Soft and Coulomb-gluon resummation in pair-production of heavy coloured particles

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Based on **M. Beneke, PF, S. Klein, C. Schwinn** **[Nucl. Phys. B828: 69-101, 2010]**, **[Nucl. Phys. B842: 414-474, 2011]** and work in preparation

Outline

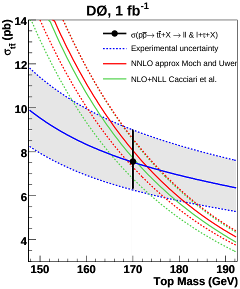
- Production of heavy coloured pairs at hadron colliders: motivation and state of the art
- Effective-theory description of pair production near threshold
- Soft and Coulomb-gluon resummation in momentum space
- Resummation for squarks and top quarks cross sections
- Summary and Outlook

Motivation

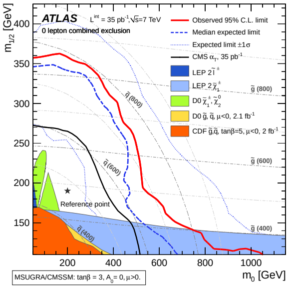
IN THIS TALK: pair production of **coloured heavy particles** at Tevatron/LHC

$$N_1 N_2 \to H(p_1) H^{'}(p_2) + X \qquad H, H^{'} = \textbf{top}, \textbf{squarks}, \text{gluinos...}$$

accurate theoretical predictions for the cross section **phenomenologically important** (sensitivity to **mass parameters**, **exclusion bounds**, **model discrimination**...)



[D0 Collaboration '09]



[ATLAS Collaboration, '11]

+ theoretically interesting due to **non-trivial colour exchange**

The standard approach: fixed-order pQCD

$$\sigma_{HH'}(s; m_H, m_{H'}) = \int_\tau^1 dz \sum_{i,j=q,\bar{q},g} \underbrace{\mathcal{L}_{ij}(z;\mu_f)}_{\text{non-pert.}} \underbrace{\hat{\sigma}_{ij\to HH'X}(\hat{s}; m_H, m_{H'}, \mu_f)}_{\text{pert. QCD}}$$

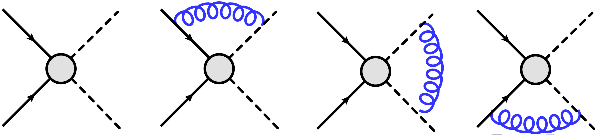
$$\tau = (m_H + m_{H'})^2/s \qquad \hat{s} = z\, s$$

- Non-perturbative physics factorized into parton density functions (**PDFs**)
  $\Rightarrow$ **extracted from experimental data**

$$\mathcal{L}_{ij}(z;\mu_f) = \int_z^1 \frac{dy}{y} f_{i/N_1}(y;\mu_f) f_{j/N_2}(z/y;\mu_f)$$

- Partonic cross section $\hat{\sigma}$ describes short-distance hard scattering of elementary DOFs
  $\Rightarrow$ **computed in standard perturbation theory (pQCD)**

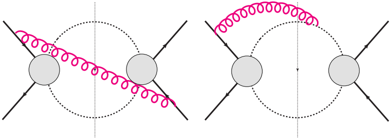
$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_s\, \hat{\sigma}^{(1)} + \ldots$$



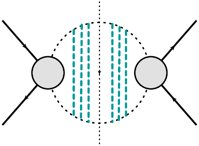
Soft-gluon and Coulomb corrections

NLO partonic cross sections enhanced near **threshold**, $\beta \equiv \sqrt{1-(m_H+m_{H'})^2/\hat{s}} \to 0$

- **Threshold logarithms**: $\sim \alpha_s^n \ln^m \beta$
  $\Leftrightarrow$ **soft-gluon exchange** between initial-initial, initial-final ($\alpha_s \ln^{2,1} \beta$) and final-final state particles ($\alpha_s \ln \beta$)



- **Coulomb corrections**: $\sim (\alpha_s/\beta)^n$
  $\Leftrightarrow$ **static interaction** of slowly-moving heavy particles (mediated by **potential gluons**...)



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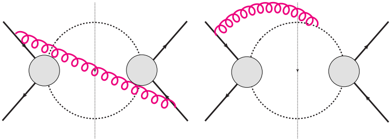

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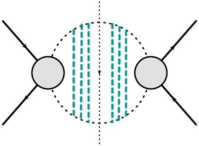

enhanced terms can spoil convergence of perturbative series $\Rightarrow$ **RESUMMATION**

$\Rightarrow$ **normalisation** of the cross section

$\Rightarrow$ reduction of dependence on the **factorisation-scale**

$\Rightarrow$ can be used to construct **higher-order approximations at fixed order in α_s**

Is resummation really important?

$\beta_{\max} = \sqrt{1-\tau}$ $\Rightarrow$ **typically β is not really small (unless $\tau \sim 1$...)**

Example: $m_t = 171.3\,\text{GeV}$, $\sqrt{s} = 7\,\text{TeV} \rightarrow \tau = 0.0025$, $\beta_{\max} = 0.999$.

Why should the threshold region be relevant at all for $\tau \ll 1$?

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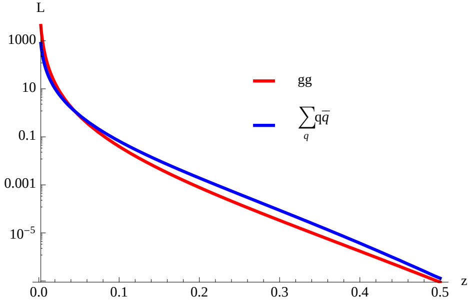

Partonic threshold region can be **dynamically enhanced** by fast drop-off of the parton luminosities al large z

generally requires a case-by-case study of the behaviour of $\mathcal{L}(\beta)\hat{\sigma}(\beta)$...

State of the art

- **$t\bar{t}$ production**
  - **NLO QCD**: Nason et al. '88; Beenakker et al. '89
  - **NLO EW**: Beenakker et al. '94; Bernreuther et al. '95; Kuhn et al. '96;...
  - **NNLO**: in progress Bonciani et al. '10; Czakon '11
  - **NLL (+NLO)**: Kidonakis et al. '96; Bonciani et al. '98; Cacciari et al. '08; Moch et al. '08; Kidonakis et al. '08;...
  - **NNLL, approx. NNLO**: Beneke, PF, Klein, Schwinn '09/'10; Ahrens et al. '10; Kidonakis '10; HATHOR Aliev et al. '10
- **Squarks, gluinos**
  - **NLO SUSY-QCD**: Beenakker et al. '96; PROSPINO, Plehn et al.
  - **NLO EW**: Bornhauser et al. '07; Hollik et al. '07-'10; Gerner et al '10
  - **NLL/approx. NNLO**: Kulesza/Motyka '09; Beenakker et al. '09/'10; Langenfeld/Moch '09/'10, Beneke, PF, Schwinn '10;

+ many works on Coulomb resummation ($\Leftrightarrow$ quarkonia physics, $e^- e^+ \to t\bar{t}$,...)

Combined soft and Coulomb resummation

$$\alpha_s/\beta \sim \alpha_s \ln\beta \sim 1 \Rightarrow \underline{\textbf{modified counting scheme}}$$

$$\hat{\sigma}_{pp'} \quad \propto \quad \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots\Big]$$

$$\times \left\{1\,(\text{LL,NLL});\, \alpha_s, \beta\,(\text{NNLL});\, \alpha_s^2, \alpha_s\beta, \beta^2\,(\text{NNNLL}); \ldots\right\}$$

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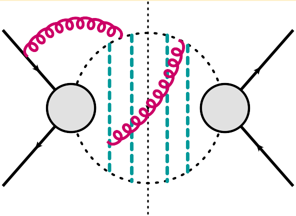
$$\times \left\{1\,\text{(LL,NLL)};\, \alpha_s, \beta\,\text{(NNLL)};\, \alpha_s^2, \alpha_s\beta, \beta^2\,\text{(NNNLL)};\ldots\right\}$$

- **non-relativistic H, H' and Coulomb gluons:** $E \sim m_H\beta^2$, $|\vec{p}| \sim m_H\beta$
- **soft gluons:** $q_s \sim m_H\beta^2$

potential and soft modes have the same energy and can "communicate" with each other

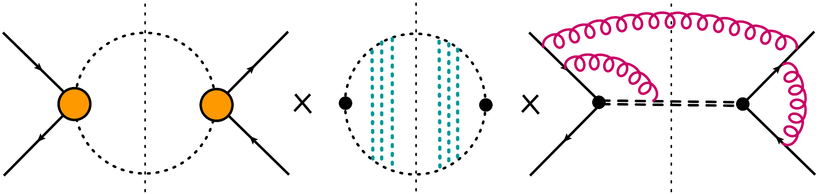


$\Rightarrow$ **structure of soft-Coulomb emission can be in principle highly non-trivial!**

Factorisation of pair production near threshold

Effective-theory description of pair production near threshold $\hat{s} \sim (m_H + m_{H'})^2$
[Beneke, PF, Schwinn, '09/'10] $\Rightarrow$ factorization of **hard**, **soft** and **Coulomb** contributions

$$\hat{\sigma}_{pp'}(\hat{s}, \mu_f) = \sum_i H_i(M, \mu_f) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega, \mu_f)$$



- hard function H_i depends on the **specific physics model and process**
- potential function J_{R_α} encodes Coulomb effects ($\sim \alpha_s^n/\beta^n$)
- process-independent soft function $W_i^{R_\alpha}$ ($\sim \alpha_s^n \ln^m \beta$)
  $\Rightarrow$ depends only on **total colour charge** R_α of the pair!

factorization valid up to NNLL and for S-wave production

EFT description of pair-production near threshold

Near threshold ($\beta \ll 1$) partonic cross section receives contributions from four different momentum regions ($M \equiv (m_H + m_{H'})/2$):

- **hard**: $k^2 \sim M^2$
- **potential** : $k_0 \sim M\beta^2$, $|\vec{k}| \sim M\beta$
- **soft**: $k_0 \sim |\vec{k}| \sim M\beta^2$
- **collinear**: $k_- \sim M$, $k_+ \sim M\beta^2$, $k_\perp \sim M\beta$

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- **collinear**: $k_- \sim M$, $k_+ \sim M\beta^2$, $k_\perp \sim M\beta$

full theory matched on an effective Lagrangian from which **hard modes** are integrated out.

$$\mathcal{L}_{\text{full}} \longrightarrow \mathcal{L}_{\text{EFT}} \equiv \mathcal{L}_{\text{SCET}} + \mathcal{L}_{\text{PNRQCD}}$$

- $\mathcal{L}_{\mathbf{SCET}}$: describes interactions of **collinear** (ξ_c, A_c) and **soft** (A_s) modes

$$\mathcal{L}_{\text{SCET}} = \bar{\xi}_c \left(in \cdot D + i\not{D}_{\perp c} \frac{1}{i\bar{n} \cdot D_c} i\not{D}_{\perp c} \right) \frac{\not{\bar{n}}}{2} \xi_c - \frac{1}{2} \text{tr} \left(F_c^{\mu\nu} F^c_{\mu\nu} \right) + \ldots$$

- $\mathcal{L}_{\mathbf{PNRQCD}}$: contains interactions of **potential** (ψ, ψ') and **soft** (A_s) modes

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi'$$
$$+ \int d^3\vec{r} \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right](x+\vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^\dagger \mathbf{T}^{(R')a} \psi' \right](x) + \ldots$$

Structure of EFT amplitudes

$$\mathcal{A}(pp' \to HH'X) = \sum_{\ell} C^{(\ell)}_{\{a;\alpha\}}(\mu_f)\; \langle HH'X|\mathcal{O}^{(\ell)}_{\{a;\alpha\}}(\mu_f)|pp'\rangle_{\text{EFT}}$$

- effective operators $\mathcal{O}^{(0)}_{\{a;\alpha\}}(\mu_f) \propto [\phi_{c;a_1,\alpha_1}\phi_{\bar{c};a_2,\alpha_2}\psi^{\dagger}_{a_3\alpha_3}\psi'^{\dagger}_{a_4\alpha_4}]$ contain collinear and non-relativistic fields $\Leftrightarrow$ **long-distance** effects.
  Operators with more fields or derivatives suppressed by extra powers of β (not required at NNLL...)

- matrix element evaluated using the EFT Lagrangian $\Rightarrow$ soft gluons interacting with everything and potential interactions between the two non-relativistic heavy particles

- hard matching coefficient $C^{(\ell)}_{\{a;\alpha\}}(\mu_f)$ encodes **short-distance** structure of pair-production process at the scale M
  - $\Rightarrow$ extracted from fixed-order calculations of on-shell amplitudes
  - $\Rightarrow$ decomposed on a suitable basis of colour-state operators: $C^{(\ell)}_{\{a;\alpha\}}(\mu_f) = C^{(\ell,i)}_{\{\alpha\}}(\mu_f)c^{(i)}_{\{a\}}$

Soft-gluon decoupling

At **leading order in β** soft gluons can be decoupled from the effective Lagrangian via field redefinitions involving soft Wilson lines (**path-order exponentials of soft gluon fields**):

$$\phi_c(x) \longrightarrow S_n^{(R)}(x_-)\phi_c^{(0)}(x)$$

$$\psi(x) \longrightarrow S_v^{(R)}(x_0)\psi^{(0)}(x) \qquad S_n^{(R)}(x) = \mathrm{P}\exp\left[ig_s \int_{-\infty}^{0} dt\, n \cdot A_s^c(x+nt)\mathbf{T}^{(R)c}\right]$$

$$S_v^{(R)\dagger}(x_0)D_s^0 S_v^{(R)}(x_0) = \partial^0 \qquad \left[\psi^\dagger \mathbf{T}^{(R)a}\psi\right](x+\vec{r}\,) = S^8_{v,ab}(x_0)\left[\psi^{(0)\dagger}\mathbf{T}^{(R)b}\psi^{(0)}\right](x+\vec{r}\,)$$

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upon field redefinition:

$$\hat{\sigma}_{pp'}(\hat{s},\mu_f) \equiv \frac{1}{2s}\int d\Phi|\mathcal{A}|^2 = \sum_{i,i'}\sum_{S=|s-s'|}^{s+s'} \underbrace{H_{ii'}^S(M,\mu_f)}_{\text{hard}} \int d\omega \sum_{R_\alpha} \underbrace{J_{R_\alpha}^S(E-\frac{\omega}{2})}_{\text{potential}} \underbrace{W_{ii'}^{R_\alpha}(\omega,\mu_f)}_{\text{soft}}$$

- $H_{ii'}^S(M,\mu_f) \propto C_{\{\alpha\}}^{(0,i)}(M,\mu_f) C_{\{\beta\}}^{(0,i')*}(M,\mu_f)\ldots$
- $J_{R_\alpha}^S(q) \propto \int d^4z e^{iq\cdot z}\langle 0|[\psi'^{(0)}\psi^{(0)}](z)[\psi^{(0)\dagger}\psi'^{(0)\dagger}](0)|0\rangle$
- $W_{ii'}^{R_\alpha}(\omega,\mu_f) = P_{\{k\}}^{R_\alpha} c_{\{a\}}^{(i)} c_{\{b\}}^{(i')} \int dz_0 e^{i\omega z_0/2}\langle 0|\overline{\mathrm{T}}[S_n^\dagger S_{\bar{n}}^\dagger S_v S_v](z)\mathrm{T}[S_{\bar{n}} S_n S_v^\dagger S_v^\dagger](0)|0\rangle$

Colour structure of the factorisation formula

The factorisation formula has a priori a **non-trivial colour structure**

- hard function is a matrix in colour-state space: $H_{ii'} \equiv H_{\{ab\}} c^{(i)}_{\{a\}} c^{(i')*}_{\{b\}}$
- potential function $J_{\{k\}}$ is projected over irreducible representations of the HH' system: $J_{\{k\}} = \sum_{R_\alpha} P^{R_\alpha}_{\{k\}} J_{R_\alpha}$, with $R \otimes R' = \sum_\alpha R_\alpha$
- soft function given by a set of colour matrices $W^{R_\alpha}_{ii'}$

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Colour basis $c^{(i)}_{\{a\}}$ can be chosen such that $W^{R_\alpha}_{ii'}$ are diagonal to all orders in α_s
[Beneke, PF, Schwinn, Nucl.Phys. B828 (2010)]

⇒ decompose initial-state and final-state product representations into **irreducible representations**:
→ Clebsch-Gordan coefficients

$$r \otimes r' = \sum_\alpha r_\alpha \to C^{r_\alpha}_{\alpha a_1 a_2} \qquad\qquad R \otimes R' = \sum_\beta R_\beta \to C^{R_\beta}_{\alpha a_1 a_2}$$

⇒ identify pairs of equivalent initial- and final-state representations $P_i = (r_\alpha, R_\beta)$

⇒ construct colour basis by contracting the Clebsches into colour-invariant combinations

$$c^{(i)}_{\{a\}} = \frac{1}{\sqrt{\dim(r_\alpha)}} C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4} \qquad\qquad P^{R_\alpha}_{\{a\}} = C^{R_\alpha *}_{\alpha a_1 a_2} C^{R_\alpha}_{\alpha a_3 a_4}$$

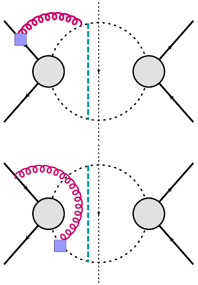
Subleading soft interactions and factorization

At NNLL subleading soft vertices in SCET and PNRQCD potentially important

$$\psi^\dagger \vec{x}\cdot\vec{E}^{\rm us}(x_0,\vec{0})\psi \qquad\qquad \bar{\xi}\left(x_\perp^\mu n_-^\nu W_c g F_{\mu\nu}^{\rm us} W_c^\dagger\right)\frac{\not{n}_+}{2}\xi$$

Subleading soft interactions not removed by field redefinitions

$\Rightarrow$ related to off-diagonal three-parton colour correlations in IR singularities of QCD amplitudes (Ferroglia, Neubert, Pecjak, Yang '09)



$$\Rightarrow \frac{\alpha_s}{\beta}\alpha_s\beta\ln\beta \sim \alpha_s^2\ln\beta$$

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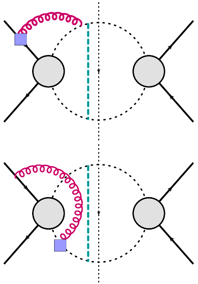

$$\Rightarrow \frac{\alpha_s}{\beta}\alpha_s\beta\ln\beta \sim \alpha_s^2\ln\beta$$

Contributions of subleading soft-collinear and soft-potential vertices vanish for the total cross section!

- **Soft-collinear**: $k_\perp$ can always be chosen to be 0
- **Soft-potential**: vanish because of rotational invariance

Soft/hard resummation in momentum space

IR structure of QCD amplitudes and scale-invariance of the hadronic cross section lead to RG evolution equations for the soft function $W_i^{R_\alpha}$ and the hard function $H_i^{R_\alpha}$
(generalisation of DY result [Becher, Neubert, Xu '07] to **arbitrary** R_α)

$$\frac{d}{d\ln\mu_f} W_i^{R_\alpha}(\omega,\mu_f) = -2\left[(C_r + C_{r'})\,\Gamma_{\text{cusp}}\ln\left(\frac{\omega}{\mu_f}\right) + 2\gamma_{H,s}^{R_\alpha} + 2\gamma_s^{r} + 2\gamma_s^{r'}\right] W_i^{R_\alpha}(\omega,\mu_f)$$
$$-2\,(C_r + C_{r'})\,\Gamma_{\text{cusp}} \int_0^\omega d\omega' \,\frac{W_i^{R_\alpha}(\omega',\mu_f) - W_i^{R_\alpha}(\omega,\mu_f)}{\omega - \omega'}$$

and similar for hard function $H_i(M,\mu_f)$

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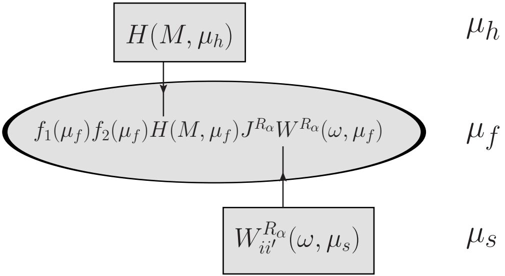
and similar for hard function $H_i(M,\mu_f)$

Resummation strategy

- Solve evolution equation in momentum space
- Evolve the function H_i from the hard scale μ_h to μ_f
- Evolve soft function $W_i^{R_\alpha}$ from a low scale μ_s to μ_f.



Resummed soft function and hard matching coefficient

Solutions to the RG evolutions equations

[Neubert, Becher, Xu '07; Beneke, PF, Schwinn '10]

$$H_i^{\text{res}}(M,\mu_f) = \exp[4S(\mu_h,\mu_f) - 2a_i^V(\mu_h,\mu_f)] \left(-\frac{M^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_f)} H_i(M,\mu_h)$$

$$W_i^{R_\alpha,\text{res}}(\omega,\mu_f) = \exp[-4S(\mu_s,\mu_f) + 2a_{W,i}^{R_\alpha}(\mu_s,\mu_f)]\tilde{s}_i^{R_\alpha}(\partial_\eta,\mu_s)\frac{1}{\omega}\left(\frac{\omega}{\mu_s}\right)^{2\eta}\theta(\omega)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha_s \frac{(C_r + C_{r'})\Gamma_{\text{cusp}}(\alpha_s)}{2\beta(\alpha_s)} \int_{\alpha_s(\nu)}^{\alpha_s} \frac{d\alpha_s'}{\beta(\alpha_s')}$$

$$a_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha_s \frac{(C_r + C_{r'})\Gamma_{\text{cusp}}(\alpha_s)}{2\beta(\alpha_s)}$$

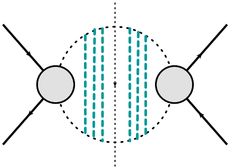
$$a_i^X(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha_s \frac{\gamma_i^X(\alpha_s)}{\beta(\alpha_s)}$$

- Resummation controlled by **cusp and soft anomalous dimensions**: Γ_{cusp}, γ_i^V, γ^r, $\gamma_{H,s}^{R_\alpha}$
- **Hard and soft scales** chosen to minimise higher-order terms in fixed-order expansions of $H_i(M,\mu_h)$ and $\tilde{s}_i^{R_\alpha}(L,\mu_s) \overset{\text{Laplace tr.}}{\longleftrightarrow} W_i^{R_\alpha}(\omega,\mu_s)$

Resummation of Coulomb corrections

Exchange of **Coulomb gluons** between the pair H, H': $\Delta\sigma^{\mathrm{Coul},(1)}/\sigma^{\mathrm{tree}} \sim \alpha_s/\beta \sim 1$

$\Rightarrow$ **Coulomb corrections must be resummed to all orders as well**

$J_{R_\alpha}(E) \Leftrightarrow$ 

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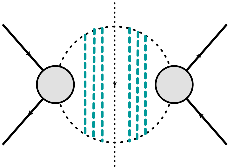
$J_{R_\alpha}(E) \Leftrightarrow$ 

Resummation of Coulomb effects well understood from **PNRQCD** and quarkonia physics.
For HH' system in **irreducible representation R_α** (and at LO in PNRQCD):

$$J_{R_\alpha}(E) = -\frac{(2m_{\mathrm{red}})^2}{2\pi}\mathrm{Im}\left\{\sqrt{-\frac{E}{2m_{\mathrm{red}}}} + \alpha_s(-D_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{8\,m_{\mathrm{red}}E}{\mu_f^2}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 - \frac{\alpha_s(-D_{R_\alpha})}{2\sqrt{-E/(2m_{\mathrm{red}})}}\right)\right]\right\}$$

$$E \equiv \sqrt{s} - M$$

Includes also bound-state contributions below threshold!

Higher-order potential corrections at NNLL

HO Coulomb and non-Coulomb corrections in PNRQCD required at NNLL/NNLO

- **Coulomb potential**:

$$\tilde{V}_{\rm C}^{(1)}(\vec{p},\vec{q}) = \frac{D_{R_\alpha}\alpha_s^2}{|\vec{q}|^2}\left(a_1 - \beta_0 \ln\frac{|\vec{q}|}{\mu_C^2}\right)$$

- **Non-Coulomb potentials**:

$$\tilde{V}_{\rm nC}^{(1)}(\vec{p},\vec{q}) = \frac{4\pi D_{R_\alpha}\alpha_s}{|\vec{q}|^2}\left[\frac{\pi\alpha_s|\vec{q}|}{4m}\left(\frac{D_{R_\alpha}}{2}+C_A\right)+\frac{|\vec{p}|^2}{m^2}+\frac{|\vec{q}|^2}{m^2}v_{\rm spin}\right]$$

$$v_{\rm spin} = 0(\text{singlet}), -\frac{2}{3}(\text{triplet})$$

contribution to NNLO total cross section

$$\Delta\sigma_{\rm nC}^{\rm NNLO} = \sigma^{(0)}\alpha_s^2 \ln\beta\left[-2D_{R_\alpha}^2(1+v_{\rm spin})+D_{R_\alpha}C_A\right]$$

Squark-antisquark production at the LHC

$$PP \to \tilde{q}\bar{\tilde{q}} + X$$

NLL soft resummation and **Coulomb resummation** to **total cross section**

$$\hat{\sigma}^{\rm NLL}_{pp'}(\hat{s},\mu_f) = \sum_i H_i(\mu_h)\; U_i(M,\mu_h,\mu_s,\mu_f) \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)} \int_0^\infty d\omega\; \frac{J_{R_\alpha}(E-\frac{\omega}{2})}{\omega} \left(\frac{\omega}{2M}\right)^{2\eta}$$

resummed cross section is matched onto the full NLO result

[Zerwas et al., '96; Langenfeld, Moch '09]

$$\hat{\sigma}^{\rm match}_{pp'}(\hat{s},\mu_f) = \left[\hat{\sigma}^{\rm NLL}_{pp'}(\hat{s},\mu_f) - \hat{\sigma}^{\rm NLL}_{pp'}(\hat{s},\mu_f)\big|_{\rm NLO}\right] + \hat{\sigma}^{\rm NLO}_{pp'}(\hat{s},\mu_f)$$

Scale choice for μ_s, μ_h and μ_C

What is a good choice for μ_s, μ_h and μ_C?

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- Choose **soft scale** such that one-loop soft corrections to the **hadronic cross section** are minimised [Becher, Neubert, Xu '07]

$$\frac{\partial}{\partial \bar{\mu}_s} \int dx_1 d_2 f(x_1, \bar{\mu}_s) f(x_2, \bar{\mu}_s) \Delta\hat{\sigma}^{S,(1)}(\hat{s}, \bar{\mu}_s) = 0$$

This choice guarantees well-behaved perturbative expansion at the low scale $\bar{\mu}_s$

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- **Coulomb scale**: set by typical **virtuality of a Coulomb gluons** $\sqrt{|q^2|} \sim m_{\tilde{q}}\beta \sim m_{\tilde{q}}\alpha_s$

$$\Rightarrow \mu_C = \max\{2m_{\tilde{q}}\beta, C_F m_{\tilde{q}}\alpha_s(\mu_C)\}$$

  $\hookrightarrow$ twice inverse Bohr radius of first bound state

Necessary to correctly resums NLL effects from running of Coulomb potential!

Squark-antisquark resummed cross section at LHC (14 TeV)

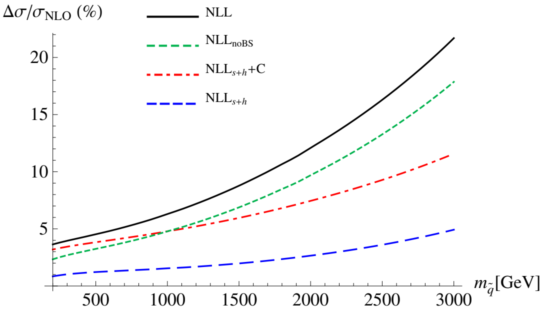
Beneke, PF, Schwinn '10

- **NLL**: full soft and Coulomb resummation (including bound-state contributions from below threshold)
- **$\mathbf{NLL_{noBS}}$**: soft and Coulomb resummation (but no bound-state contribution)
- **$\mathbf{NLL_{s+h} + C}$**: soft resummation + Coulomb resummation (no interference terms)
- **$\mathbf{NLL_{s+h}}$**: soft resummation only

$$\kappa_{\rm NLL} = \frac{\sigma_{\rm NLL} - \sigma_{\rm NLO}}{\sigma_{\rm NLO}}$$

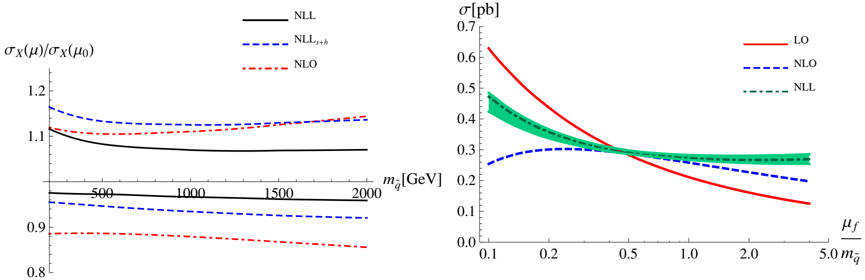
Setup:

- MSTW2008 PDFs
- equal squark masses
- no stops
- $m_{\tilde{g}} = 1.25 m_{\tilde{q}}$



Factorisation-scale dependence

Resummation usually leads to significant **reduction of scale dependence** of NLO result:



LHC@14TeV

$m_{\tilde{q}} = 1$ TeV

All scales varied by a factor 2 around the default values, and uncertainties summed in quadrature

Comparison to Mellin-space resummation

Resummation of threshold logs can be also performed in Mellin-moment space
[Sterman '87; Catani, Trentadue '89]

$$\sigma_{HH'}(N) \equiv \int \tau^{N-1} \sigma_{HH'}(\tau) = \sum_{ij} \mathcal{L}_{ij}(N+1) \hat{\sigma}_{ij \to HH'}(N)$$

$$\log \beta \Leftrightarrow \log N$$

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Compare NLL squark resummation in momentum space to [Beenakker et al. JHEP 0912:041, 2009] (**Mellin-space formalism**, only soft resummation, no Coulomb effects)

	$\sigma(pp \to \tilde{q}\tilde{q})$(pb), $\sqrt{s} = 14$ TeV			
$m_{\tilde{q}}$[GeV]	NLO	$\text{NLL}_{\text{Mellin}}$	NLL_s	NLL
200	1.3×10^3	1.31×10^3 (1%)	1.31×10^3 (1%)	1.34×10^3 (3.4%)
500	1.6×10^1	1.61×10^1 (1.2%)	1.62×10^1 (1.3%)	1.67×10^1 (4.2%)
1000	2.89×10^{-1}	2.93×10^{-1}(1.7%)	2.94×10^{-1} (1.7%)	3.06×10^{-1}(5.8%)
2000	1.11×10^{-3}	1.14×10^{-3}(3.4%)	1.14×10^{-3} (3.1%)	1.24×10^{-3} (11%)
3000	7.13×10^{-6}	7.59×10^{-6}(6.4)%	7.54×10^{-6} (5.8%)	8.61×10^{-6} (21%)

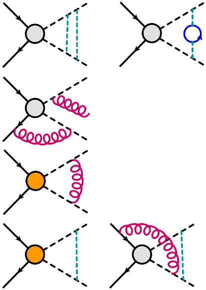
- Good agreement of momentum-space and Mellin-moment resummation
- **Full soft-Coulomb resummation generally larger than pure soft resummation!**

All ingredients for **NNLL resummation** of $t\bar{t}$ cross section known

- 1-loop colour-separated hard functions $H_i^{(1)}$ [Czakon, Mitov '09]
- 2-**loop soft anomalous dimension** [Beneke, PF, Schwinn '09; Czakon, Mitov, Sterman '09]
- NLO Coulomb and non-Coulomb potentials [Beneke, Signer, Smirnov '99]

Can be used to construct approx. NNLO containing all terms singular in β
[Beneke, PF, Czakon, Mitov, Schwinn '09; HATHOR Aliev et al. '10]

$$
\begin{aligned}
\hat{\sigma}^{\text{NLO}}_{\text{approx.}} = & \frac{k_{\text{LO}}^2}{\beta^2} + \frac{1}{\beta}\left[k_{\text{NLO},1}\ln\beta + k_{\text{NLO},0}\right] + k_{\text{n-C}}\ln\beta \\
& + c_{S,4}^{(2)}\ln^4\beta + c_{S,3}^{(2)}\ln^3\beta + c_{S,2}^{(2)}\ln^2\beta + \mathbf{c}_{\mathbf{S},\mathbf{1}}^{(\mathbf{2})}\ln\beta \\
& + H^{(1)}\left[c_{S,2}^{(1)}\ln^2\beta + c_{S,1}^{(1)}\ln\beta\right] \\
& + \frac{k_{\text{LO}}}{\beta}\left[c_{S,2}^{(1)}\ln^2\beta + c_{S,1}^{(1)}\ln\beta + c_{S,0}^{(1)} + H^{(1)}\right]
\end{aligned}
$$

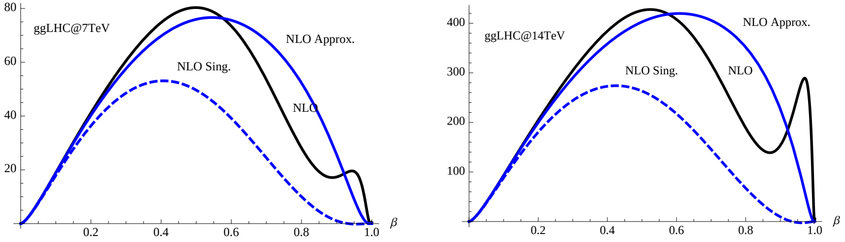


Contribution of threshold-enhanced terms

At LHC $\sqrt{s} \gg 2m_t \Rightarrow$ **How good is the threshold approximation?**
can study the approximation at the NLO level...

Plot $8\beta m_t^2/(s(1-\beta^2)^2)\mathcal{L}_{gg}(\beta)\hat{\sigma}_{tt}(\beta)$:

- **NLO**: exact NLO result
- **NLO sing.**: only singular terms in β
- **NLO approx.**: singular terms + O(1) term ($\Leftrightarrow H_i^{(1)}$)



NLO sing. is good approximation only up to $\beta \sim 0.3$
However: expect NNLO approximation to be better (more singular terms at $O(\alpha_s^2)$...)

$m_t = 173.1$ GeV, $\mu_f = m_t$, MSTW2008NNLO
Beneke, PF, Klein, Schwinn, **PRELIMINARY!**

$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC@7	LHC@10	LHC@14
NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}	380^{+44+17}_{-46-17}	842^{+97+30}_{-97-32}
NLO+NLL	$6.57^{+0.52+0.33}_{-0.30-0.24}$	151^{+23+8}_{-12-8}	382^{+60+17}_{-32-18}	$848^{+136+30}_{-75-32}$
NLO+NNLL	$6.77^{+0.27+0.35}_{-0.48-0.25}$	155^{+4+8}_{-9-9}	390^{+14+17}_{-26-18}	858^{+35+31}_{-64-33}
$\mathrm{NNLO_{app}}(\beta)$	$7.10^{+0.0+0.36}_{-0.26-0.26}$	162^{+2+9}_{-3-9}	407^{+9+17}_{-5-18}	895^{+24+31}_{-6-33}
$\mathrm{NNLO_{app}}(\beta)$+NNLL	$7.13^{+0.22+0.36}_{-0.24-0.26}$	162^{+4+9}_{-1-9}	405^{+14+17}_{-2-18}	892^{+38+31}_{-3-33}
$\mathrm{NNLO_{app}}(\beta)$+NNLL+BS	$7.14^{+0.14+0.36}_{-0.22-0.26}$	162^{+4+9}_{-1-9}	407^{+14+17}_{-2-18}	896^{+38+31}_{-3-33}

- Combined soft-Coulomb resummation for $t\bar{t}$ total cross section
- All scales $(\mu_f, \mu_h, \mu_s, \mu_C)$ varied in interval $0.5\tilde{\mu}_i < \mu_i < 2\tilde{\mu}_i$
- Fixed μ_s from minimising $\Delta\sigma^{\mathrm{NLO}}_{\mathrm{soft}}$: $\Rightarrow$ $\mu_s = 85, 146, 174, 202$ GeV for Tevatron, LHC@7, LHC@10, LHC@14. **No large scale hierarchy**
  $\Leftrightarrow$ treatment of soft scale should probably different...

Summary

- Factorisation formula for pair-production near threshold in **SCET+PNRQCD**
  - ⇒ Valid for **arbitrary colour representations**
  - ⇒ Proves decoupling of **hard**, **soft** and **Coulomb** modes
  - ⇒ **Diagonal in colour-space** to all orders in α_s
- Simultaneous resummation of threshold logarithms and Coulomb singularities
  - ⇒ Directly in **momentum space** via RG evolution equations
- Application to **squark-antisquark production** at the LHC
  - ⇒ NLL corrections $\sim 4-20\%$ for $m_{\tilde{q}} \sim 200\text{GeV} - 3\text{TeV}$
  - ⇒ Reduction of factorisation-scale dependence
- NNLL resummation of $t\bar{t}$ **total cross section**
  - ⇒ All $O(\alpha_s^2)$ **terms singular in** β included
  - ⇒ NNLL corrections beyond NNLO very small

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Things still to do

- Could be applied to **more processes**, ex. gluino pair production (larger colour charges...)
- More satisfactory treatment of scales (**running soft scale**...)

Backup slides

Construction of the colour basis

EXAMPLE: $t\bar{t}$ **(squark-antisquark) production in gluon fusion** $(8 \otimes 8 \to 3 \otimes \bar{3})$

- **Irreducible representations**:

$$3 \otimes \bar{3} = 1 + 8 \qquad\qquad 8 \otimes 8 = 1 + 8_S + 8_A + 10 + \bar{10} + 27$$

- **Pairs of equivalent representations**: $P_i = \{(1,1), (8_S, 8), (8_A, 8)\}$
- **Clebsch-Gordan coefficients**:

$$3 \otimes \bar{3} \;:\; C^{(1)}_{a_1 a_2} = \frac{1}{\sqrt{N_C}} \delta_{a_1 a_2} \,, \qquad C^{(8)}_{\alpha a_1 a_2} = \sqrt{2} T^{\alpha}_{a_2 a_1}$$

$$8 \otimes 8 \;:\; C^{(1)}_{a_1 a_2} = \frac{1}{\sqrt{D_A}} \delta_{a_1 a_2} \,, \qquad C^{(8_S)}_{\alpha a_1 a_2} = \frac{1}{2\sqrt{B_F}} D^{\alpha}_{a_2 a_1} \,, \qquad C^{(8_A)}_{\alpha a_1 a_2} = \frac{1}{2\sqrt{N_C}} F^{\alpha}_{a_2 a_1} \,, \quad \ldots$$

- **Colour basis**:

$$c^{(1)}_{\{a\}} = \frac{1}{\sqrt{N_C D_A}} \delta_{a_1 a_3} \delta_{a_2 a_4} \qquad c^{(2)}_{\{a\}} = \frac{1}{\sqrt{2 D_A B_F}} D^{\alpha}_{a_2 a_1} T^{\alpha}_{a_3 a_4} \qquad c^{(3)}_{\{a\}} = \sqrt{\frac{2}{N_C D_A}} F^{\alpha}_{a_2 a_1} T^{\alpha}_{a_3 a_4}$$

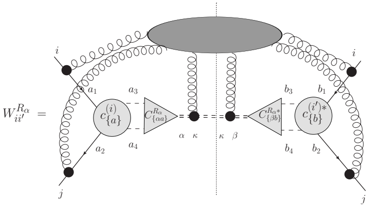
Colour structure of the soft function

- The basis $c^{(i)}_{\{a\}}$ diagonalises $W^{R_\alpha}_{ii'}$ to all orders in α_s (extends results for $t\bar{t}$, squarks, gluinos at one-loop [Kidonakis, Sterman '97; Kulesza, Motyka '08])
  - Follows from completeness and orthonormality properties of the Clebsch-Gordan coefficients (+ Bose symmetry of the soft function)

$$\sum_{R_\alpha} C^{R_\alpha *}_{\alpha a_1 a_2} C^{R_\alpha}_{\alpha b_1 b_2} = \delta_{a_1 b_1}\delta_{a_2 b_2} \qquad C^{R_\alpha *}_{\alpha a_1 a_2} C^{R_\beta}_{\beta a_1 a_2} = \delta_{\alpha\beta}\delta_{R_\alpha R_\beta}$$

  - The diagonal element $W^{R_\alpha}_{ii}$ is non-vanishing only if R_α is equivalent to either irreducible representations in the pair $P_i = (r_\alpha, R_\beta)$

- $W^{R_\alpha}_{ii'}$ can be rewritten as the soft function of a single scalar in the representation R_α $\Rightarrow$ soft radiation emitted off the total colour charge of the pair



Spefic to the threshold region:

$v_1 = v_2 = v$

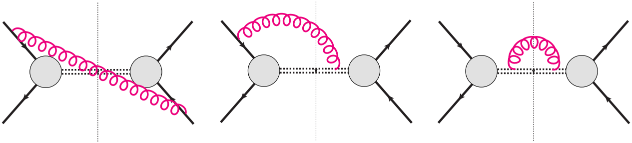
$$C^{R_\alpha}_{\alpha a_1 a_2} S^{(R)}_{v,a_1 b_1} S^{(R')}_{v,a_2 b_2} = S^{(R_\alpha)}_{v,\alpha\beta} C^{R_\alpha}_{\beta,b_1 b_2}$$

One-loop soft function and anomalous dimensions

One-loop massive soft anomalous dimension $\gamma_{H,s}^{R_\alpha}$ extracted from one-loop soft function:

$$W^{R_\alpha}_{\{a\alpha,b\beta\}}(z_0,\mu_f) = 1 + \left(-ig_s \mathbf{T}^{(R_\alpha)c}_{\beta\kappa}\right)\left(ig_s \mathbf{T}^{(r)d}_{a_1 i}\right)\int_0^\infty ds \int_{-\infty}^0 dt \langle 0|\overline{\mathrm{T}}[v\cdot A^c_v(v(z_0+s))]\mathrm{T}[n\cdot A^d_n(tn)]|0\rangle + \ldots$$

Equivalently, in terms of **soft Feynman diagrams:**



One-loop soft function for arbitrary initial and final-state particles:

$$W_i^{R_\alpha,(1)}(L,\mu_f) = (C_r + C_{r'})\left(\frac{2}{\epsilon^2}+\frac{2}{\epsilon}L + L^2 + \frac{\pi^2}{6}\right) + 2C_{R_\alpha}\left(\frac{1}{\epsilon}+L+2\right)$$

$$L = 2\ln\left(\frac{iz_0\mu_f e^{\gamma_E}}{2}\right)$$

$\Rightarrow \gamma_{H,s}^{R_\alpha} = -2C_{R_\alpha}$ with C_{R_α} Casimir invariant for the representation R_α

Agrees with [Kulesza, Motyka '08]

Determination of the two-loop anomalous dimension

Two-loop massive anomalous dimension recently extracted from existing results:
[Beneke, PF, Schwinn '09]

- **IR structure of UV regularised amplitudes in QCD** [Becher, Neubert '09]

$$\Gamma(\{k\},\{m\},\mu_f) = \sum_{(i,j)} \frac{\mathbf{T}_i\cdot\mathbf{T}_j}{2}\,\gamma_{\rm cusp}\,\ln\left(\frac{\mu_f^2}{-s_{ij}}\right) + \sum_i \gamma^{r_i} + \sum_I \gamma_{H,s}^{R_I}$$
$$- \sum_{(I,J)} \frac{\mathbf{T}_I\cdot\mathbf{T}_J}{2}\,\gamma_{\rm cusp}(\beta_{IJ}) + \sum_{I,j} \mathbf{T}_I\cdot\mathbf{T}_j\,\gamma_{\rm cusp}\,\ln\left(\frac{m_J\mu_f}{-s_{Ij}}\right) + \textbf{3-parton corr.}$$

3-parton correlations ($\propto f^{abc}\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_K^c$) vanish for $2 \to 1$ processes
[Mitov, Sterman, Sung '09; Becher, Neubert '09]

- **Two loop results for HQET form factors** [Korchemsky et al. '92; Kidonakis '09]

$$\gamma_{H,s}^{(1),R_\alpha} = C_{R_\alpha}\left[-C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{9}T_f n_f\right]$$

Agrees with direct calculation by [Czakon, Mitov, Sterman '09]

Alternative approaches

- **Pair invariant-mass distribution** $d\sigma(t\bar{t}+X)/dM_{t\bar{t}}$
  [Kidonakis, Sterman '97; Ahrens et al. '10]

$$\left[\frac{\ln^n(1-z)}{(1-z)}\right]_+ \qquad z = \frac{M_{t\bar{t}}^2}{\hat{s}}$$

- **One-particle inclusive cross section** $d\sigma(t+X)/ds_4$:
  [Laenen, Oderda, Sterman '98; Kidonakis '10]

$$\left[\frac{\ln^n(s_4/m_t^2)}{s_4}\right] \qquad s_4 = p_X^2 - m_t^2$$

	$\sigma_{t\bar{t}}$[pb]	Tevatron	LHC@7	LHC@10	LHC@14
BFKS	NLO	$6.50^{+0.32+0.33}_{-0.70-0.24}$	150^{+18+8}_{-19-8}	380^{+44+17}_{-46-17}	842^{+97+30}_{-97-32}
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Kidonakis '10 (1PI)	NNLO(β)	$7.08^{+0.0+0.36}_{-0.24-0.27}$	163^{+7+9}_{-5-9}	415^{+17+18}_{-21-19}	920^{+50+33}_{-39-35}
Ahrens et al. '10	NLO+NNLL	$6.48^{+0.17+0.32}_{-0.21-0.25}$	146^{+7+8}_{-7-8}	368^{+20+19}_{-14-15}	813^{+50+30}_{-36-35}
	NNLO(β)	$6.55^{+0.32+0.33}_{-0.41-0.24}$	149^{+10+8}_{-9-8}	377^{+28+16}_{-23-18}	832^{+65+31}_{-50-29}