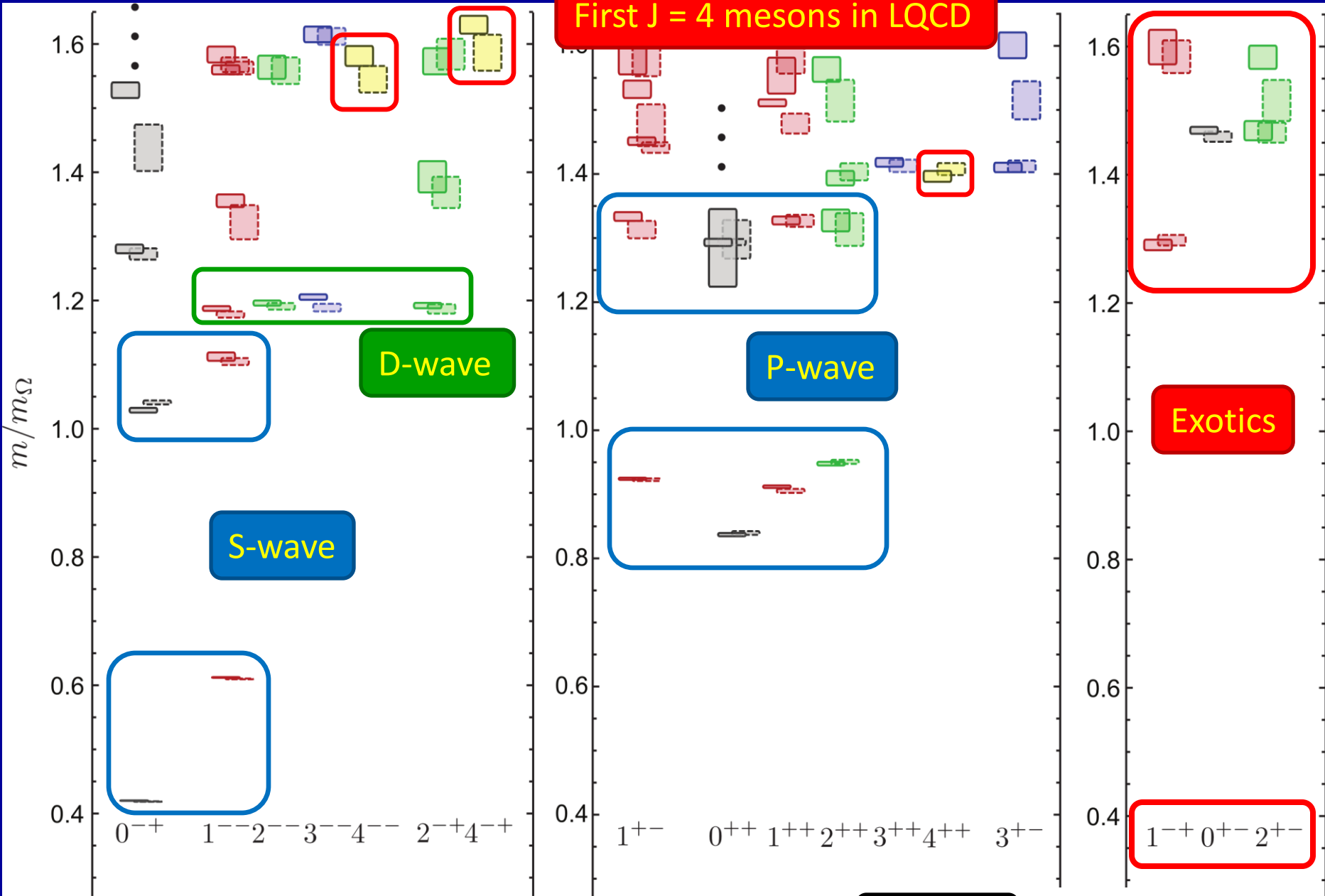
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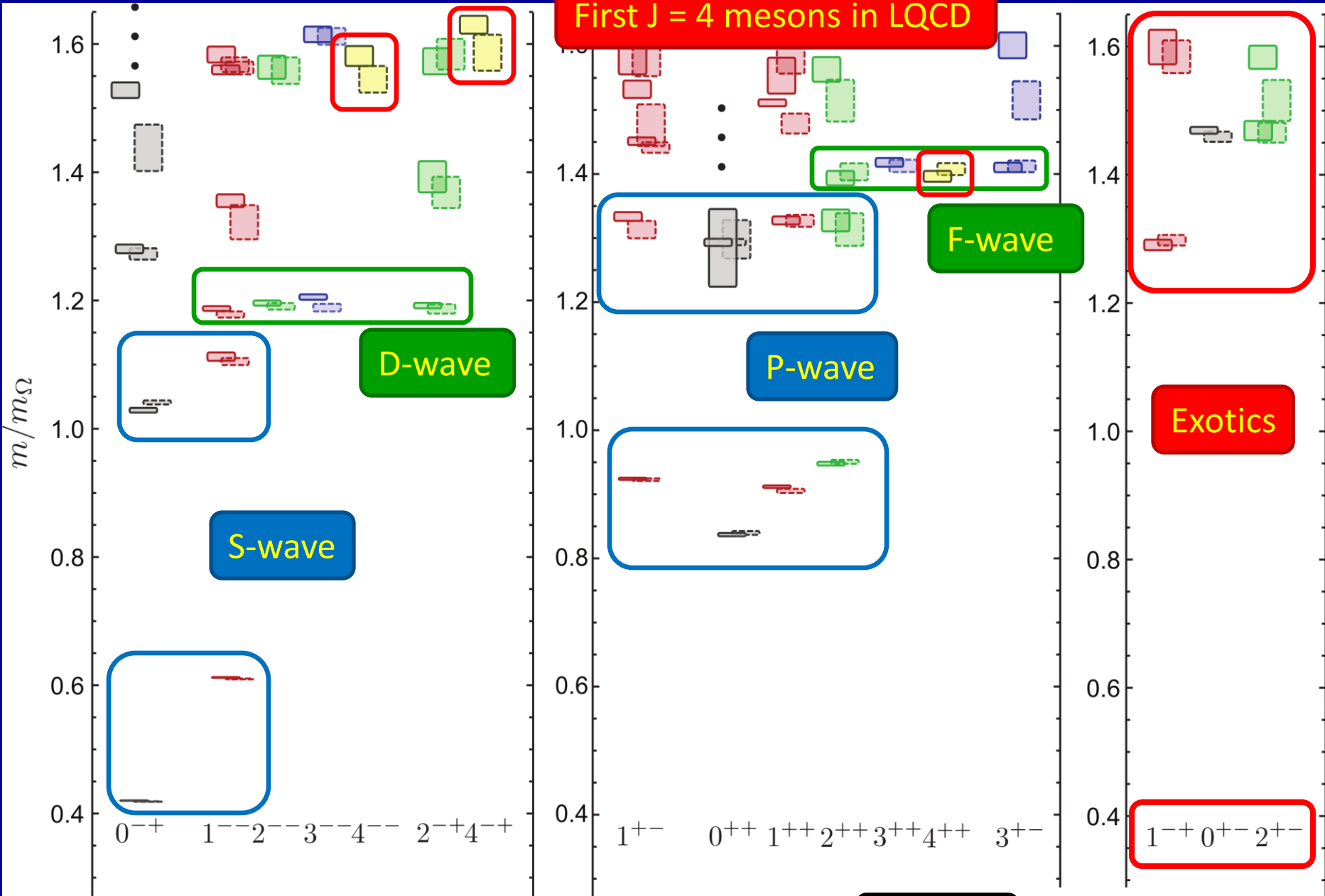
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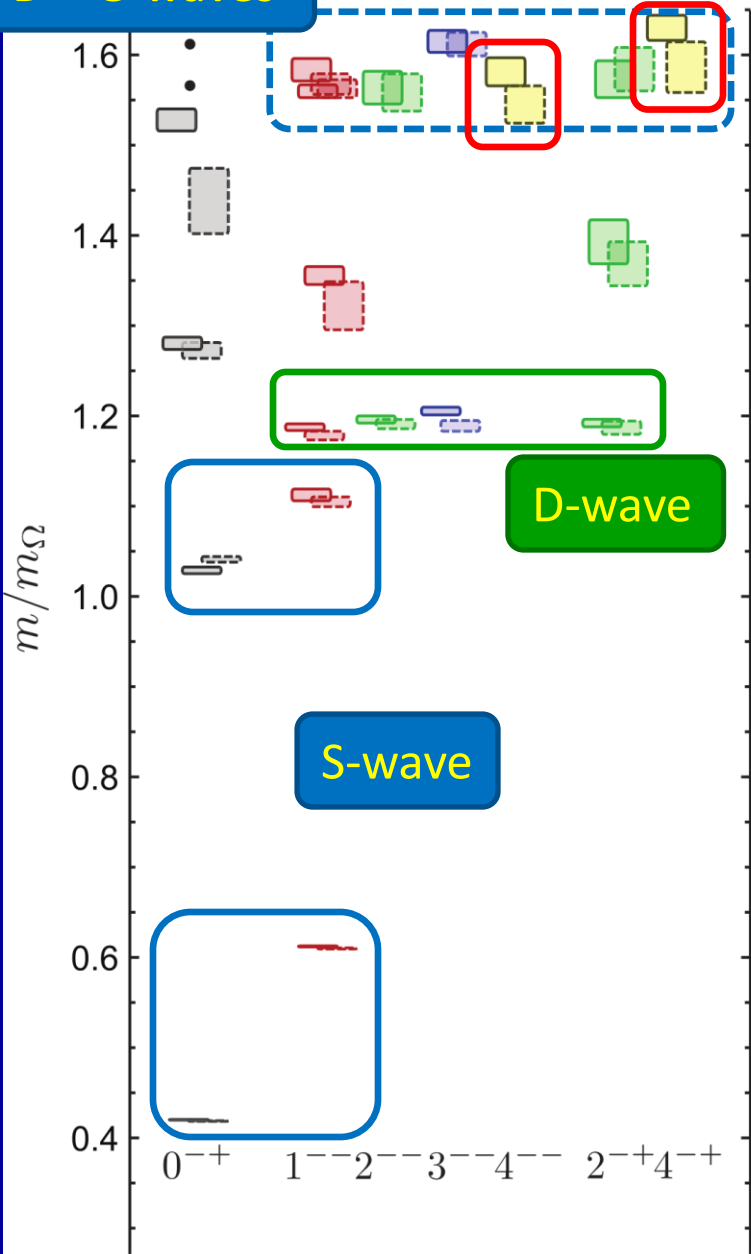
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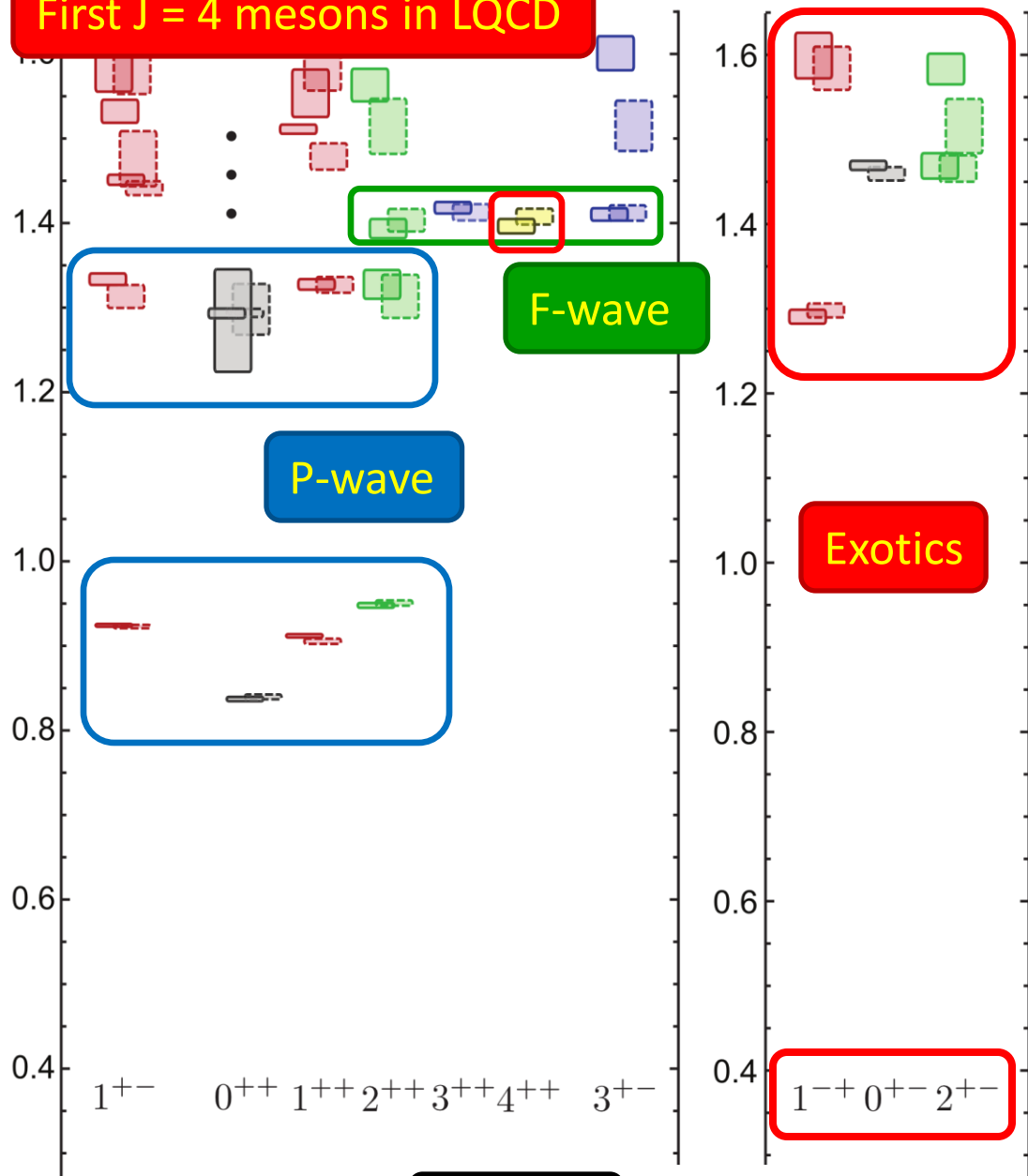
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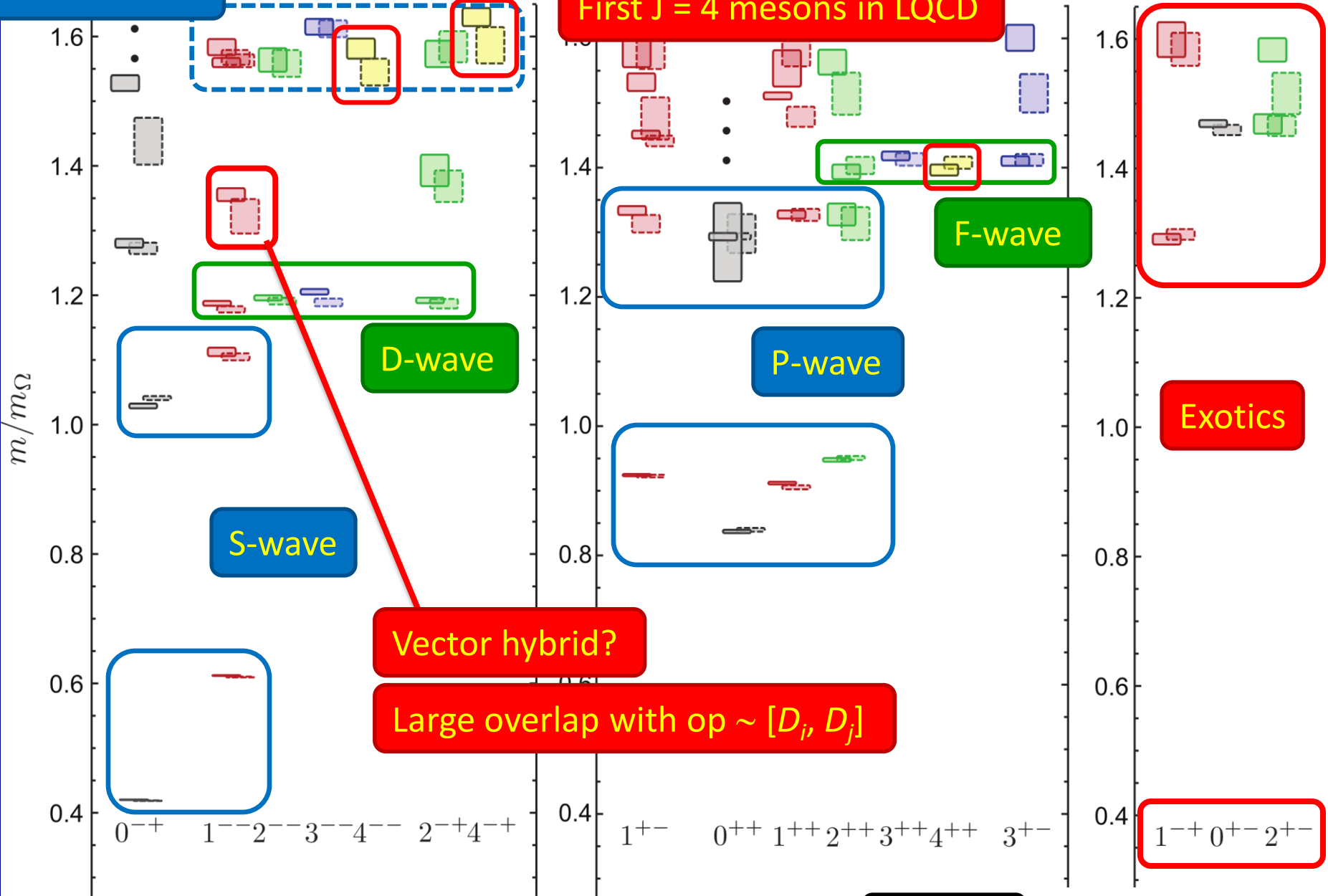


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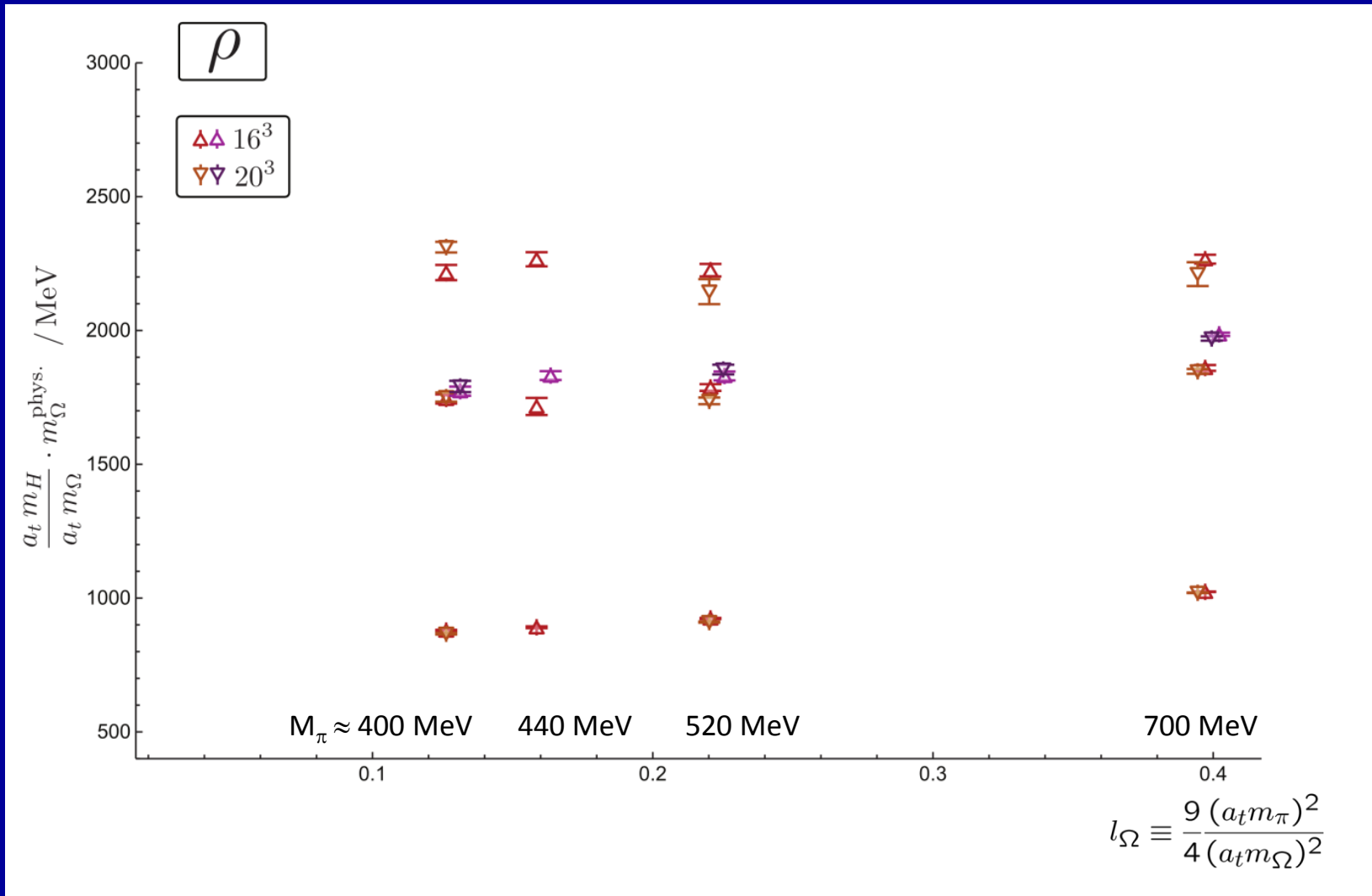
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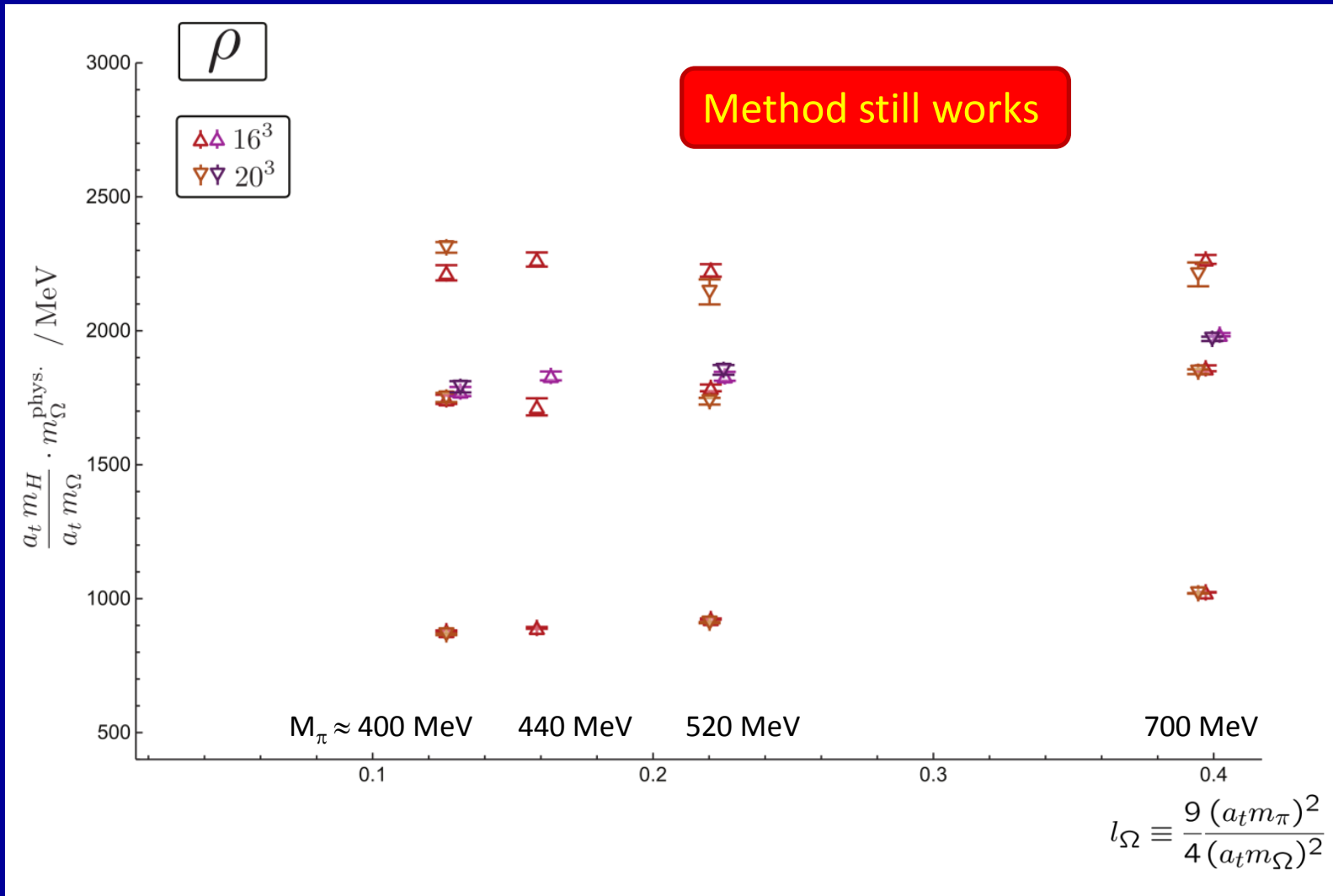
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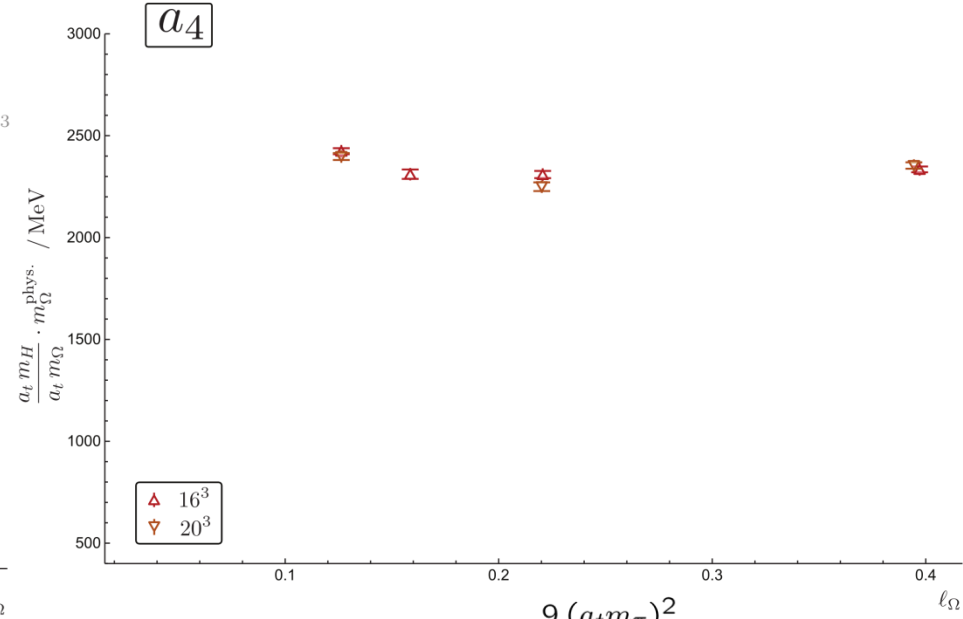
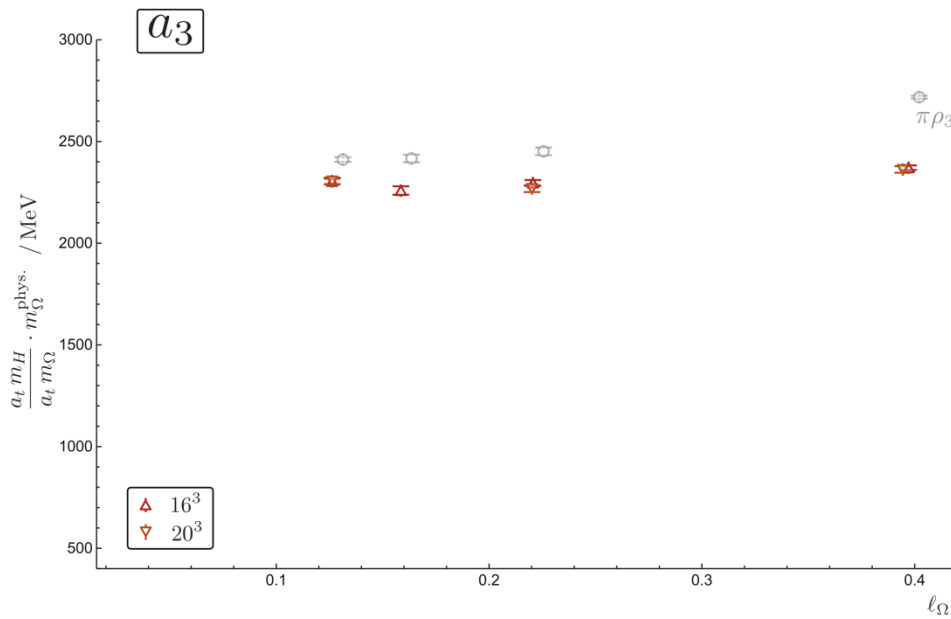
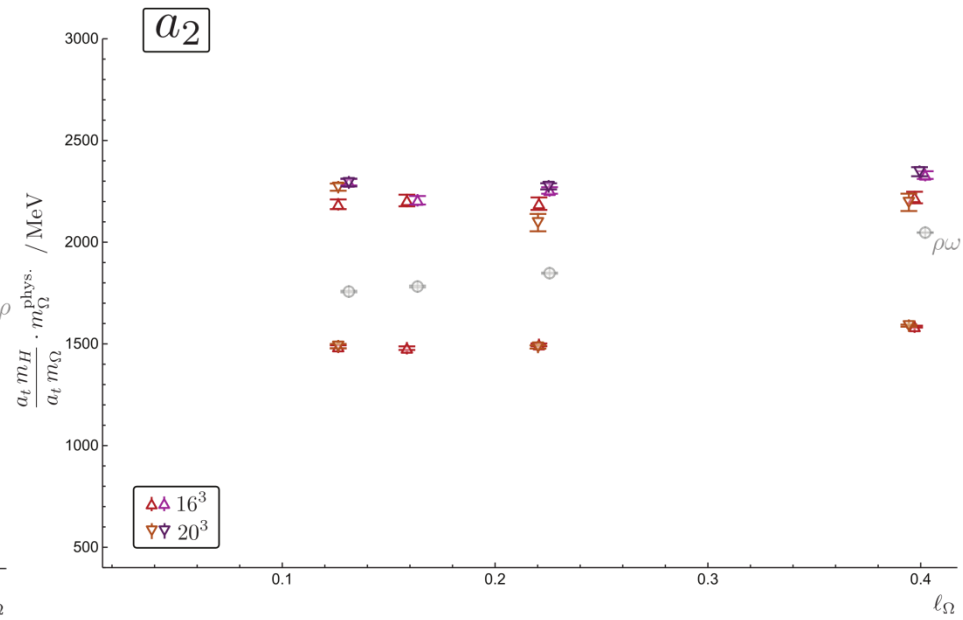
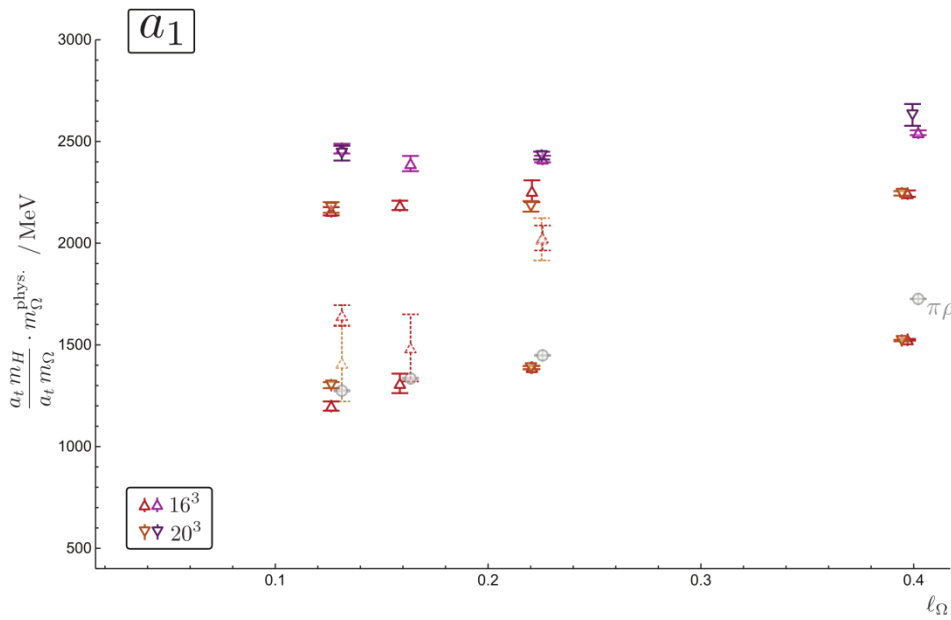
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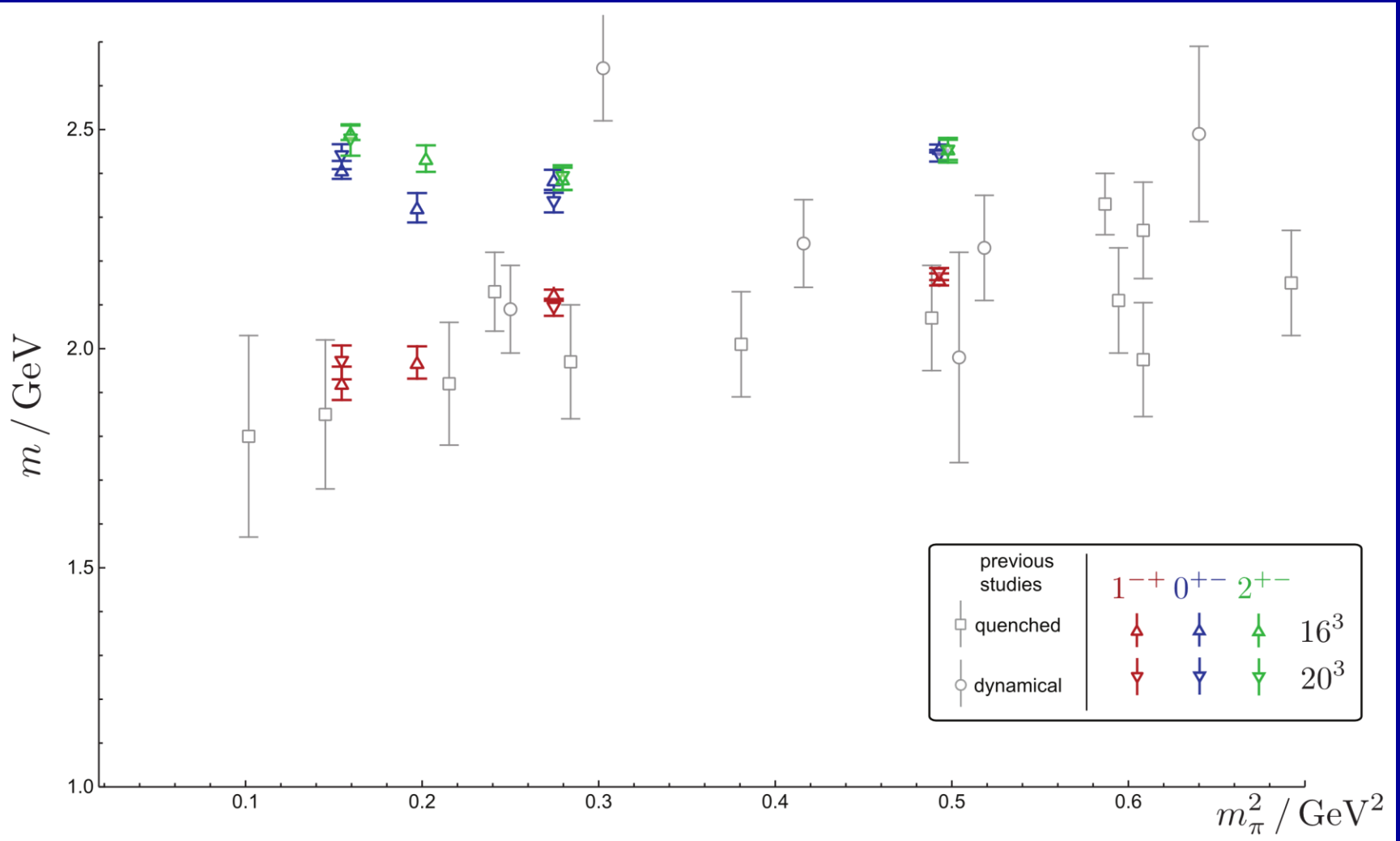
Lower pion masses





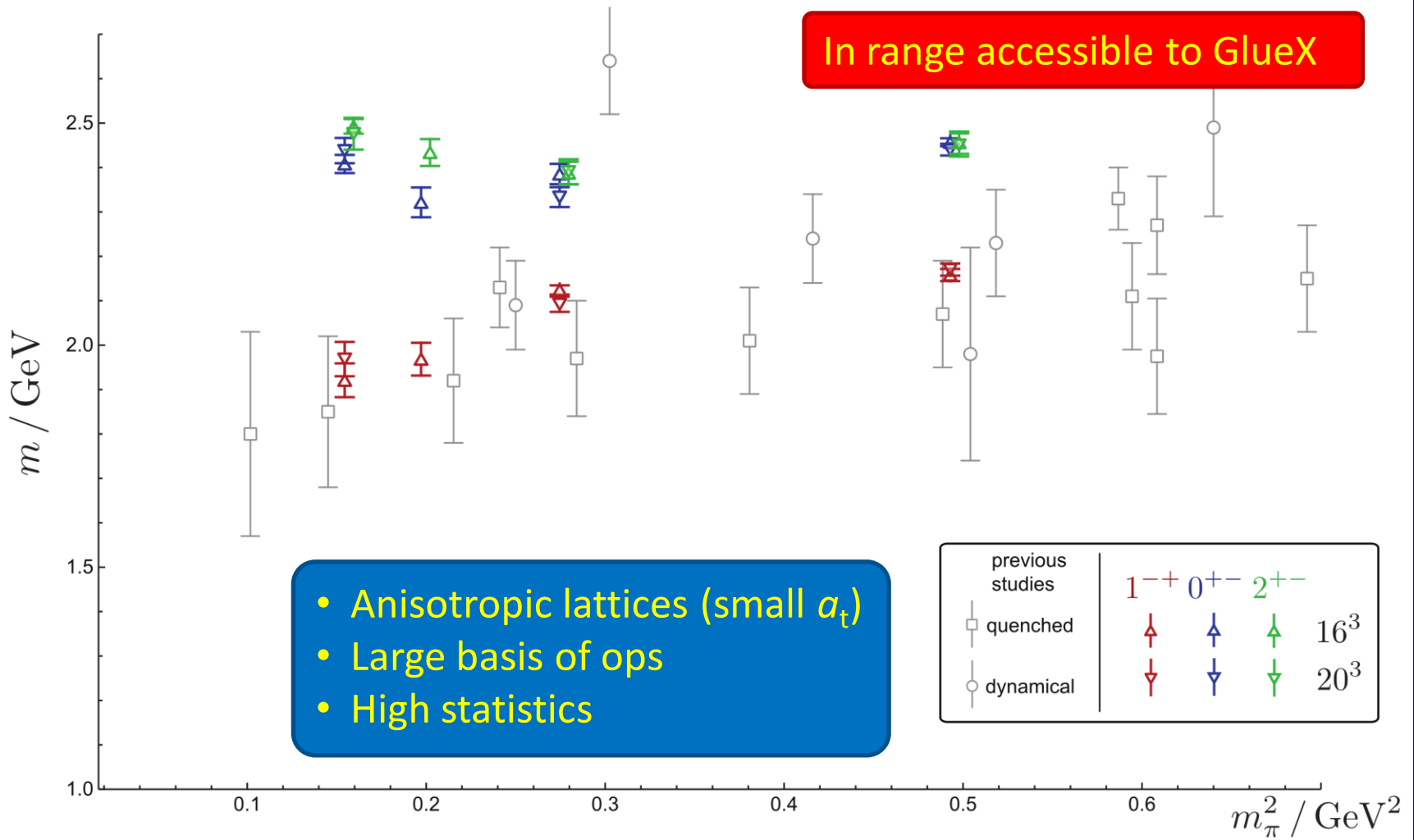
$$l_\Omega \equiv \frac{9 (a_t m_\pi)^2}{4 (a_t m_\Omega)^2}$$

Exotics summary



Exotics summary

In range accessible to GlueX



Kaons

Lower the light quark mass ($N_f = 2+1$) — SU(3) sym breaking

M_π / MeV	700	520	440	400
M_K / M_π	1	1.2	1.3	1.4

c.f. physical
 $M_K / M_\pi = 3.5$

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No longer is C-parity a good quantum number for kaons (or a gen. of C-parity)

Combine J^{P+} and J^{P-} operators

Physically, axial kaons [$K_1(1270)$, $K_1(1400)$] are a mixture
Suggested mixing angle $\approx 45^\circ$ (combination of exp and models)

But...

Kaons

Lower th

M_π / M_K

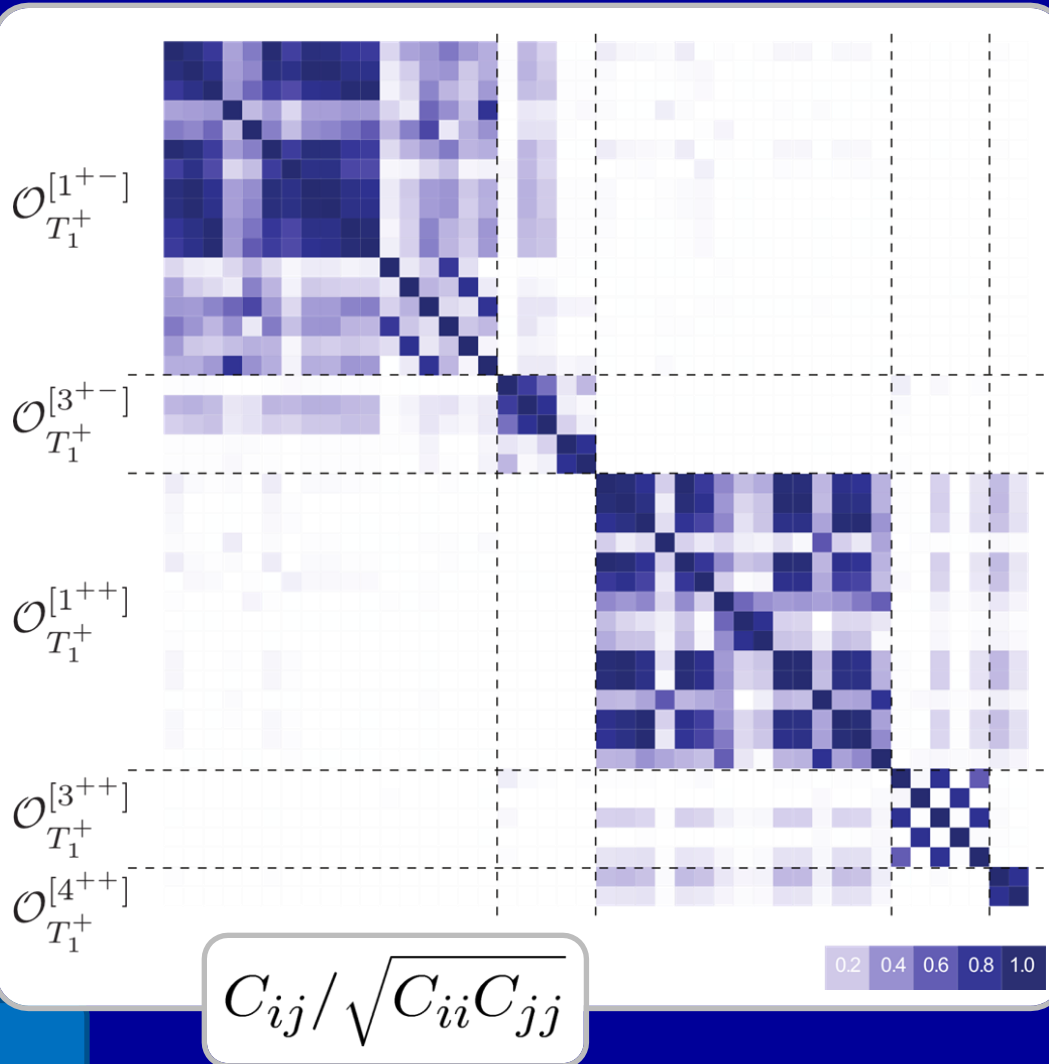
M_K / M_π

No long

Combin

Physical
Suggest

But...



16^3

$M_\pi \approx 400 \text{ MeV}$

$M_K / M_\pi \approx 1.4$

c.f. physical

$M_K / M_\pi = 3.5$

or a gen. of C-parity)

odels)

Kaons – Operator Overlaps

16^3

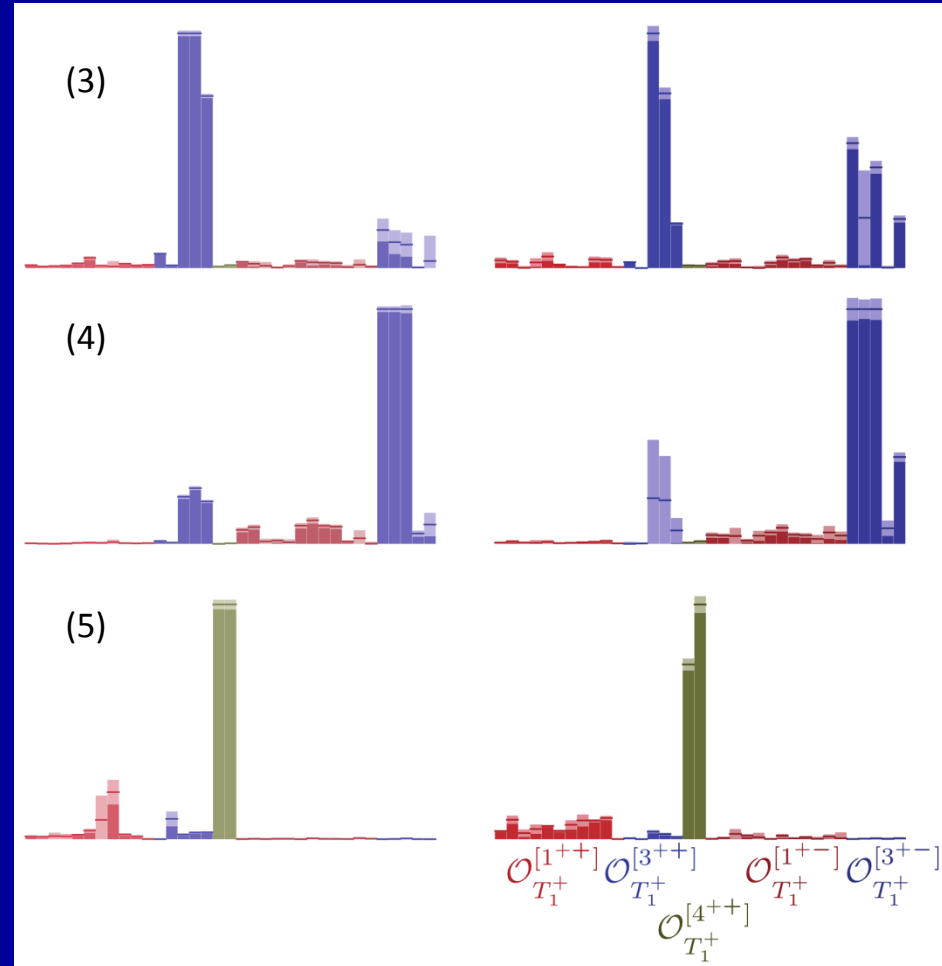
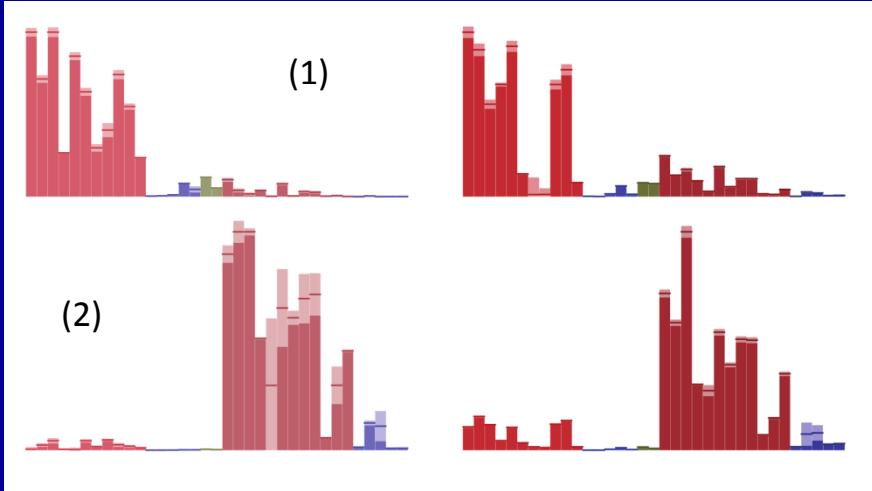
$M_\pi \approx 520 \text{ MeV}$

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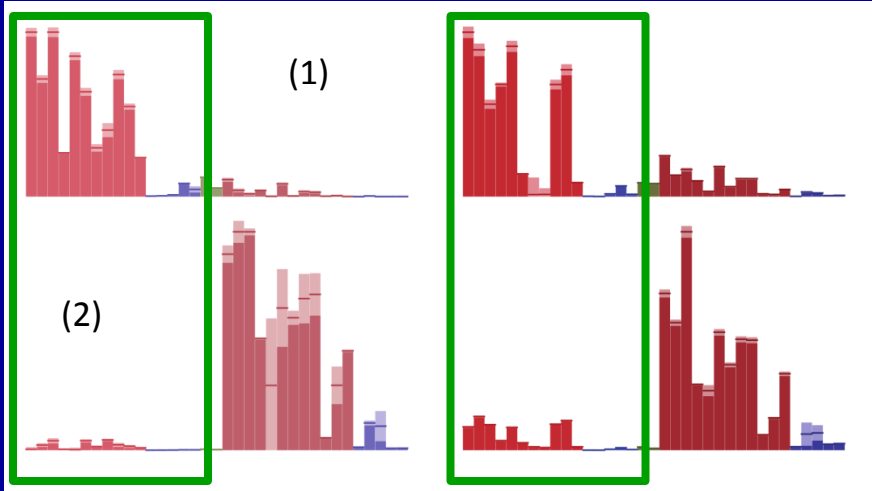
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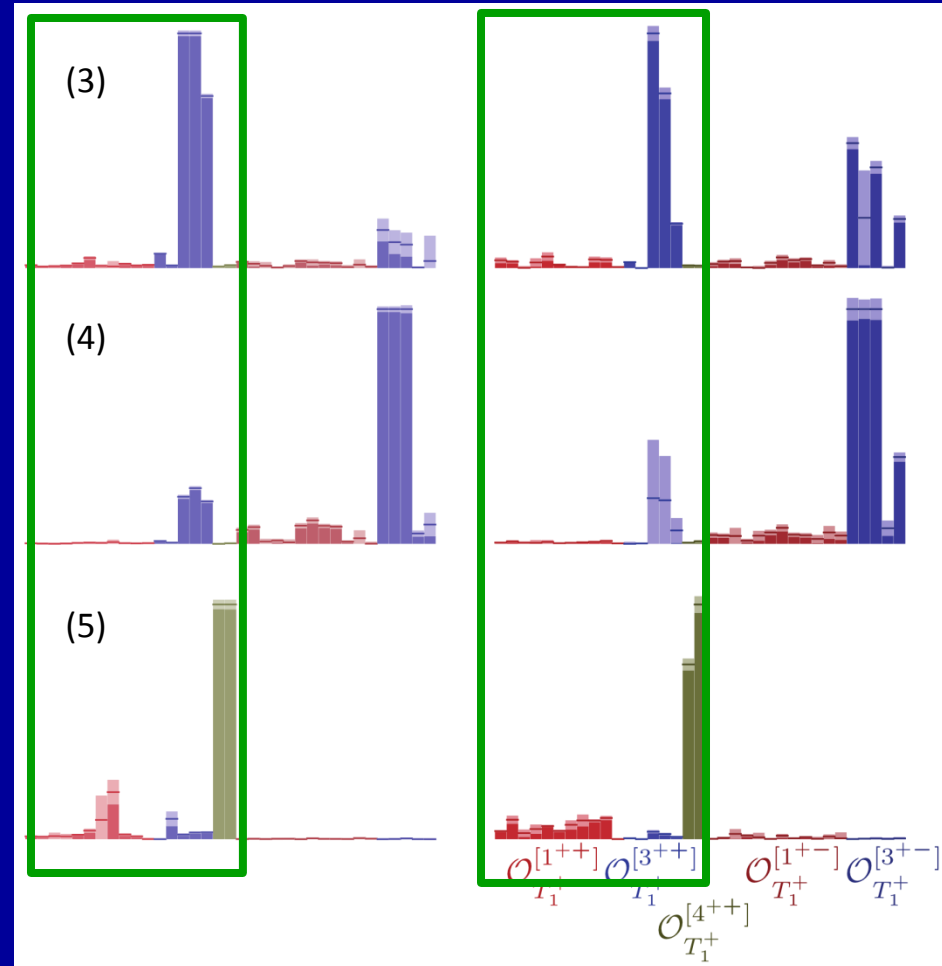
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J^{++}



Kaons – Operator Overlaps

16^3

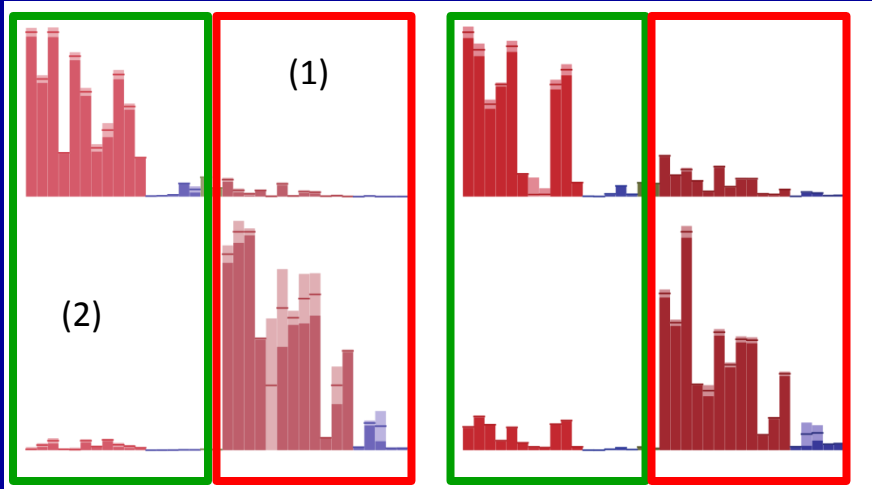
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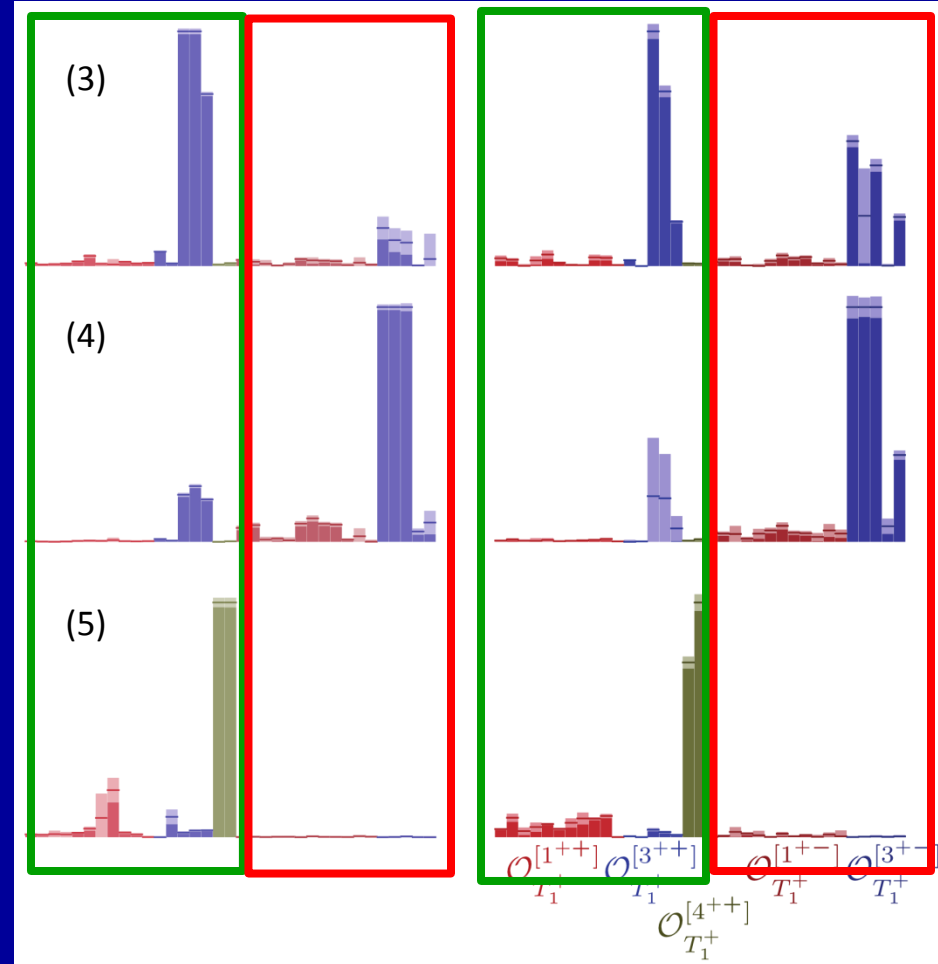
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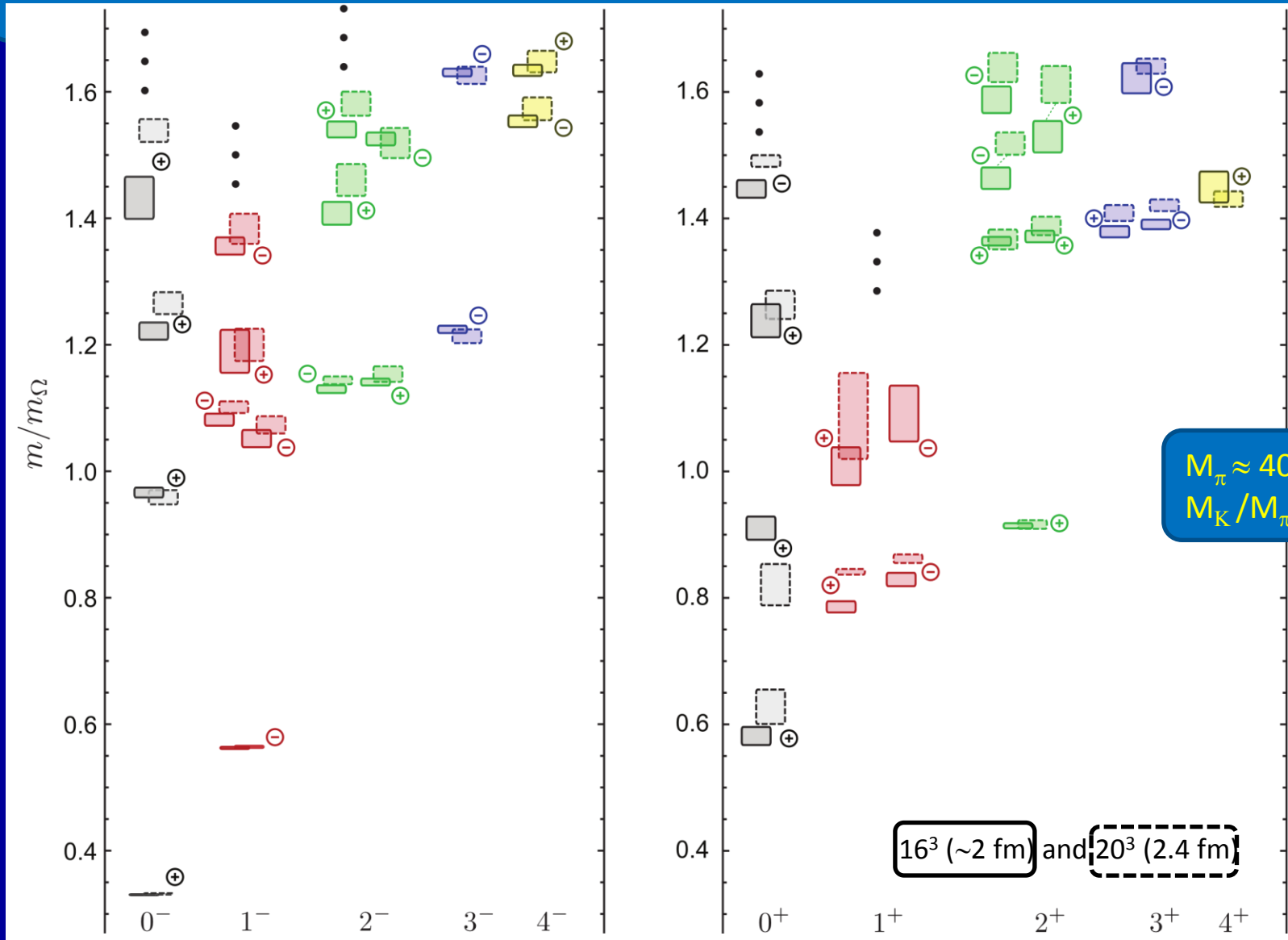


J^{++}

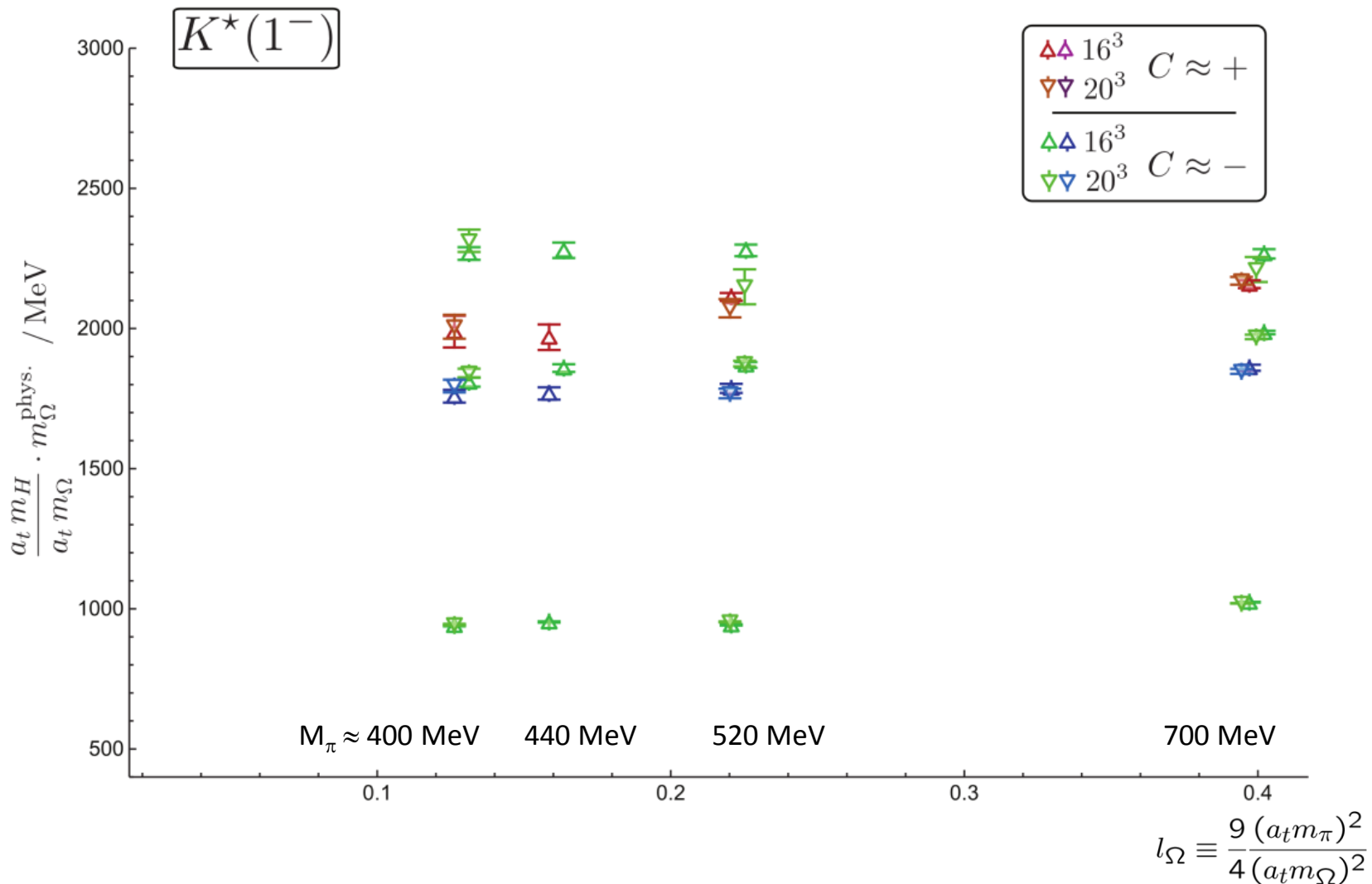
J^{+-}



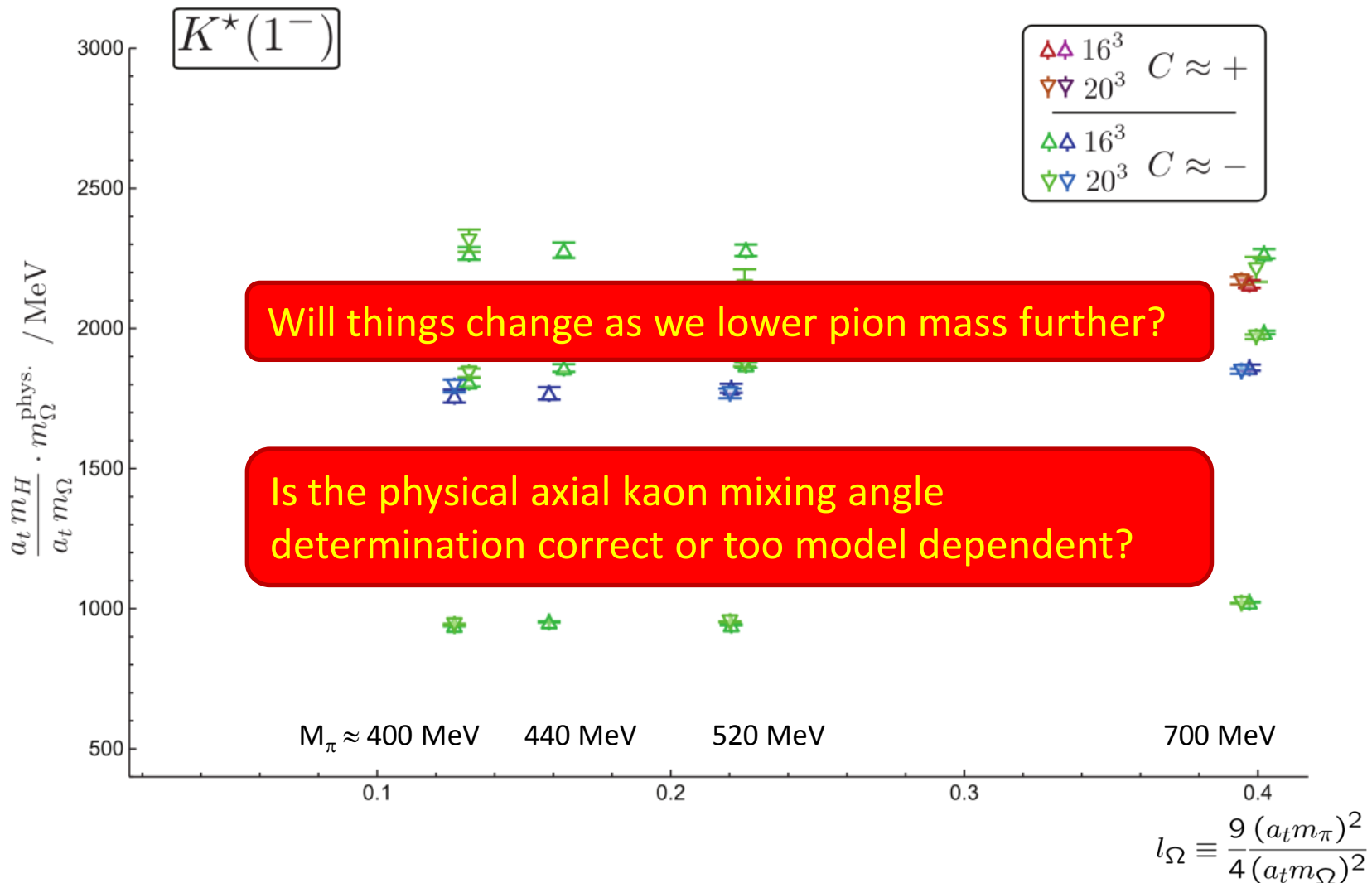
Kaons - spectrum



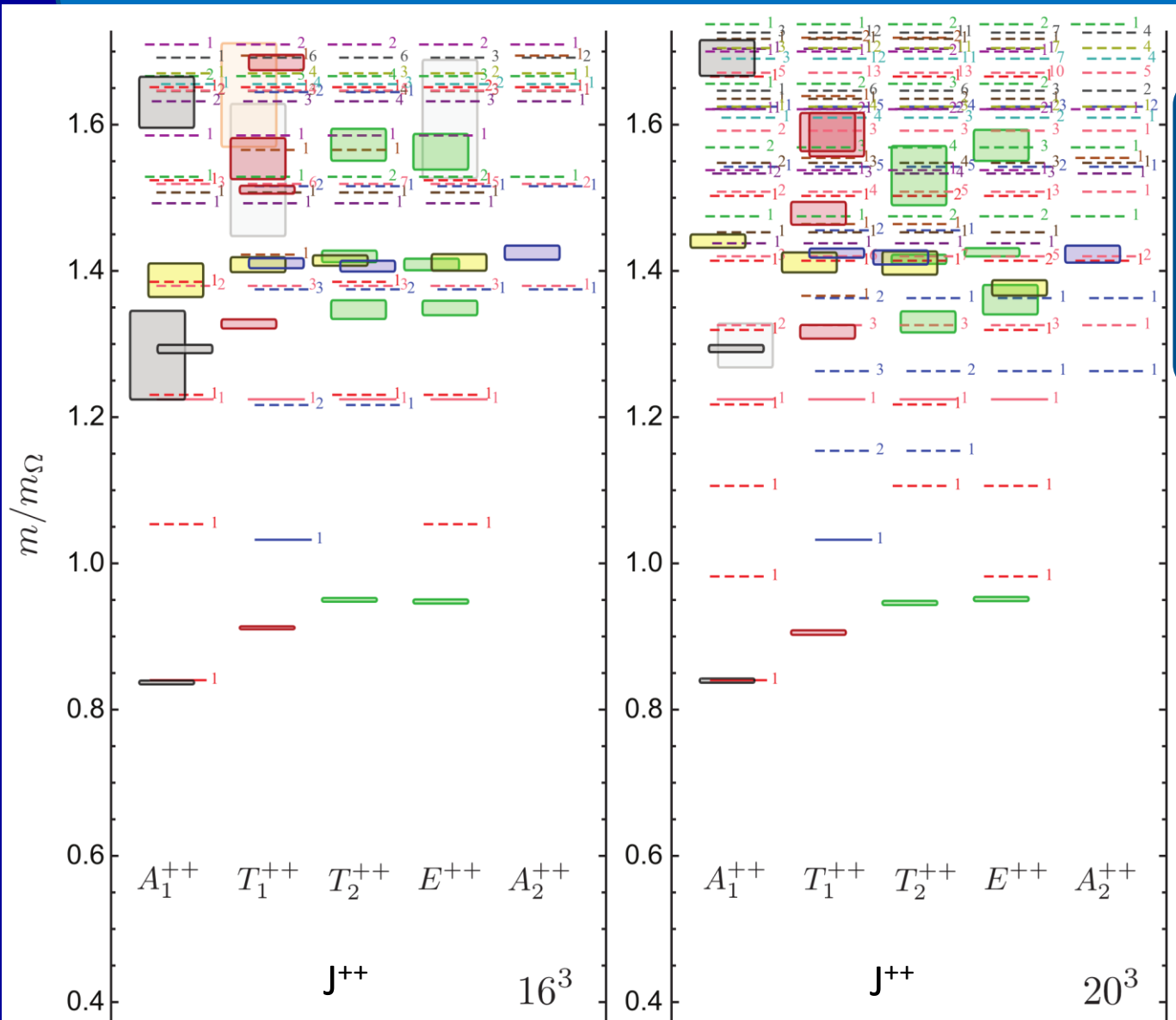
Kaons – Various pion masses



Kaons – Various pion masses

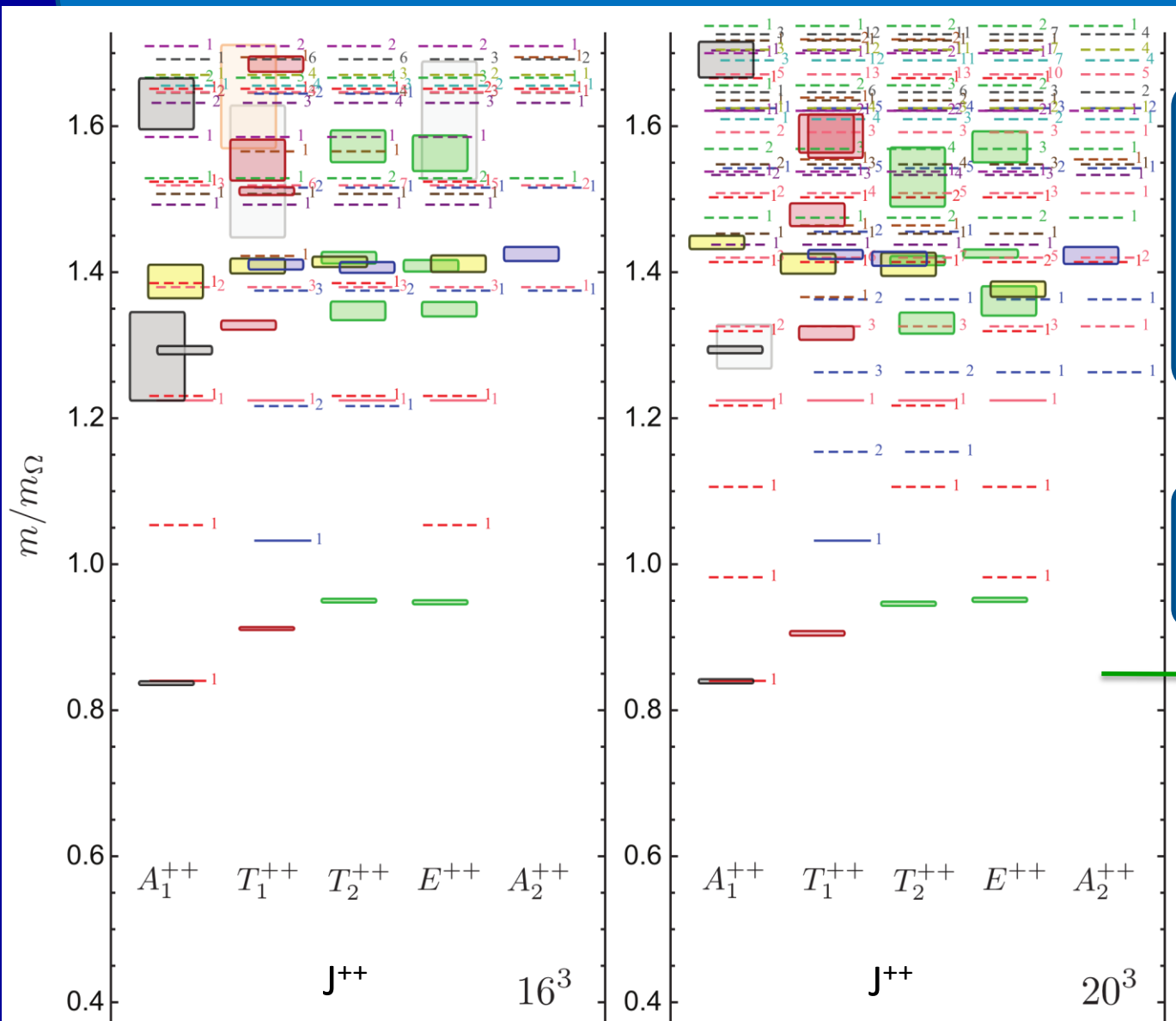


Multi-particle states?



Finite box – discrete allowed momenta
 → discrete spectrum of multiparticle states

Multi-particle states?



Finite box – discrete allowed momenta
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Expect two-meson states above $2m_\pi$

$2m_\pi \sim 0.85 m_\Omega$

Where are they?

Charmonium

“Hydrogen atom” of meson spectroscopy

Potential models, effective field theories, QCD sum rules, ...

New and improved measurements at BABAR, Belle, BES, CLEO-c

Charmonium

“Hydrogen atom” of meson spectroscopy

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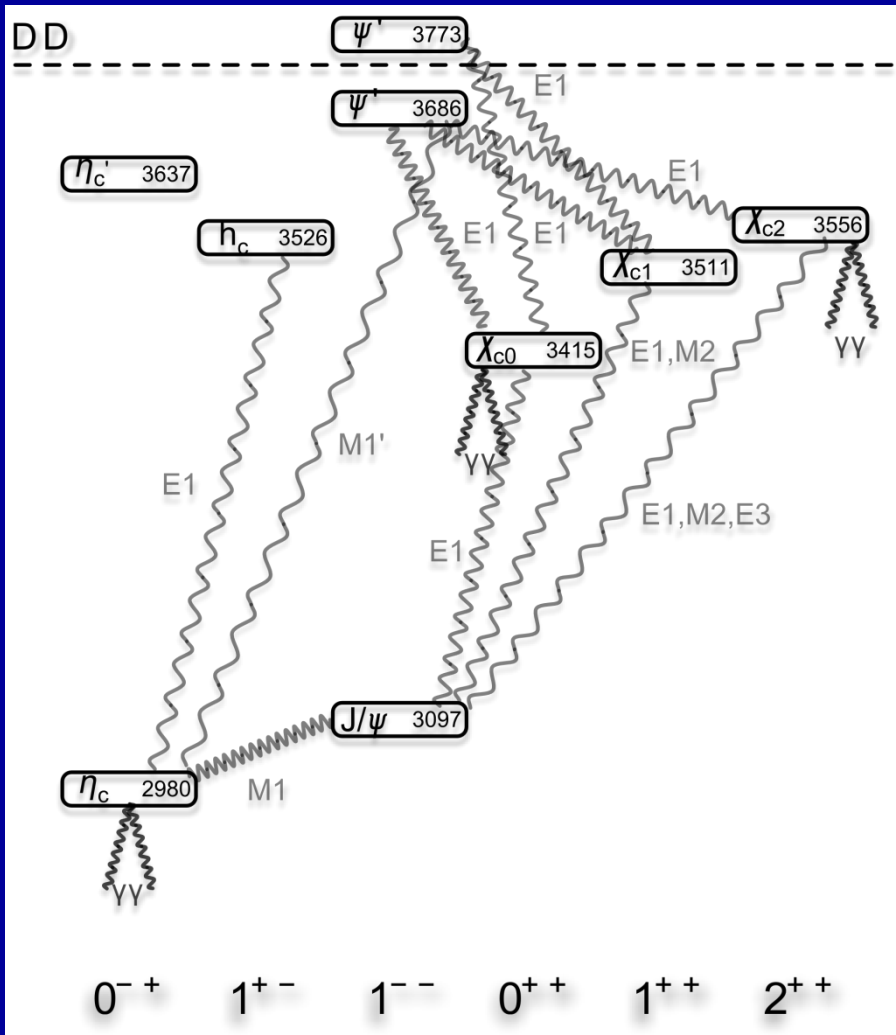
New and improved measurements at BABAR, Belle, BES, CLEO-c

New resonances not easily described by quark model

Theoretical speculation: hybrids, multiquark/molecular mesons, ...

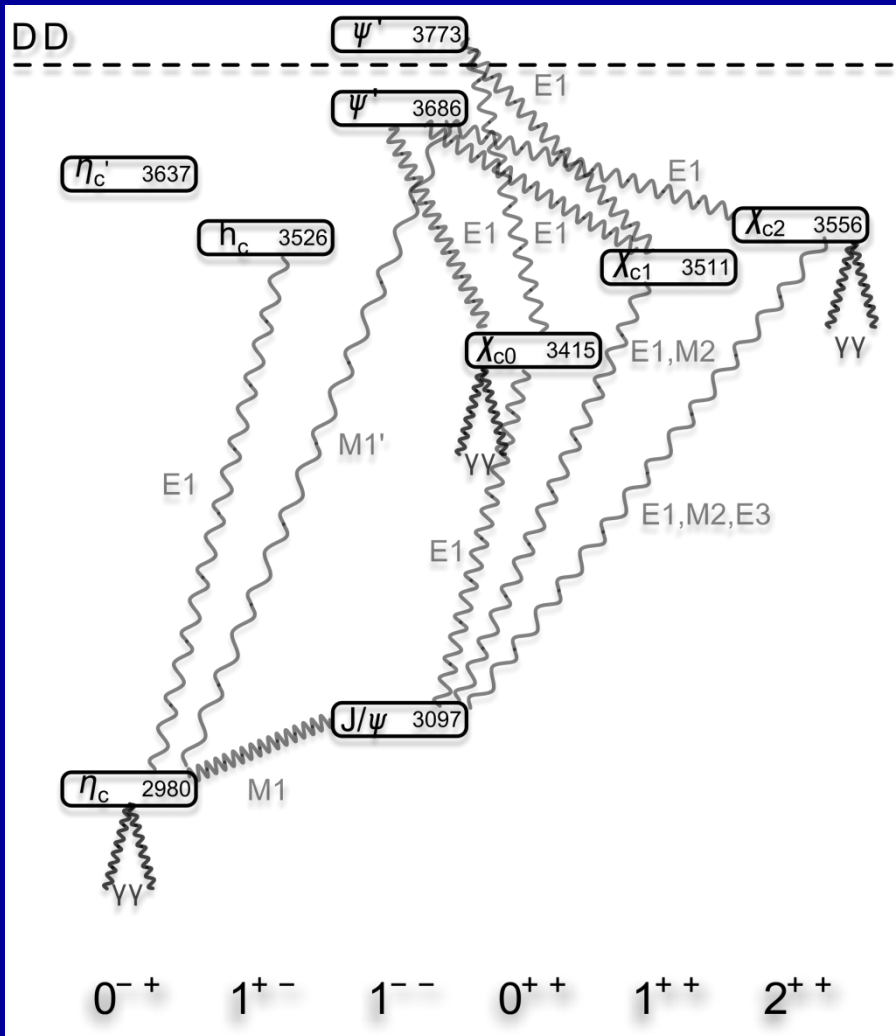
As yet, no exotic J^{PC} observed (1^{-+} , 0^{+-} , 2^{+-})

Charmonium radiative transitions



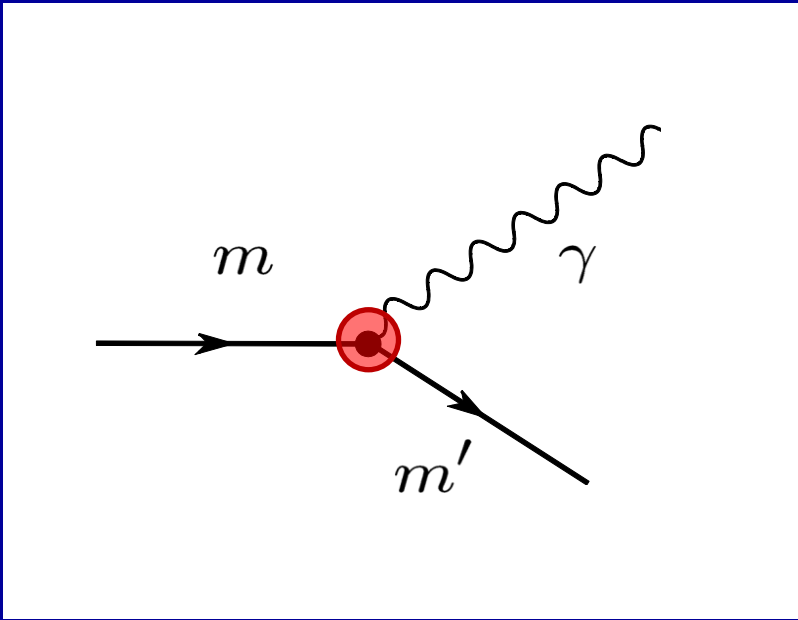
Below DD threshold radiative transitions have significant BRs

Charmonium radiative transitions

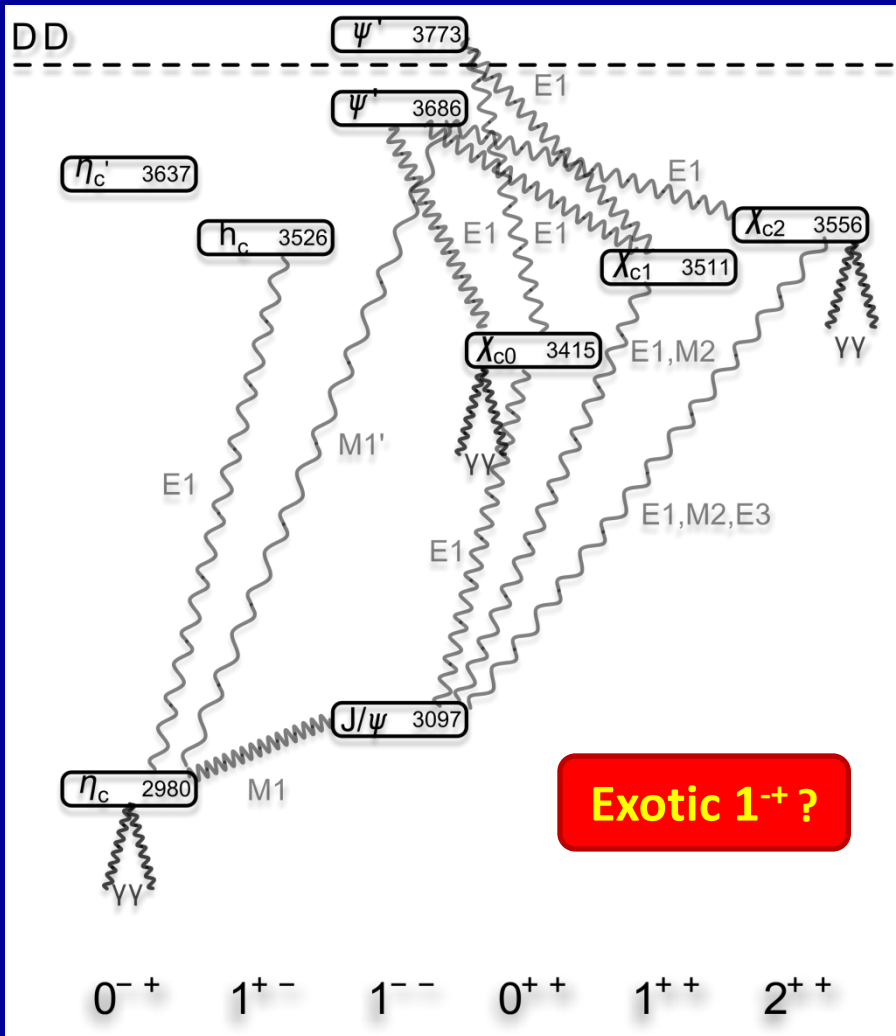


Below DD threshold radiative transitions have significant BRs

Meson – Photon coupling

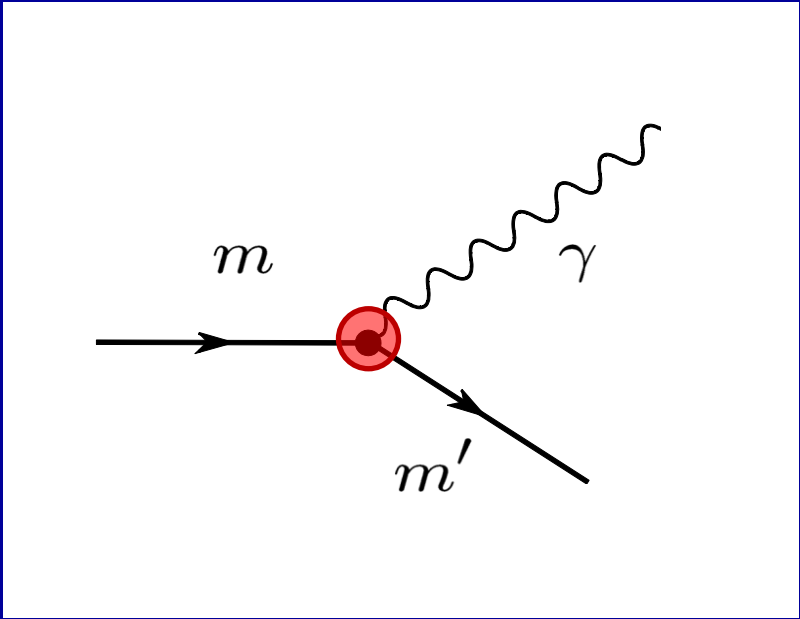


Charmonium radiative transitions



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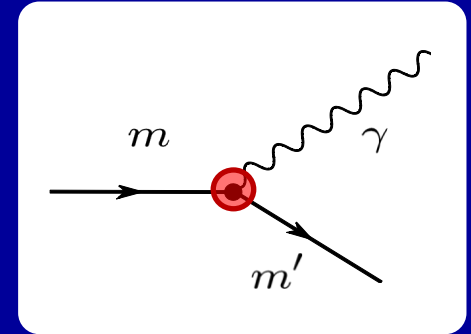
Meson – Photon coupling



Photocouplings

Charmonium (quenched) – testing method

$$C_{ij}(t_f, t, t_i) = \langle 0 | O_i(t_f) \bar{\psi}(t) \gamma^\mu \psi(t) O_j(t_i) | 0 \rangle$$

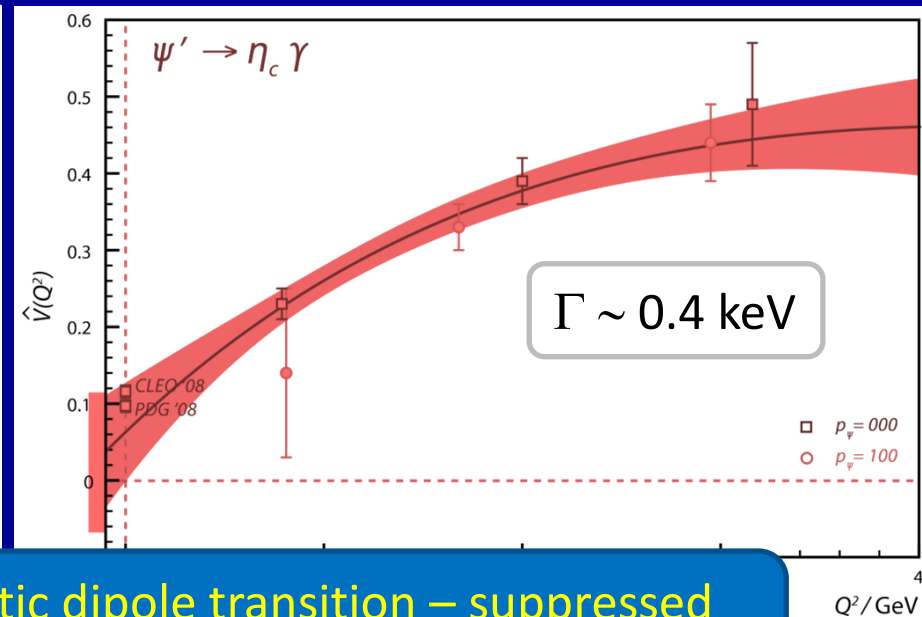
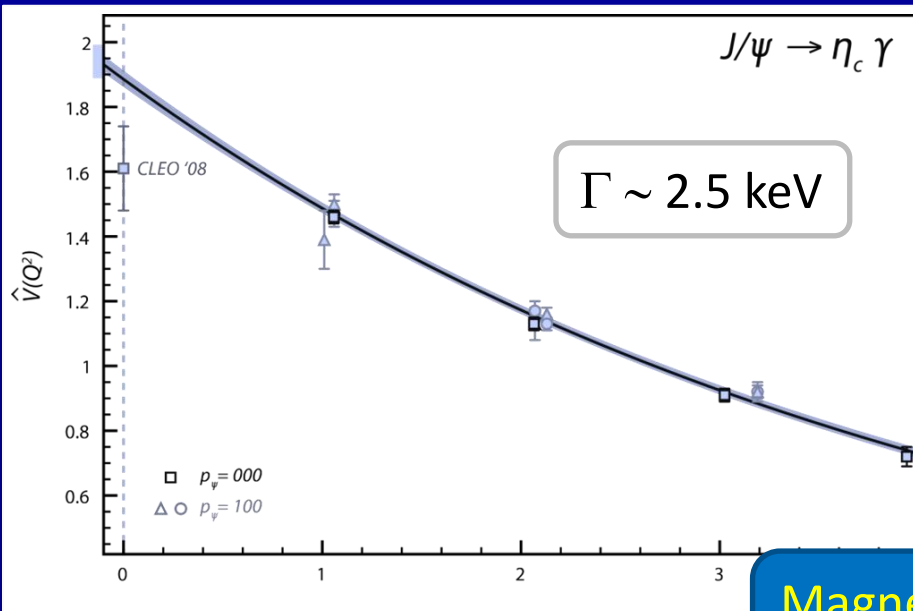
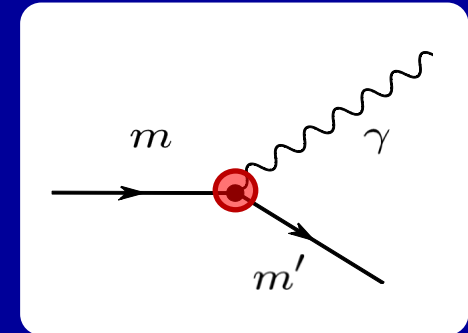


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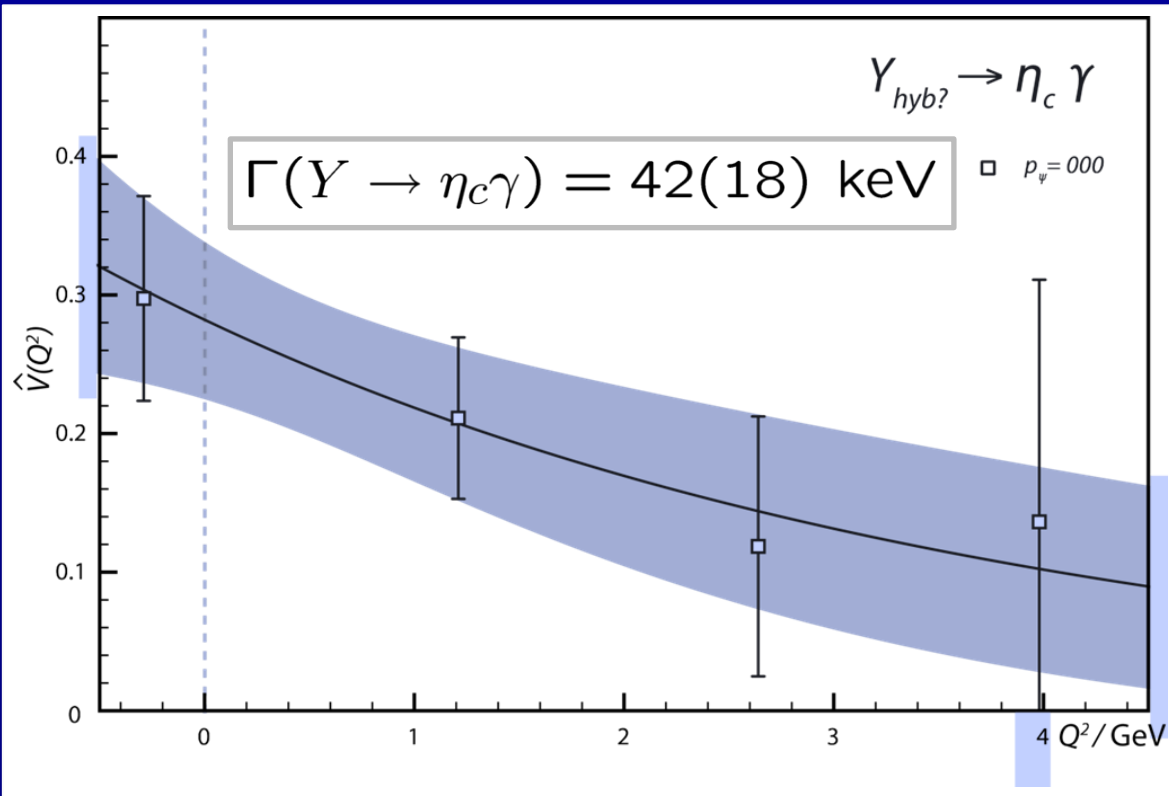
Conventional vector – pseudoscalar transition



Magnetic dipole transition – suppressed
(in quark model spin flip $\sim 1/m_c$)

PR D79 094504 (2009)

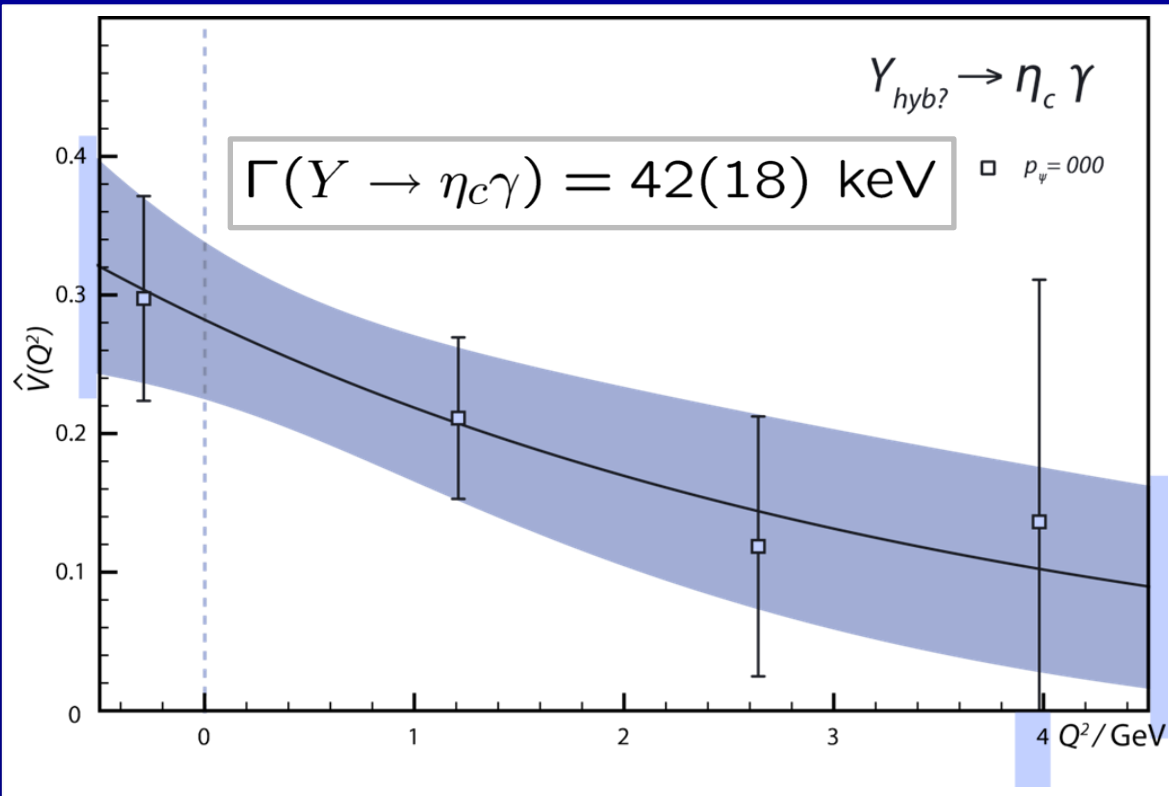
Photocouplings



Much larger than other
 $1^- \rightarrow 0^+$ M_1 transitions

$\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2 \text{ keV}$

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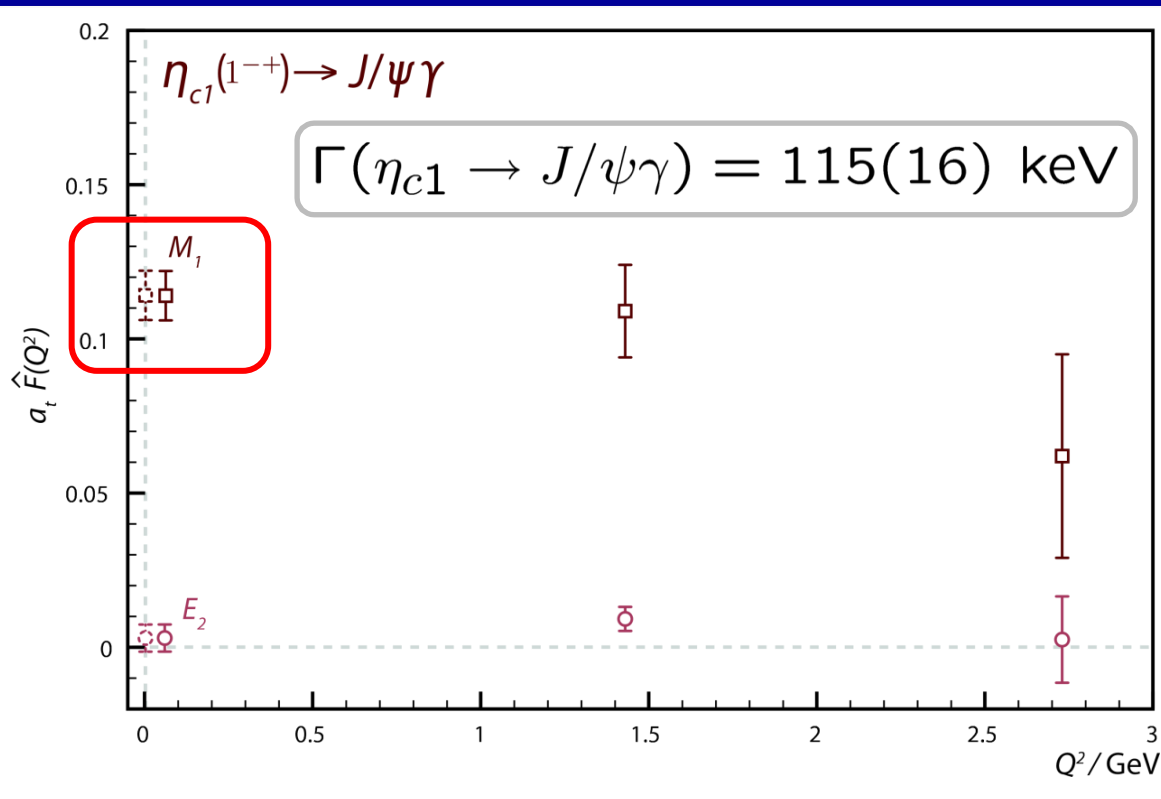
Spectrum analysis
suggests a vector hybrid
(spin-singlet)

Spin-singlet hybrid:
 M_1 with no spin flip

c.f. flux tube model 30 – 60 keV

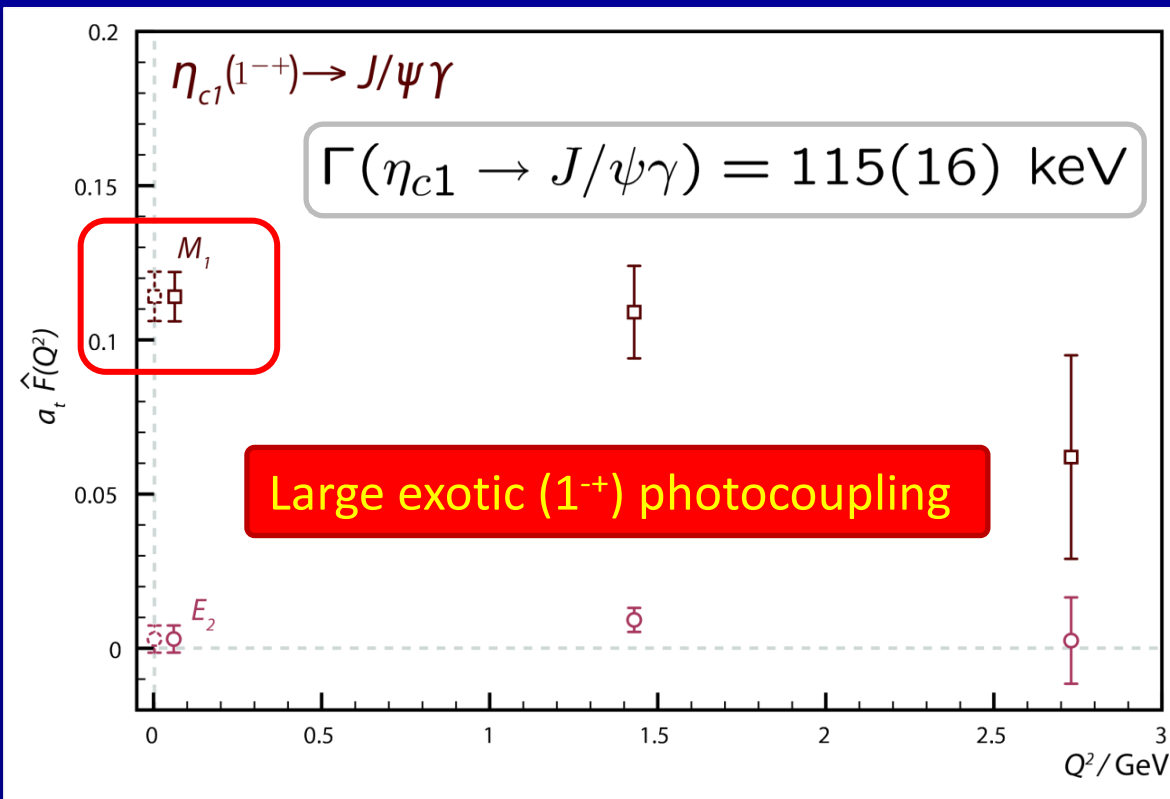
Photocouplings

Exotic meson photocoupling



Photocouplings

Exotic meson photocoupling



Same scale as many measured conventional charmonium transitions

BUT very large for an M_1 transition

$$\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2 \text{ keV}$$

Suggests a spin-triplet hybrid

Analogous to 1^{--} hybrid to pseudoscalar trans: M_1 with no spin flip

More charmonium results

Tensor – Vector transitions $\chi_{c2}, \chi'_{c2}, \chi''_{c2} \rightarrow J/\psi\gamma$

Identify $1^3P_2, 1^3F_2, 2^3P_2$ tensors from hierarchy of multipoles E_1, M_2, E_3

Vector – Pseudoscalar $J/\psi, \psi', \psi'' \rightarrow \eta_c\gamma$

Scalar – Vector $\chi_{c0} \rightarrow J/\psi\gamma \quad \psi', \psi'' \rightarrow \chi_{c0}\gamma$

Axial – Vector $\chi_{c1}, \chi'_{c1} \rightarrow J/\psi\gamma$

Summary and Outlook

Summary

- Our first results on light mesons – **technology and method work**
- **Spin identification** is possible using operator overlaps
- **First spin 4 meson** extracted and confidently identified on lattice
- **Exotics** (and non-exotic **hybrids**)
- Isovectors and kaons
- **Excited radiative transitions** in charmonium (incl. exotic meson)

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Outlook – ongoing work

- Multi-meson operators – resonance physics
- Disconnected diagrams – isoscalars and multi-mesons
- Baryons
- Photocouplings
- Lighter pion masses and larger volumes

