# Desperately Seeking SUSY?

by

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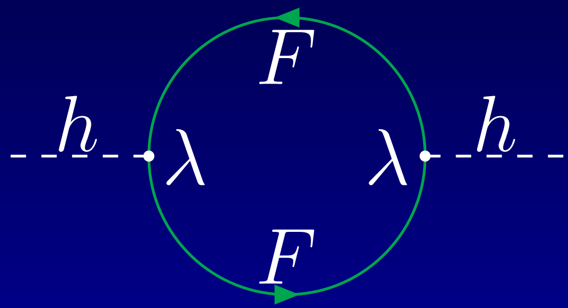
## Talk outline

- Weak scale SUSY
- Fits and 2011 LHC searches
- mAMSB interpretation
- 2012 searches

*Please ask questions while I'm talking*

# Technical Hierarchy Problem

A problem with light, fundamental scalars. Their mass receives quantum corrections from heavy particles in the theory:




$$\sim -\frac{c\lambda^2}{16\pi^2}\int \frac{d^n k}{k^2 - m_F^2} + \ldots$$

Quantum correction to Higgs mass:

$$m_h^{phys} = m_h^{tree} + \mathcal{O}(m_F/100).$$

$m_F \sim 10^{19}$ GeV$/c^2$ is *heaviest mass scale* present. Higgs is eaten by $W, Z$ to give $O(M_{W,Z}) \sim 90$ GeV$/c^2 \Rightarrow m_h^{tot} \lesssim 1$ TeV$/c^2$.

# Symmetry

Standard model gauge symmetry is *internal*, but supersymmetry (SUSY) is a space-time symmetry. We call extra SUSY generators $Q, \bar{Q}$.

$$
\begin{aligned}
Q|\text{fermion}\rangle &\rightarrow |\text{boson}\rangle \\
Q|\text{boson}\rangle &\rightarrow |\text{fermion}\rangle
\end{aligned}
$$

In the simplest form of SUSY, we have multiplets

$$
\left( \begin{array}{c} \text{spin } 0 \\ \text{spin } 1/2 \end{array} \right), \qquad \left( \begin{array}{c} \text{spin } 1/2 \\ \text{spin } 1 \end{array} \right),
$$

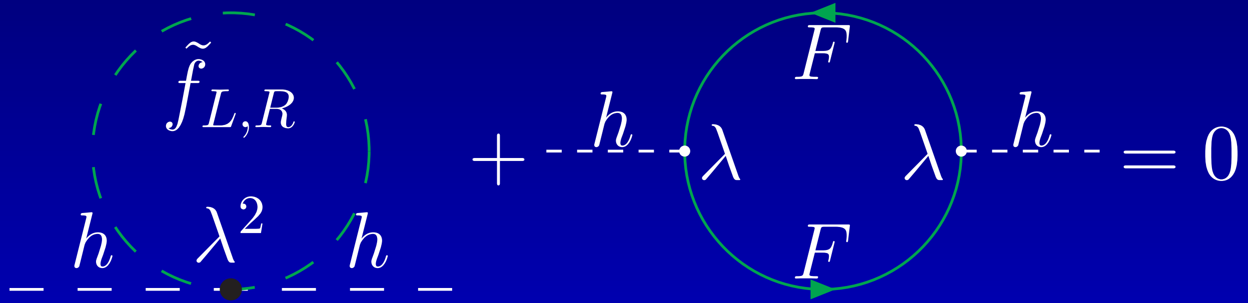
where each spin component in the multiplet should have identical quantum numbers (except spin).

# Supersymmetric Solution

Exact supersymmetry adds 2 scalars $\tilde{f}_{L,R}$ for every massive fermion with

$$m_{\tilde{f}_{L,R}} = m_F$$

and they couple to $h$ with the same strength:




$Q$: Where are the selectrons?

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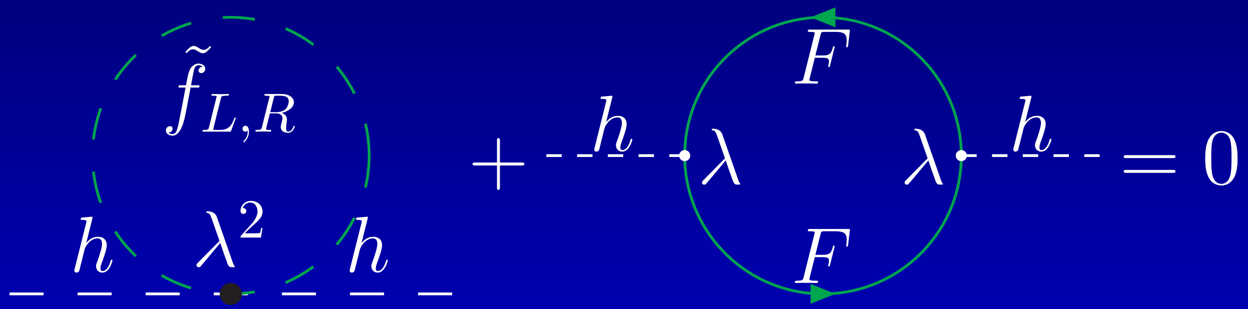


$\mathcal{Q}$: Where are the selectrons?
$\mathcal{A}$: SUSY must be softly broken.

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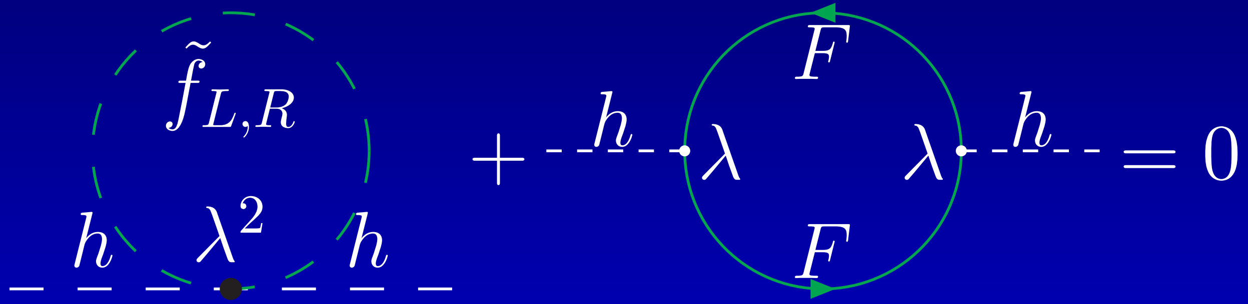


When we break SUSY, we must make sure that we don't reintroduce the naturalness problem: "soft breaking".

# Soft breaking

Make scalar partners heavier than fermions:

$$m^2_{\tilde{f}_{L,R}} = m^2_F + \delta^2$$

Then we find a quantum correction to $m_h$ of (Drees)

$$\Delta m_h^2 \sim \frac{\lambda^2}{16\pi^2}\left(4\delta^2 + 2\delta^2 \ln \frac{m_F^2}{\mu^2}\right) + O(\delta^4).$$

So, if $\delta \stackrel{<}{\sim} O(1)$ TeV/$c^2$, there's no fine tuning in $m_h$.

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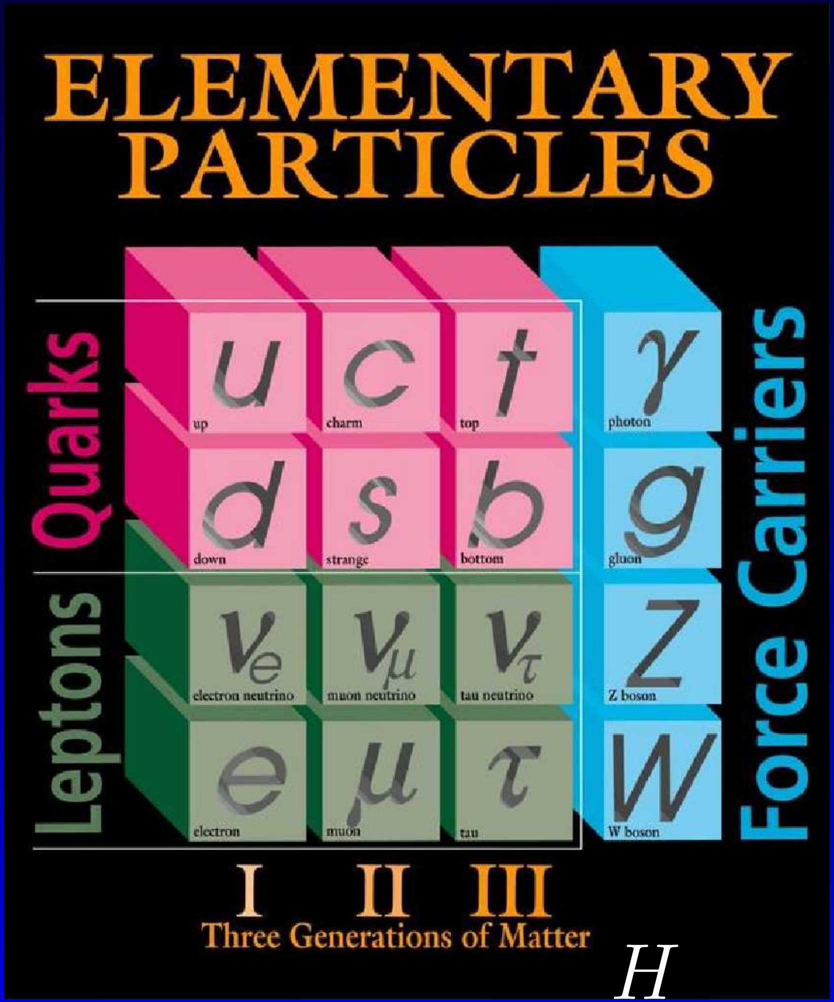
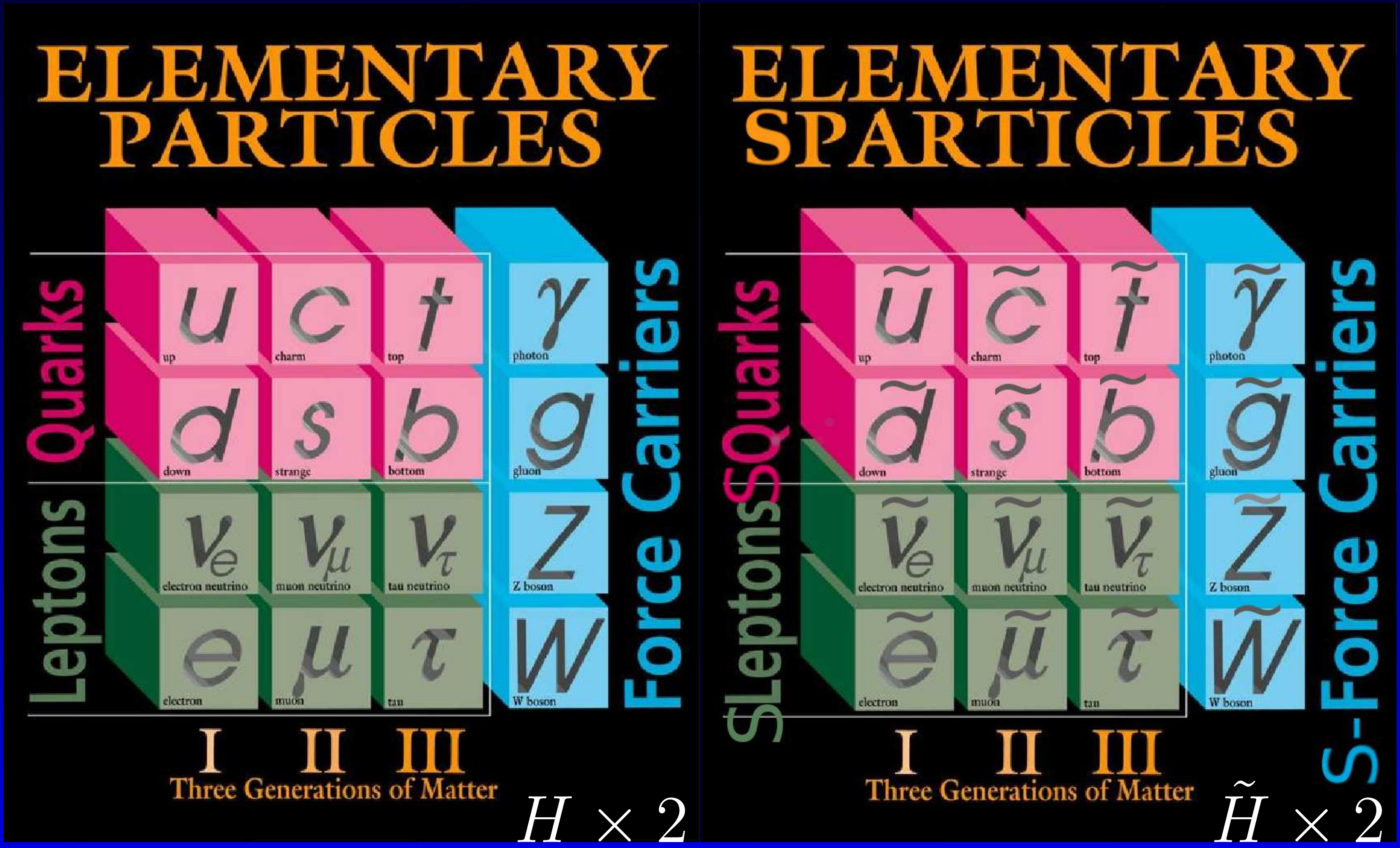
So, if $\delta \stackrel{<}{\sim} O(1)$ TeV/$c^2$, there's no fine tuning in $m_h$. *We should see supersymmetric particles in the Large Hadron Collider.*

# Supersymmetric Copies




$H$

# Supersymmetric Copies



# Broken Symmetry

3 components of the Higgs particles are eaten by $W^{\pm}, Z^0$, leaving us with 5 physical states:

$$h^0, H^0(\text{CP+}), \qquad A^0(\text{CP-}), \qquad H^{\pm}$$

SUSY breaking and electroweak breaking imply particles with identical quantum numbers mix:

$$\begin{aligned}
(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0) &\rightarrow \chi^0_{1,2,3,4} \\
(\tilde{t}_L, \tilde{t}_R) &\rightarrow \tilde{t}_{1,2} \\
(\tilde{b}_L, \tilde{b}_R) &\rightarrow \tilde{b}_{1,2} \\
(\tilde{\tau}_L, \tilde{\tau}_R) &\rightarrow \tilde{\tau}_{1,2} \\
(\tilde{W}^{\pm}, \tilde{H}^{\pm}) &\rightarrow \chi^{\pm}_{1,2}
\end{aligned}$$

# Universality

Reduces number of SUSY breaking parameters from 100 to 3:

- $\tan\beta \equiv v_2/v_1$
- $m_0$, the common scalar mass (flavour).
- $M_{1/2}$, the common gaugino mass (GUT/string).
- $A_0$, the common trilinear coupling (flavour).

These conditions should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

$$\propto \ 1/(16\pi^2)\ln(M_X/M_Z).$$

Also, Higgs potential parameter sgn($\mu$)=$\pm 1$.

# Implementation

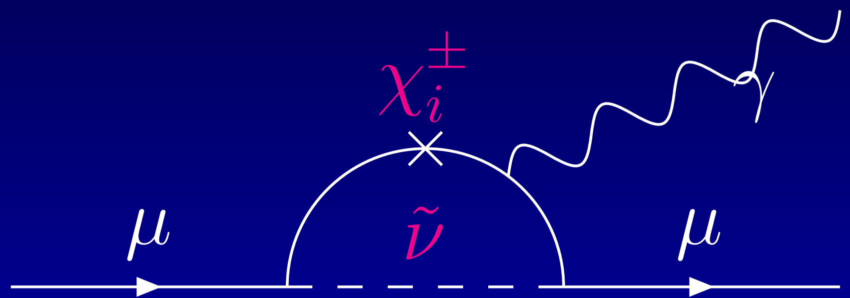
We use

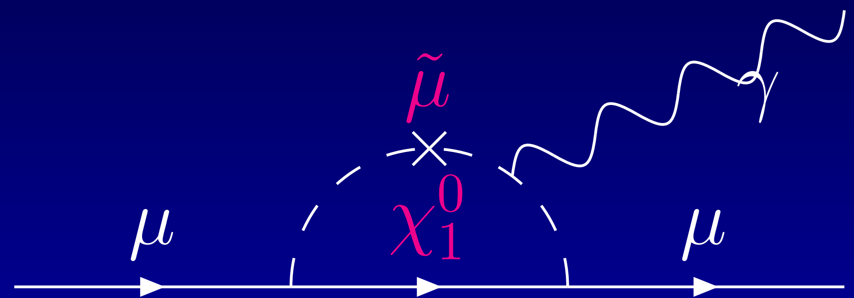
- *95% C.L. direct search constraints*
- $\Omega_{DM}h^2 = 0.1143 \pm 0.02$ `micrOMEGAs`
- $\delta(g-2)_\mu/2 = (29.5 \pm 8.8) \times 10^{-10}$ Stöckinger *et al*
- $B-$physics observables including `SusyBSG` $BR[b \to s\gamma]_{E_\gamma > 1.6 \text{ GeV}} = (3.52 \pm 0.38) \times 10^{-4}$, $\text{BR}(B_s \to \mu\mu) < 1.1 \times 10^{-8}$ `micrOMEGAs`
- Electroweak data W Hollik, A Weber *et al*

$$2 \ln \mathcal{L} = -\sum_i \chi_i^2 + c = \sum_i \frac{(p_i - e_i)^2}{\sigma_i^2} + c$$

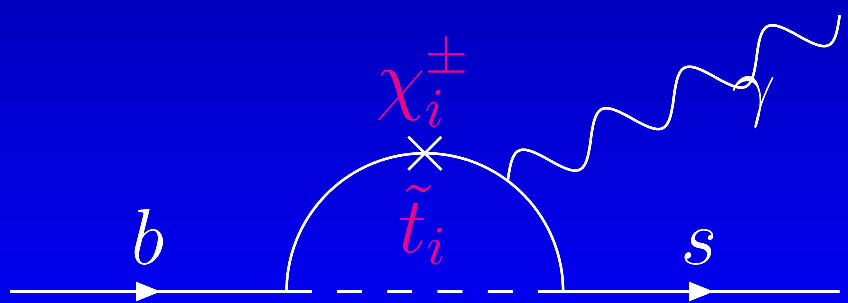
# Additional observables

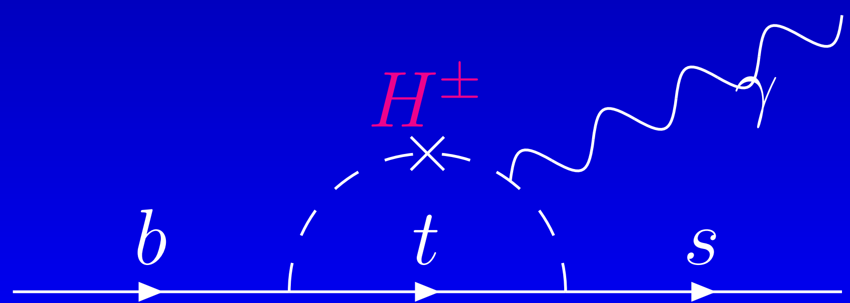
$$\delta \frac{(g-2)_\mu}{2} \sim 13 \times 10^{-10} \left( \frac{100\ \mathrm{GeV}}{M_{SUSY}} \right)^2 \tan\beta$$



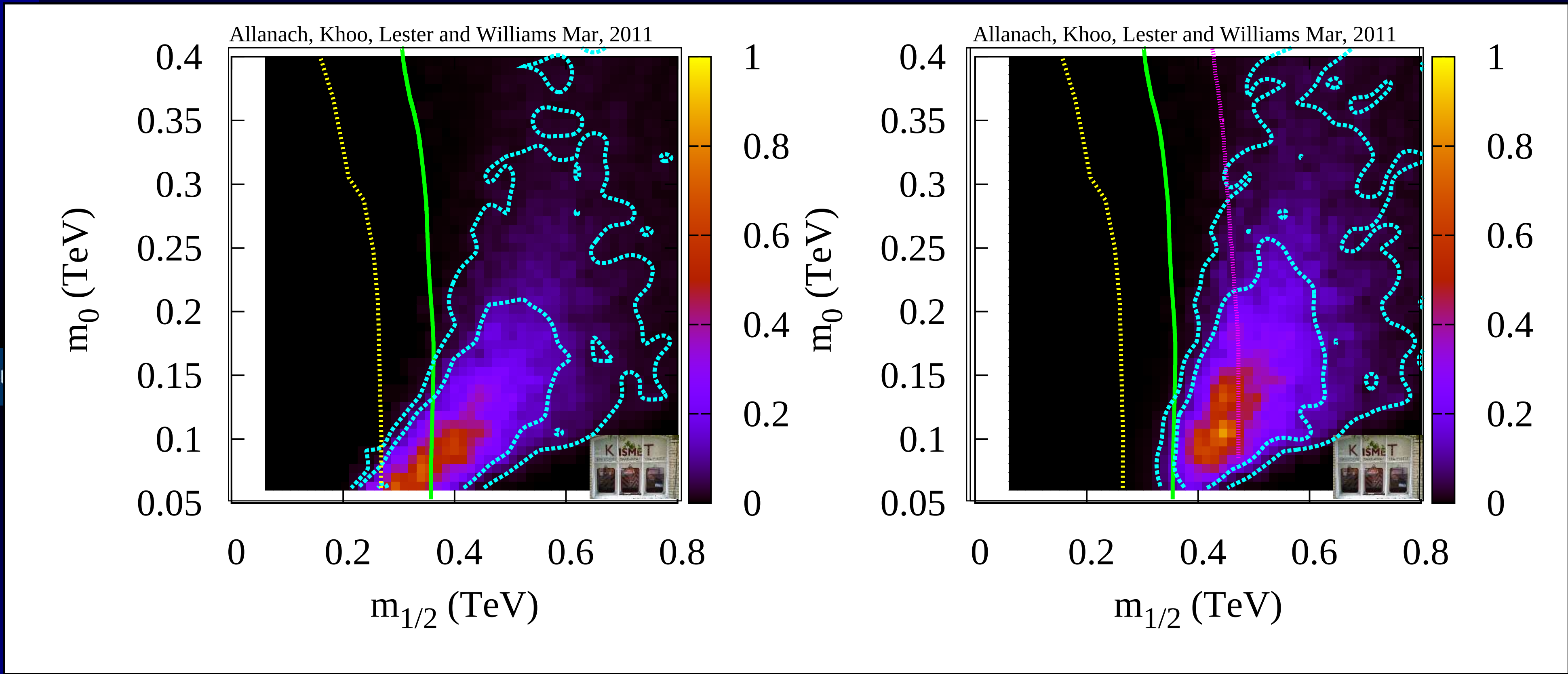




$$BR[b \to s\gamma] \propto \tan\beta (M_W/M_{SUSY})^2$$



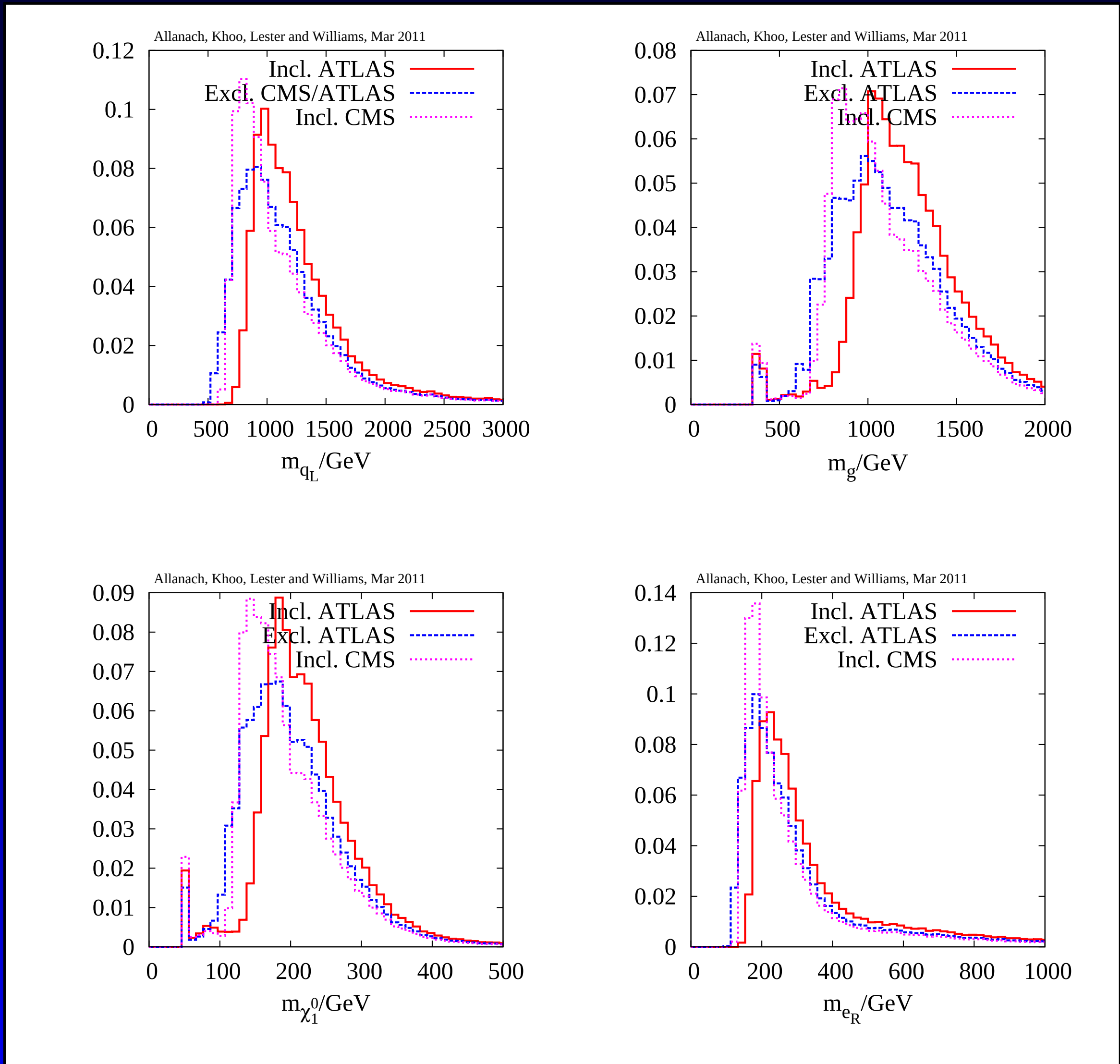



# ATLAS Weighted Fits



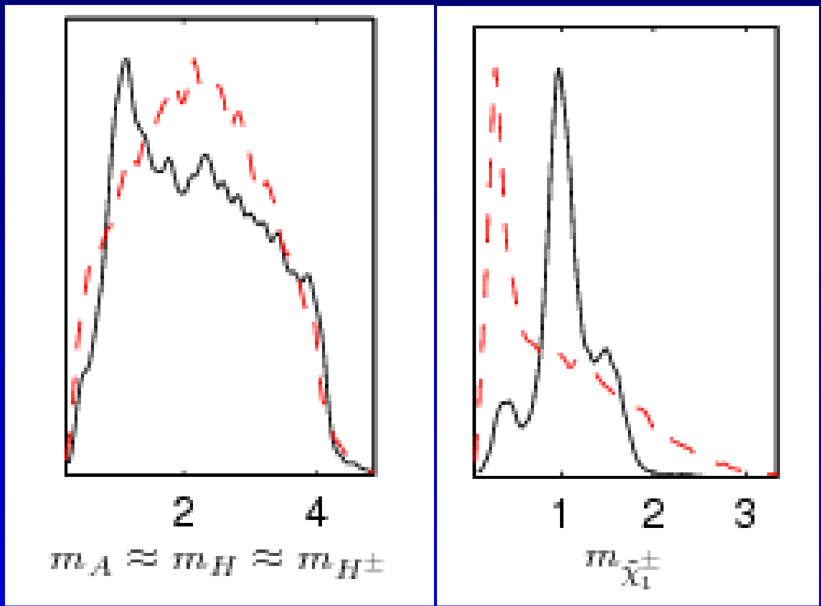

Again, we assume $A_0$-$\tan\beta$ independence and interpolate across $m_0$ and $m_{1/2}$. CMS 35 pb$^{-1}$, ATLAS 35 pb$^{-1}$, CMS 1 fb$^{-1}$

# CMS/ATLAS Weighted Fits

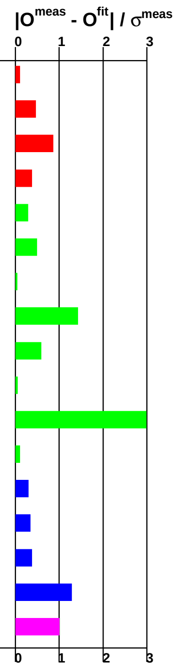


# pMSSM Fits

25 pMSSM input parameters are: $M_{1,2,3}$, $A_{t,b,\tau,\mu}$, $m_{H_{1,2}}$, $\tan\beta$, $m_{\tilde d_{R,L}} = m_{\tilde s_{R,L}}$, $m_{\tilde u_{R,L}} = m_{\tilde c_{R,L}}$, $m_{\tilde e_{R,L}} = m_{\tilde \mu_{R,L}}$, $m_{\tilde t, \tilde b, \tilde \tau_{R,L}}$ $m_t$, $m_b(m_b)$ $\alpha_s(M_Z)^{\overline{MS}}$, $\alpha^{-1}(M_Z)^{\overline{MS}}$, $M_Z$. Combined Bayesian fit[a]:

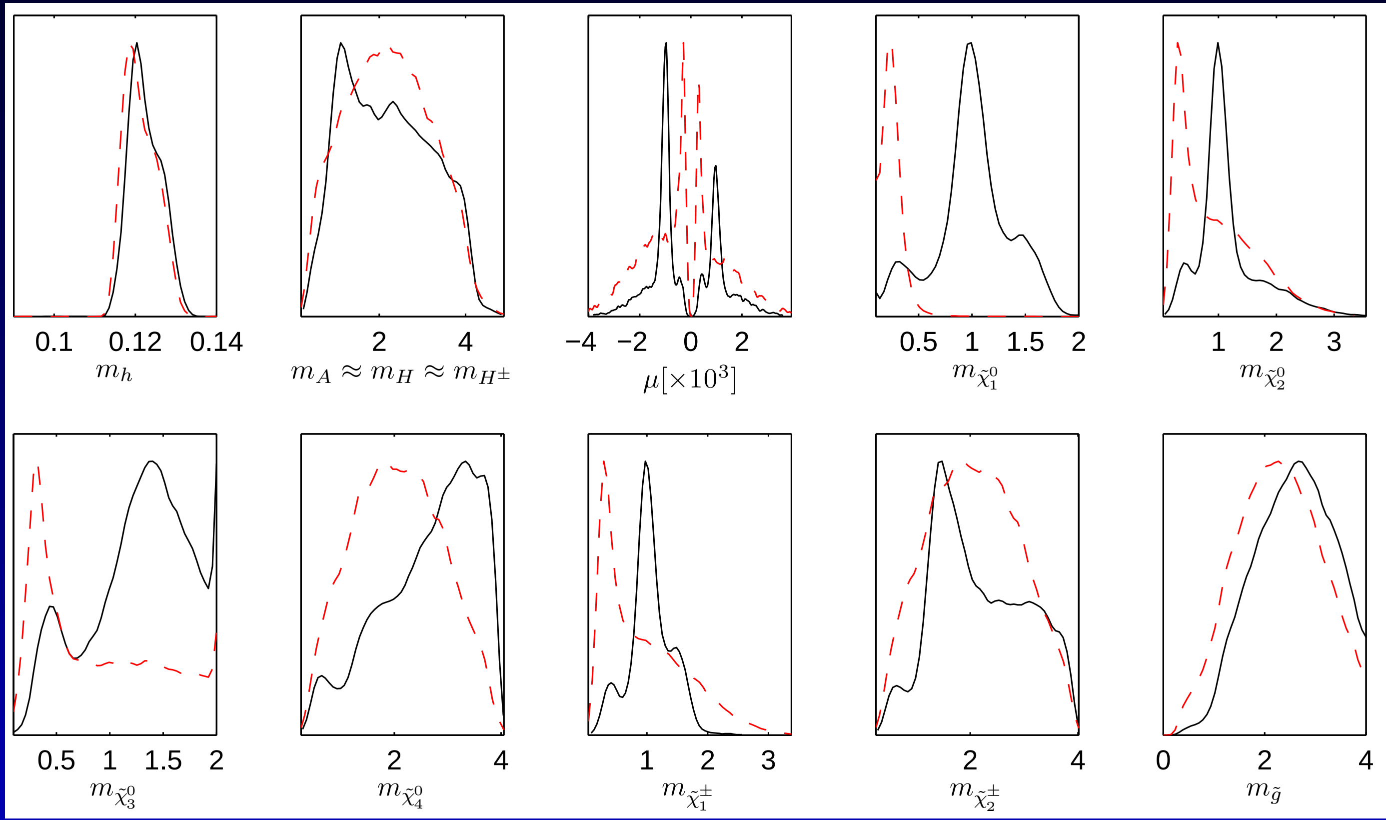



| Observable | Measurement | Fit(Log) |
|---|---|---|
| $m_W$ [GeV] | $80.399 \pm 0.025$ | 80.402 |
| $\Gamma_Z$ [GeV] | $2.4952 \pm 0.0025$ | 2.4964 |
| $\sin^2\theta^{eff}_{lep}$ | $0.2324 \pm 0.0012$ | 0.2314 |
| $\delta(g-2)_\mu \times 10^{10}$ | $30.20 \pm 9.02$ | 26.74 |
| $R_l^0$ | $20.767 \pm 0.025$ | 20.760 |
| $R_b$ | $0.21629 \pm 0.00066$ | 0.21962 |
| $R_c$ | $0.1721 \pm 0.0030$ | 0.1723 |
| $A_e$ | $0.1513 \pm 0.0021$ | 0.1483 |
| $A_b$ | $0.923 \pm 0.020$ | 0.935 |
| $A_c$ | $0.670 \pm 0.027$ | 0.685 |
| $A^b_{FB}$ | $0.0992 \pm 0.0016$ | 0.1040 |
| $A^c_{FB}$ | $0.071 \pm 0.035$ | 0.074 |
| $BR(B \to X_s\gamma) \times 10^4$ | $3.55 \pm 0.42$ | 3.42 |
| $R_{BR(B_u \to \tau\nu)}$ | $1.11 \pm 0.32$ | 1.00 |
| $R_{\Delta M_{B_s}}$ | $1.15 \pm 0.40$ | 1.00 |
| $\Delta_{0-}$ | $0.0375 \pm 0.0289$ | 0.0748 |
| $\Omega_{CDM}h^2$ | $0.11 \pm 0.02$ | 0.13 |




[a] S.S. AbdusSalam, BCA, F. Quevedo, F. Feroz, M. Hobson, PRD81 (2010) 985012, `arXiv:0904.2548`

# Spectrum




Obtained with `MultiNest`[a] algorithm in 16 CPU years. Prior dependence is *useful*: which predictions are robust?

[a] Feroz, Hobson `arxiv:0704.3704`

# CMSSM at $1fb^{-1}$ is Getting Heavier

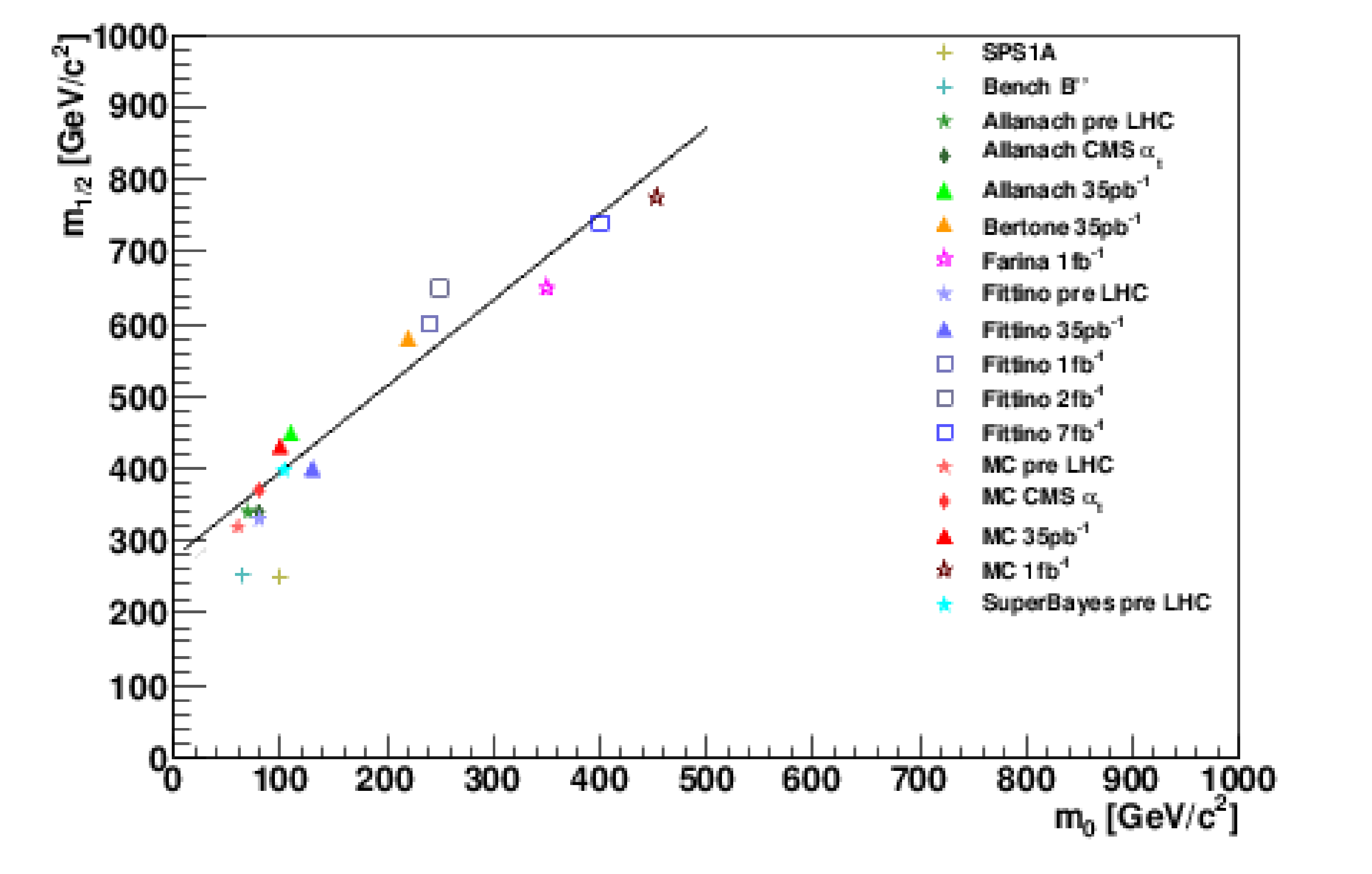
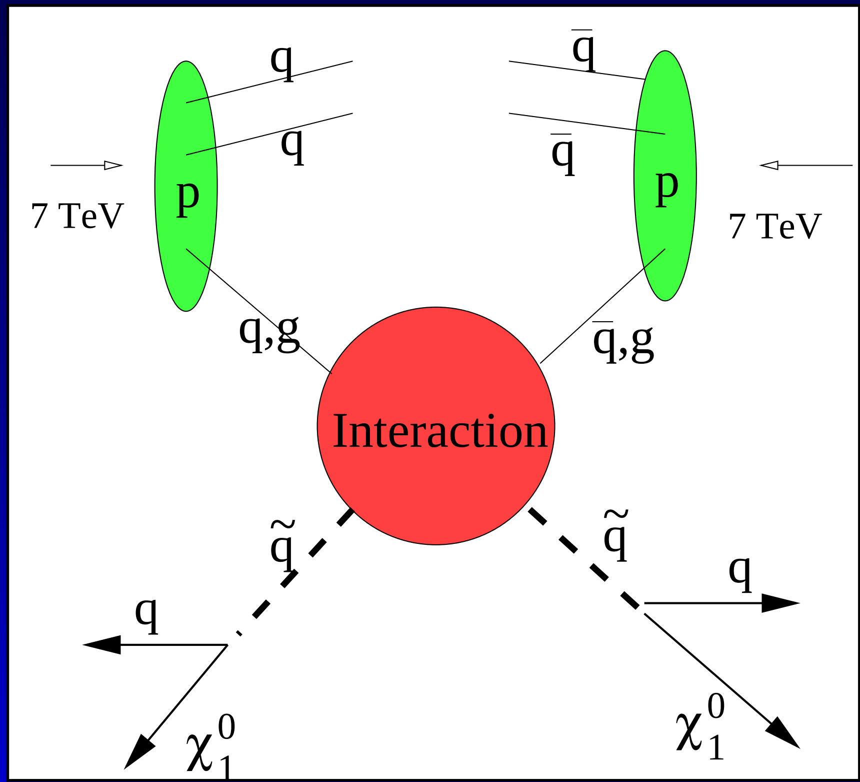



Figure 1: CMSSM good/best fit-points arXiv:1109.3859

# Collider SUSY Dark Matter Production

Strong sparticle production and decay to dark matter particles.




*Any (light enough) dark matter candidate that couples to hadrons can be produced at the LHC*

# Searches and the CMSSM

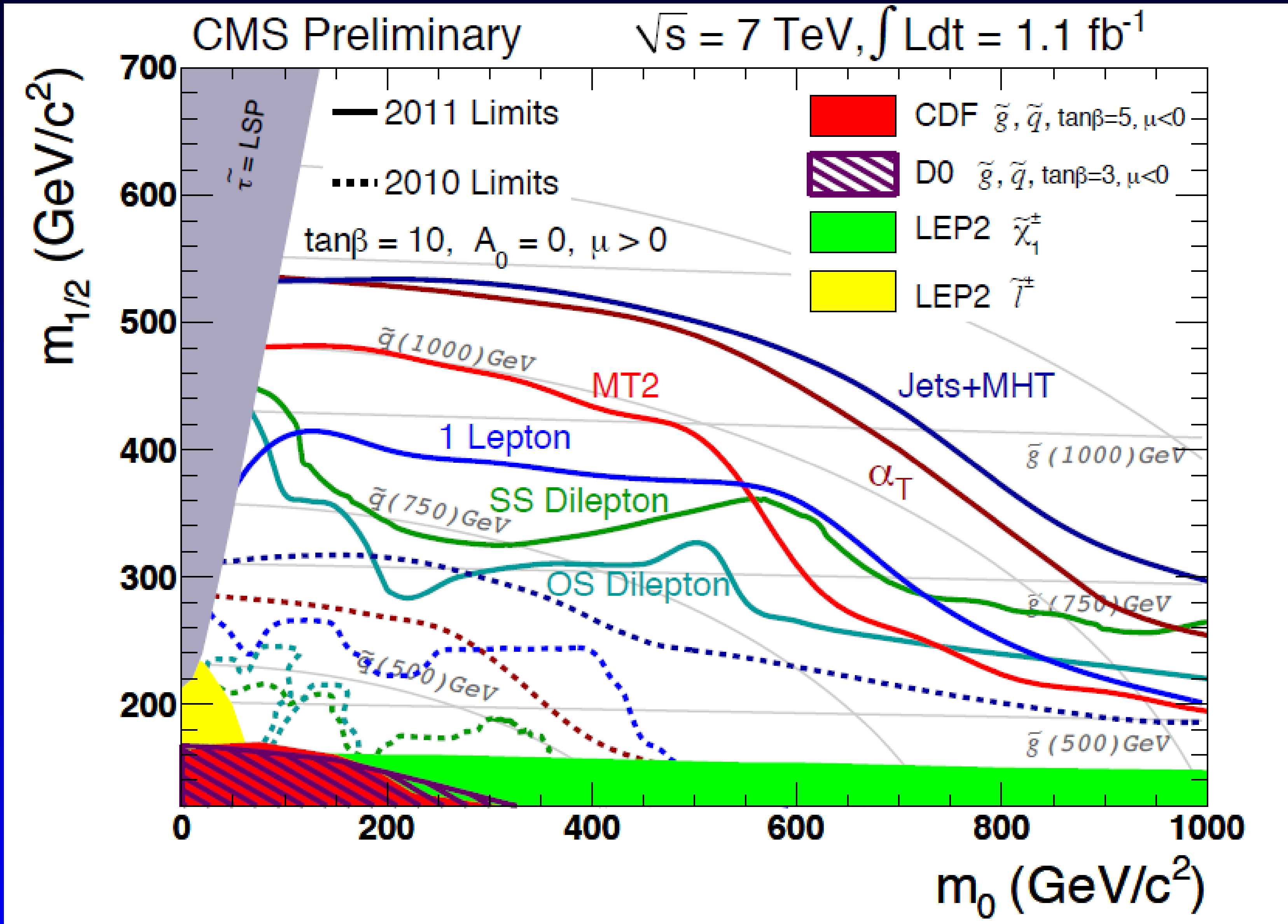





Figure 2: CMSSM CMS exclusions

# SPS Points Savaged at 165 $pb^{-1}$

| Benchmark point | Model scenario | $\sigma/$pb | | | | status |
|---|---|---|---|---|---|---|
| | | A | B | C | D | ATLAS 35, 165/pb |
| ATLAS Limits | | 1.3 | 0.35 | 1.1 | 0.11 | |
| SPS 1a [52] | CMSSM | 2.031 | 0.933 | 1.731 | 0.418 | A,B,C,D |
| SPS 1b [52] | CMSSM | 0.120 | 0.089 | 0.098 | 0.067 | 165/pb |
| SPS 2 [52] | CMSSM | 0.674 | 0.388 | 0.584 | 0.243 | B,D |
| SPS 3 [52] | CMSSM | 0.123 | 0.093 | 0.097 | 0.067 | 165/pb |
| SPS 4 [52] | CMSSM | 0.334 | 0.199 | 0.309 | 0.144 | D |
| SPS 5 [52] | CMSSM | 0.606 | 0.328 | 0.541 | 0.190 | D |
| SPS 6 [52] | CMSSM (non-universal $m_{1/2}$) | 0.721 | 0.416 | 0.584 | 0.226 | B,D |
| SPS 7 [52] | mGMSB ($\tilde{\tau}_1$ NLSP) | 0.022 | 0.016 | 0.023 | 0.015 | allowed |
| SPS 8 [52] | mGMSB ($\tilde{\chi}_1^0$ NLSP) | 0.021 | 0.011 | 0.022 | 0.009 | allowed |
| SPS 9 [52] | mAMSB | $0.019^*$ | $0.004^*$ | $0.006^*$ | $0.002^*$ | allowed |

Figure 3: Dolan, Grellscheid, Jaeckel, Khoze, Richardson, arXiv:1104.0585

# Benchmarks

Currently we[a] have devised SUSY benchmark models. 1109.3859

- CMSSM, NUHM, mAMSB, mGMSB, RPV and some simplified models (via pMSSM) are defined.
- Defining interesting *parameter planes*: identifying important parameters which control the masses of sparticles in each case.
- Discrete set of points along monotonic lines: next point for the experiments to study is defined as the lightest one that is not ruled out to 95% CL.

[a] S.S. AbdusSalam, BCA H. Dreiner, J. Ellis, S. Heinemeyer, M. Krämer, M. Mangano, K.A. Olive, S. Rogerson, L. Roszkowski, M. Schlaffer, G. Weiglein

# Benchmarks Example: mAMSB

Plane: $(m_{\text{aux}}, m_0)$ with $\tan\beta = 10, \mu > 0$.
Line: $m_0 = 0.0075 m_{\text{aux}}$, $\tan\beta = 10, \mu > 0$.
Points:

| Point | $m_{\text{aux}}$ | $m_0$ | $m_{\tilde{g}}$ | $\langle m_{\tilde{q}} \rangle$ | $m_{\tilde{t}_1}$ | $m_{\tilde{b}_1}$ | BR($\tilde{g} \to \tilde{t}t$) | BR($\tilde{g} \to \tilde{b}b$) |
|---|---|---|---|---|---|---|---|---|
| mAMSB1.1 | $4 \times 10^4$ | 300 | 890 | 880 | 630 | 765 | 69 | 29 |
| mAMSB1.2 | $5 \times 10^4$ | 375 | 1085 | 1080 | 780 | 940 | 74 | 25 |
| mAMSB1.3 | $6 \times 10^4$ | 450 | 1280 | 1280 | 925 | 1110 | 76 | 24 |
| mAMSB1.N | ... | ... | ... | ... | ... | ... | ... | ... |

Figure 4: mAMSB benchmark points arXiv:1109.3859.

Next point for consideration is *the next one that hasn't been yet ruled out to 95%*

# ATLAS 0-lepton, jets and $\not{p}_T$ search

ATLAS use cuts on different variables to search for SUSY:

- jet $p_T$s
- $m_{eff} = \sum p_T^{(j)} + |\not{p}_T|$
- ${m_T^{(i)}}^2({\mathbf{p}_T}^{(i)}, {\not{\mathbf{q}}_T}^{(i)}) \equiv 2\left|{\mathbf{p}_T}^{(i)}\right|\left|{\not{\mathbf{q}}_T}^{(i)}\right| - 2{\mathbf{p}_T}^{(i)} \cdot {\not{\mathbf{q}}_T}^{(i)}$ where ${\not{\mathbf{q}}_T}^{(i)}$ is the transverse momentum of particle $(i)$. For each event, it is a lower bound on $m(NLSP)$.

$$M_{T2}({\mathbf{p}_T}^{(1)}, {\mathbf{p}_T}^{(2)}, \not{\mathbf{p}}_T) \equiv \min_{\sum \not{\mathbf{q}}_T = \not{\mathbf{p}}_T} \left\{ \max\left( m_T^{(1)}, m_T^{(2)} \right) \right\}$$

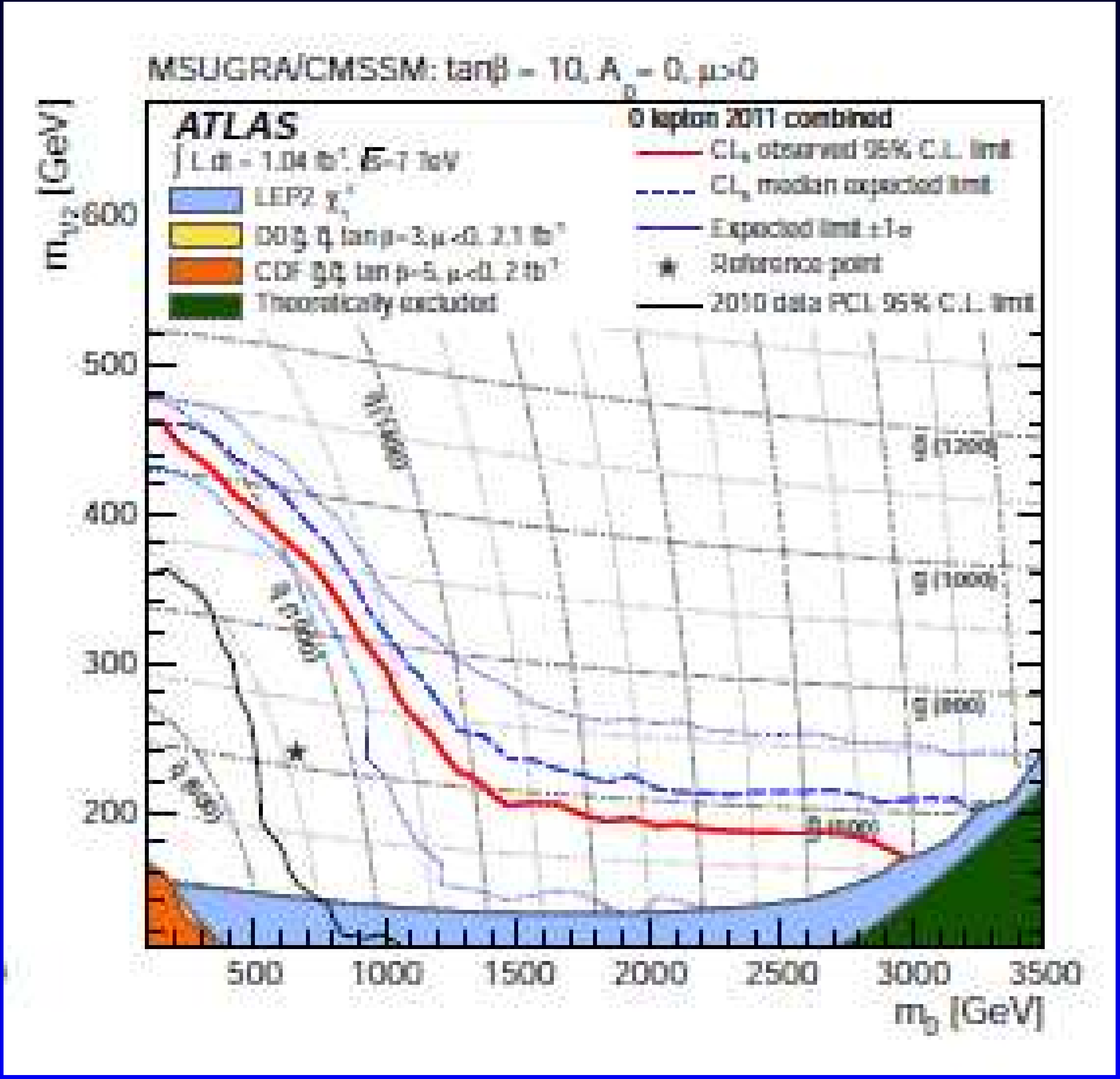
# ATLAS 1 fb$^{-1}$ 0-lepton Search Results

| | $\geq 2$ jets | $\geq 3$ jets | $\geq 4$ jets | $\geq 4$ jets' | High mass |
|---|---|---|---|---|---|
| $p_T(j_1)$ | $> 130$ GeV | $> 130$ GeV | $> 130$ GeV | $> 130$ GeV | $> 130$ GeV |
| $p_T(j_2)$ | $> 40$ GeV | $> 40$ GeV | $> 40$ GeV | $> 40$ GeV | $> 80$ GeV |
| $p_T(j_3)$ | – | $> 40$ GeV | $> 40$ GeV | $> 40$ GeV | $> 80$ GeV |
| $p_T(j_4)$ | – | – | $> 40$ GeV | $> 40$ GeV | $> 80$ GeV |
| $\lvert p_T^{miss}\rvert$ | $> 130$ GeV | $> 130$ GeV | $> 130$ GeV | $> 130$ GeV | $> 130$ GeV |
| $\Delta\phi$ | $> 0.4$ | $> 0.4$ | $> 0.4$ | $> 0.4$ | $> 0.4$ |
| $p_T^{miss}/m_{eff}$ | $> 0.3$ | $> 0.25$ | $> 0.25$ | $> 0.25$ | $> 0.2$ |
| $m_{eff}$ | $> 1000$ GeV | $> 1000$ GeV | $> 500$ GeV | $> 1000$ GeV | $> 1100$ GeV |
| Observed | 58 | 59 | 1118 | 40 | 18 |
| Background | $62.4\pm4.4\pm9.3$ | $54.9\pm3.9\pm7.1$ | $1015\pm41\pm144$ | $33.9\pm2.9\pm6.2$ | $13.1\pm1.9\pm2.5$ |
| $\sigma \times A \times \epsilon$/fb | 22 | 25 | 429 | 27 | 17 |

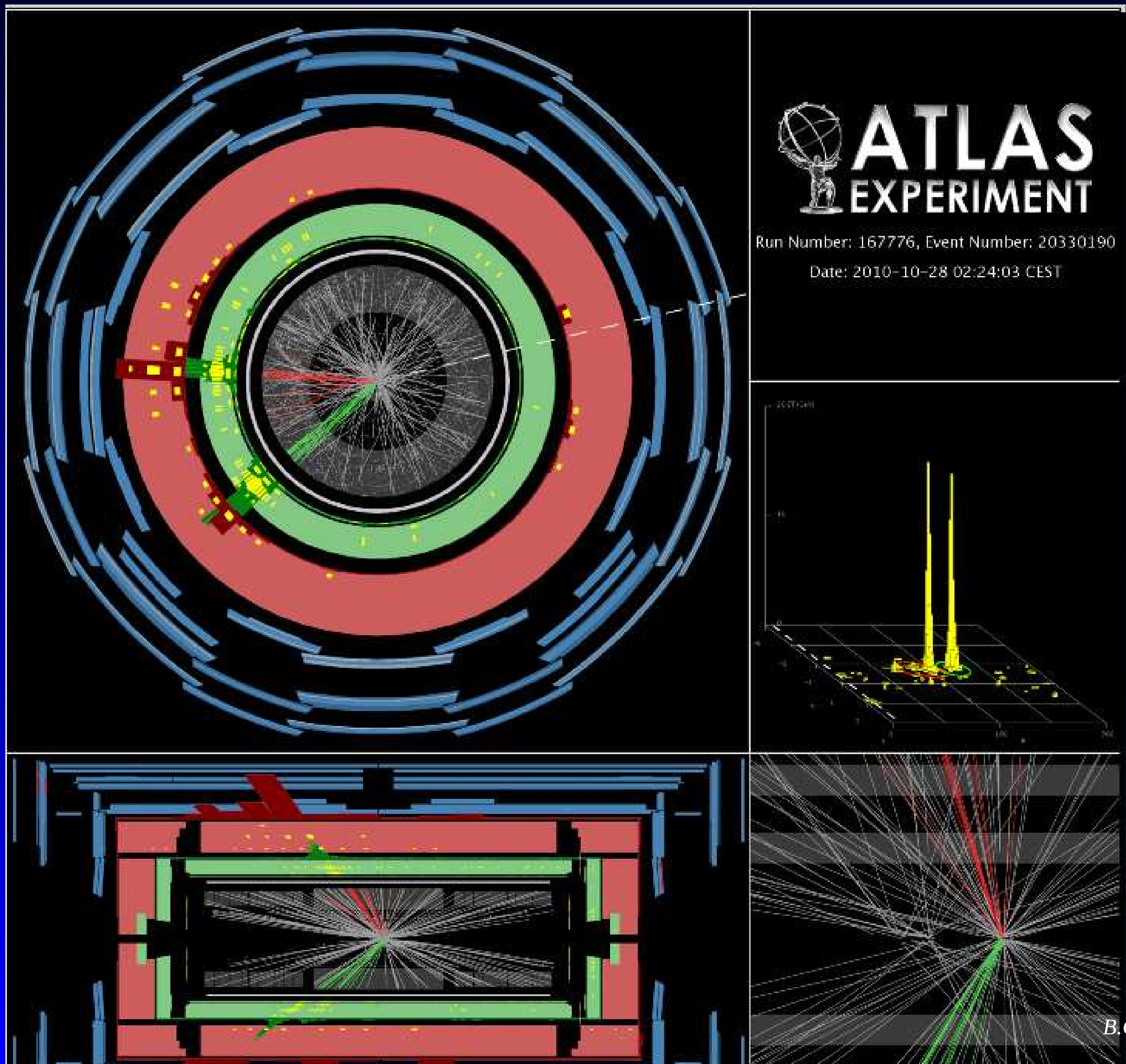
At any point in parameter space, one chooses the set of cuts with the greatest expected sensitivity[a].

[a] ATLAS, arxiv:1109.6572

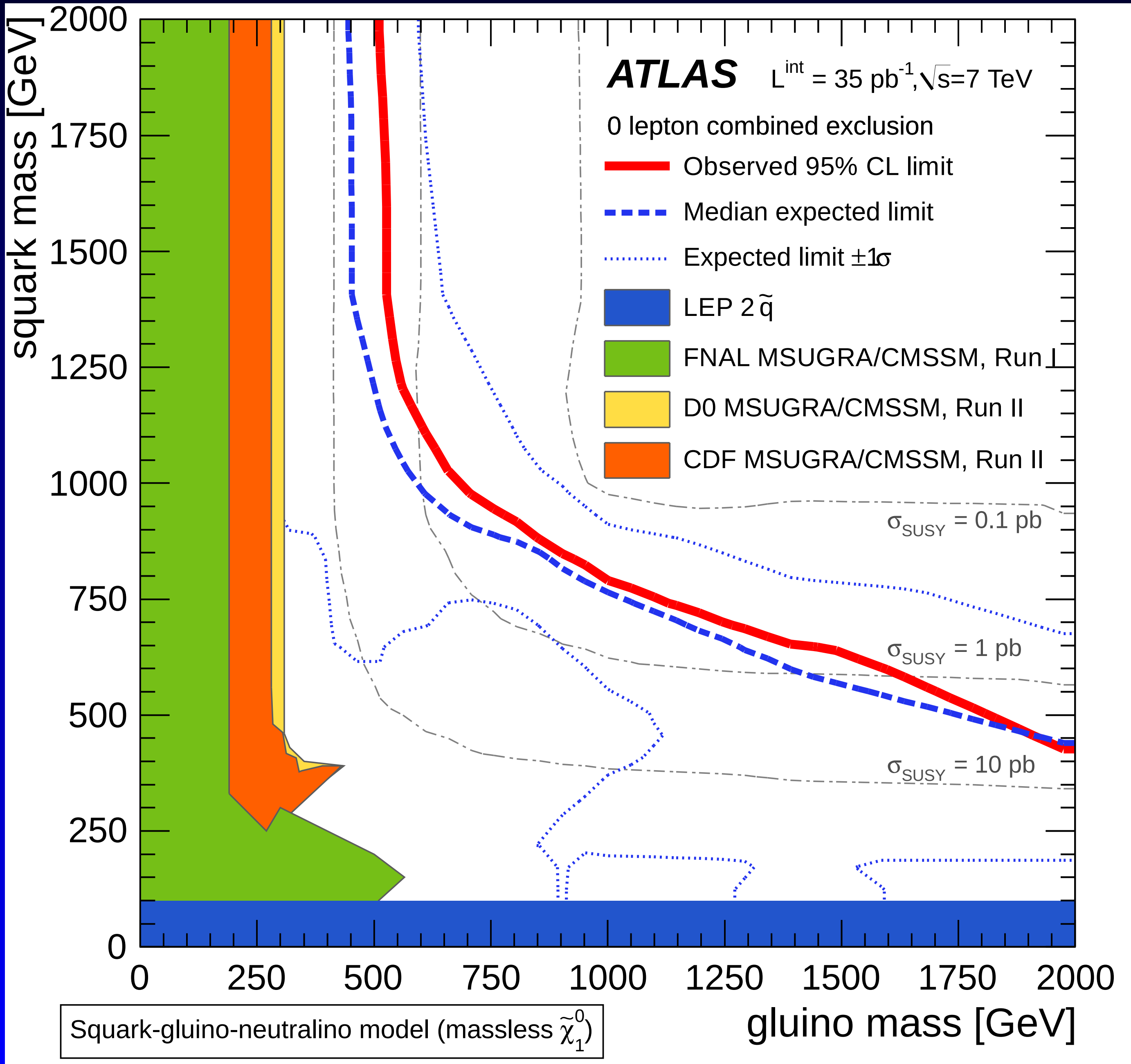
# 0-lepton CMSSM Exclusion



# Candidate Event: High $E_T(j)$



# MSSM Exclusion: Simplified Model



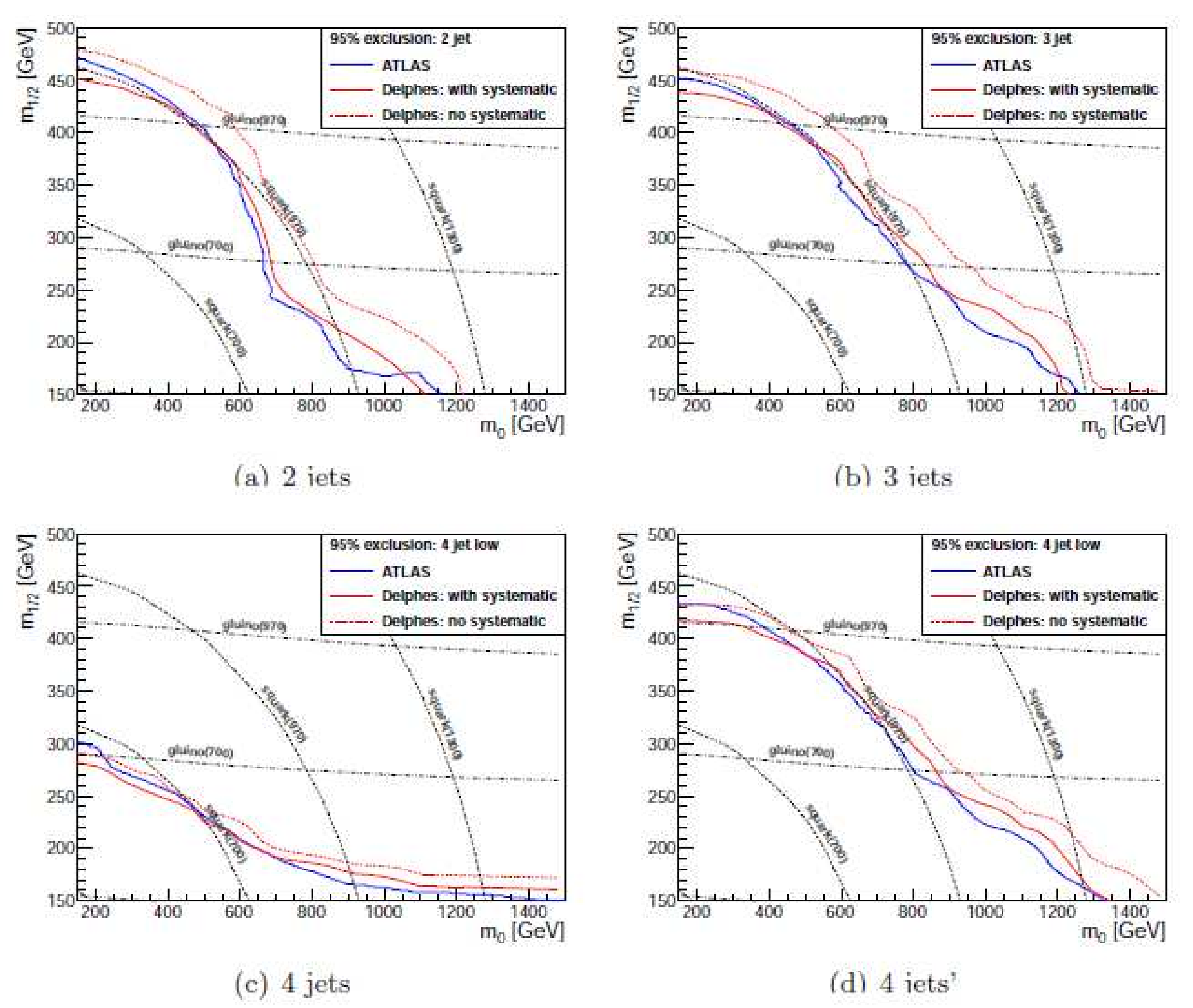
# Intepretation

The results give a lower limit of 1020 GeV for $m_{\tilde{g}} = m_{\tilde{q}}$ in the CMSSM. We wish to *reinterpret* the search in mAMSB, to find the exclusion there (and study if mAMSB evades the search).

We simulate *signal* only, with `HERWIG++-2.5.1`, and use ATLAS' upper limits on $\sigma \times A \times \epsilon$. However *we have to fit the signal systematics*.

This becomes more involved when you want to do a fit and reconstruct the likelihood. To validate then, you need also details on the statistics.

# ATLAS Validation

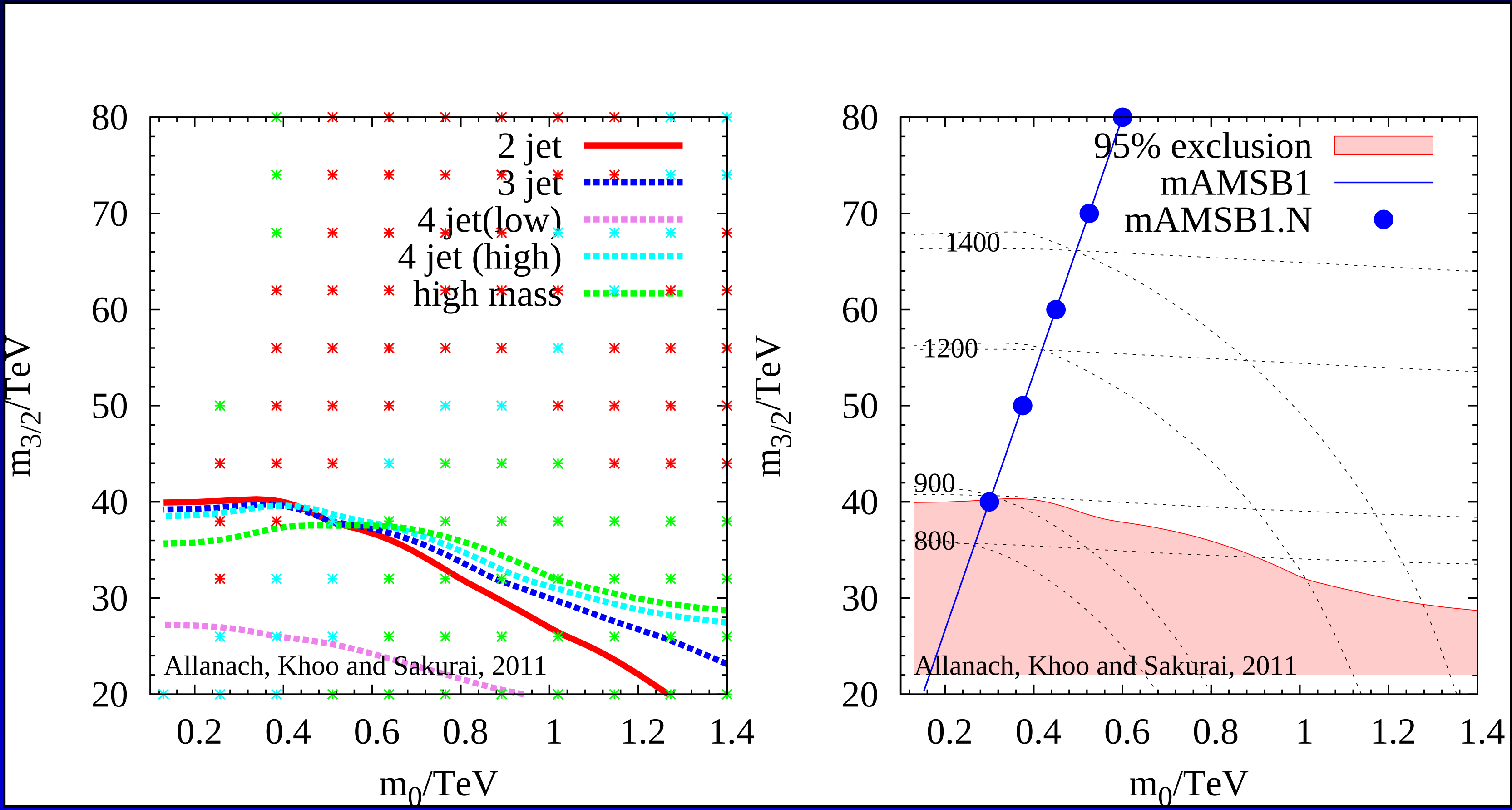



(a) 2 jets (b) 3 jets

(c) 4 jets (d) 4 jets'

# mAMSB Exclusion

Interpret ATLAS exclusion in a different model: mAMSB.



# Natural SUSY

The particles coupling the most strongly to the higgs are the *stops*[a]. Minimising the MSSM Higgs potential,

$$-\frac{M_Z^2}{2} = |\mu|^2 + m_{H_2}^2,$$
$$\delta m_{H_2}^2 = \frac{-3h_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln \left( \frac{\Lambda_{UV}}{m_{\tilde{t}}} \right)$$

Applying that there should be no cancellation implies that

$$m_{\tilde{t}} \stackrel{<}{\sim} 700 \text{ GeV}, \qquad m_{\tilde{g}} \stackrel{<}{\sim} 1000 \text{ GeV}.$$

[a] M. Papucci, J. T. Ruderman and A. Weiler, arXiv:1110.6926;

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\end{aligned}
$$

Experimental $\not{E}_T$ searches currently rule out

$$m_{\tilde{t}} \lesssim 500 \text{ GeV}, \qquad m_{\tilde{g}} \lesssim 800 \text{ GeV}.$$

[a] M. Papucci, J. T. Ruderman and A. Weiler, `arXiv:1110.6926`; C. Brust, A. Katz, S. Lawrence and R. Sundrum, `arXiv:1110.6670`

# Definition of R-Parity

$\mathcal{Q}$: How is $W_{\not R_P}$ normally banned?
$\mathcal{A}$: By defining discrete symmetry $R_p$

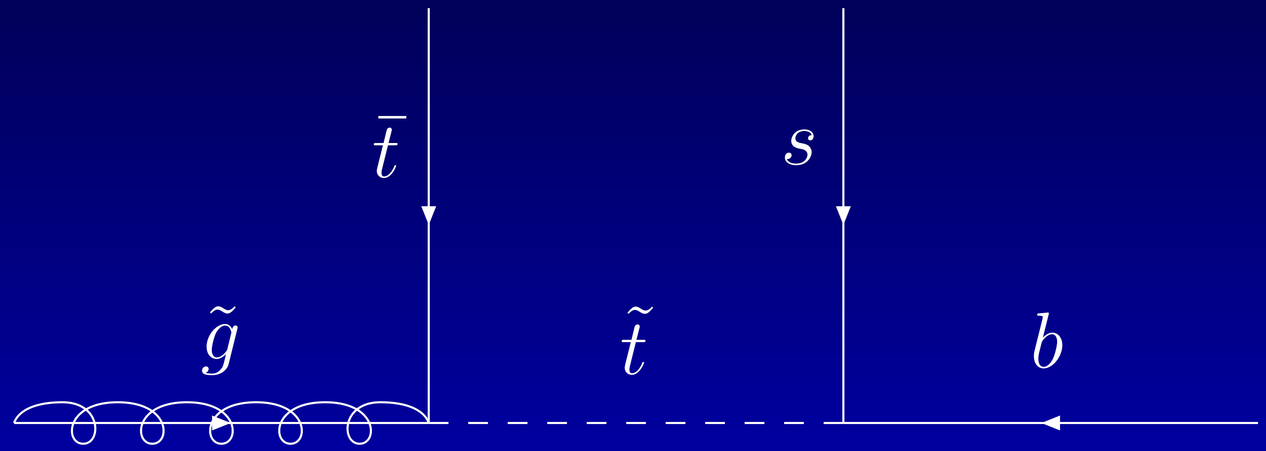
$$R_p = (-\mathbf{1})^{3B+L+2S}.$$

$\rightarrow$ SM fields have $R_p = +1$ and superpartners have $R_p = -1$. There are two important consequences:

- Because initial states in colliders are $R_p$ EVEN, we can only pair produce SUSY particles
- The *lightest superpartner is stable*

# Baryon Number Violation

Can lead to natural SUSY with light stops and gluinos that hasn't been excluded yet *because there is no large $\not{E}_T$ signature*[a].




$\tilde{g}\tilde{g}$ production dominates: we have some ideas on how to reinterpret current *non-SUSY* searches in terms of this model.

[a] BCA and Ben Gripaios, coming soon

# Shopping List

Things that the CMS/ATLAS always provide that we need:

- Cuts and numbers of events observed past them
- Expected background numbers with systematic errors

We could really do with:

- Keeping in mind: we can't combine analyses that use the same events: much better to keep the events disjoint. Doesn't preclude fully inclusive analysis, but make the others as disjoint as possible.
- Likelihood versus predicted number of events past cuts (before efficiency correction). Ideally, sanitized `RooStats`

# Shopping List II

Failing that, then we must calculate the likelihood:

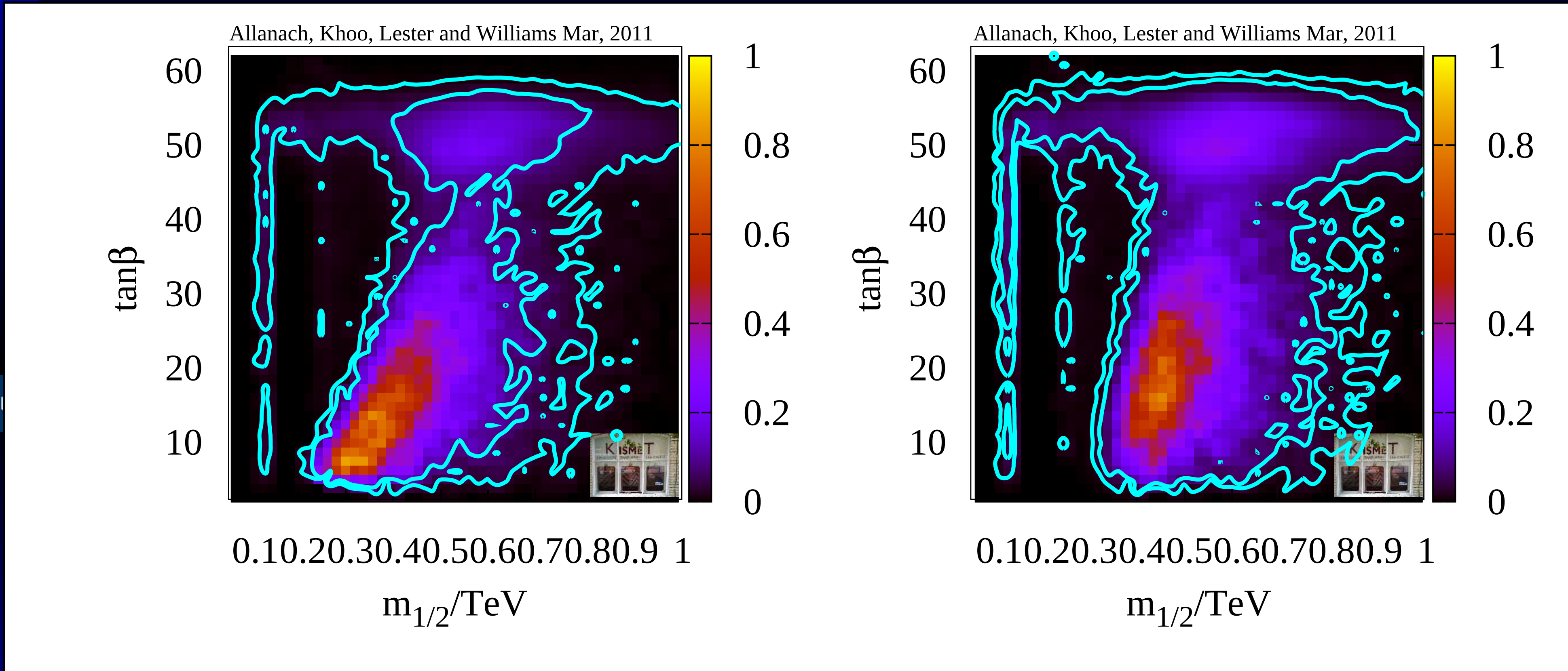
- Systematic errors on signals: perhaps at least a range over parameter space in one model. Ideally, it would be parameterised in terms of important quantities.
- Other contours (eg 1/5 sigma exclusion contours) so we can check our likelihood away from 95% excluded region.
- Numbers in histogram plots attached to `arXiv` publication

# Summary

- LHC has ruled out a significant portion of the most natural part of SUSY parameter space, but will require *nothing* at 14 TeV to dissuade most fans.
- Constrained models are now getting even more squeezed. Experiments are inviting theorists to do the interpretation in terms of their favourite SUSY breaking model.
- Current *missing energy type* searches reach squark and gluino masses of 1020 GeV (CMSSM), 900 GeV (mAMSB). Too early to give up on SUSY though.
- Another possibility: we haven't seen SUSY yet because $R-$parity is violated

# Supplementary Material

# Log Fits

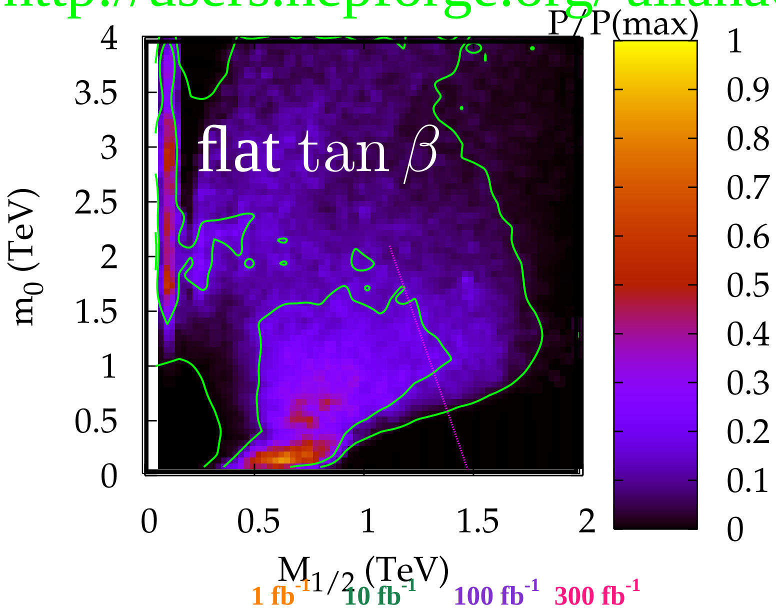


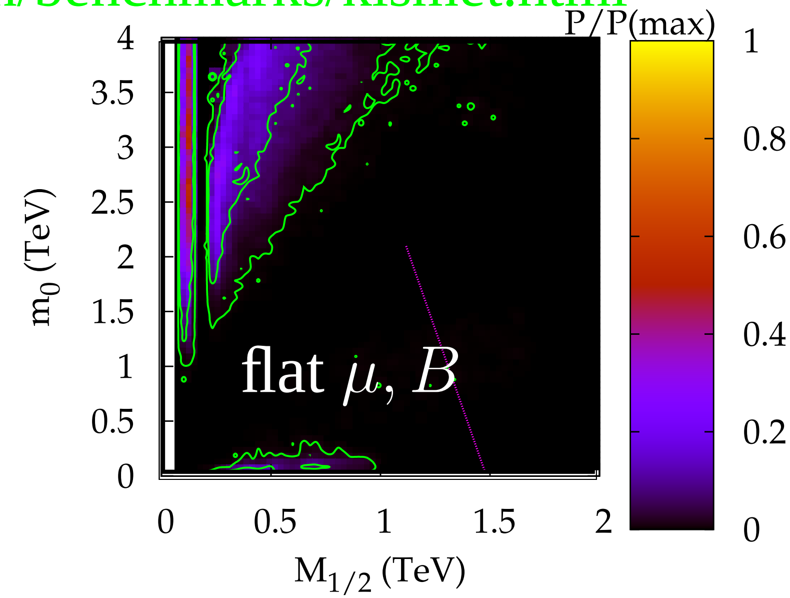

Before (left) and after (right) ATLAS 0-lepton exclusion limits.

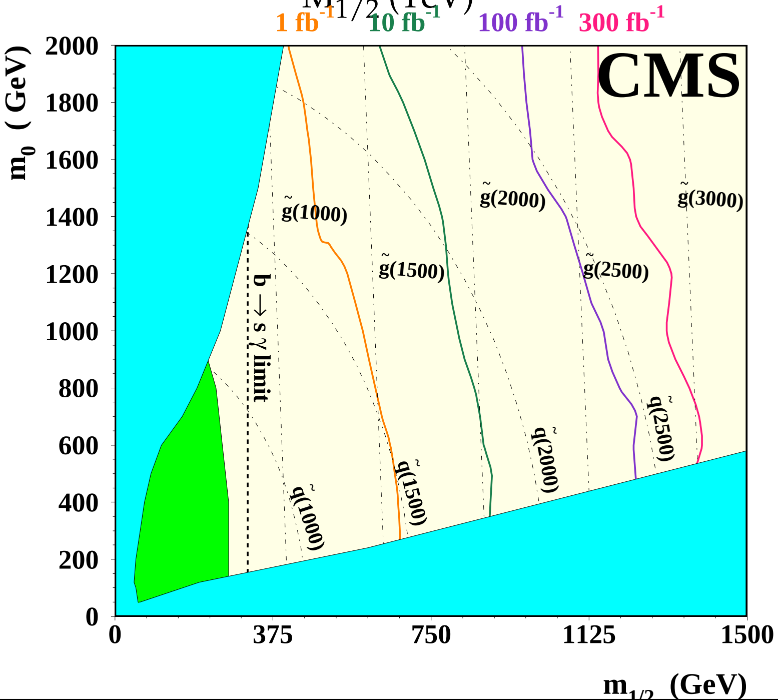

# Killer Inference for Susy METeorology

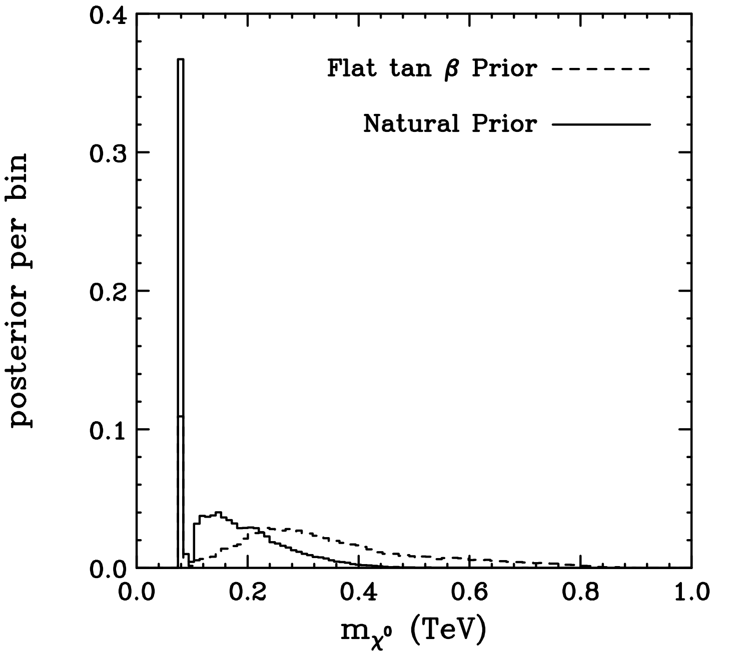
BCA, Cranmer, Weber, Lester, arXiv:0705.0487

http://users.hepforge.org/~allanach/benchmarks/kismet.html



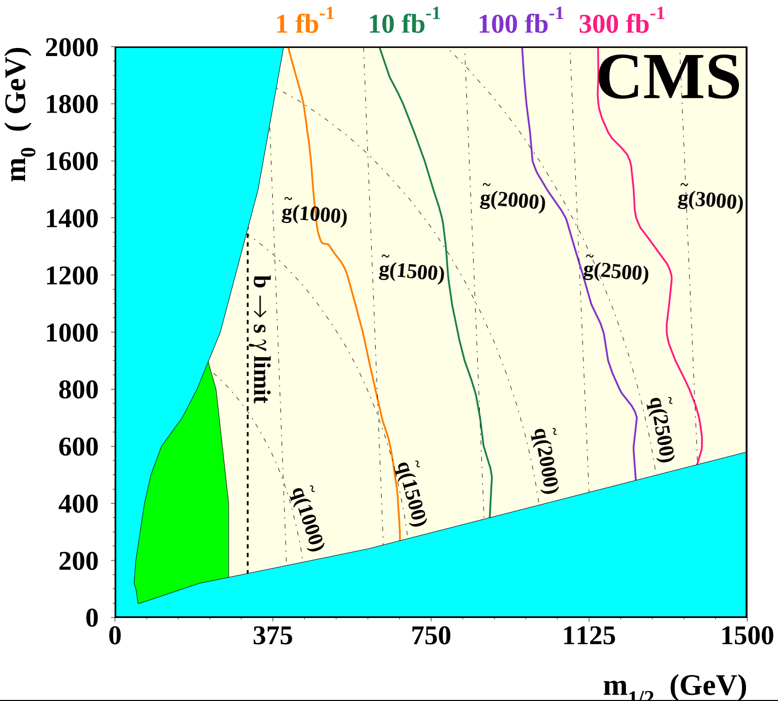



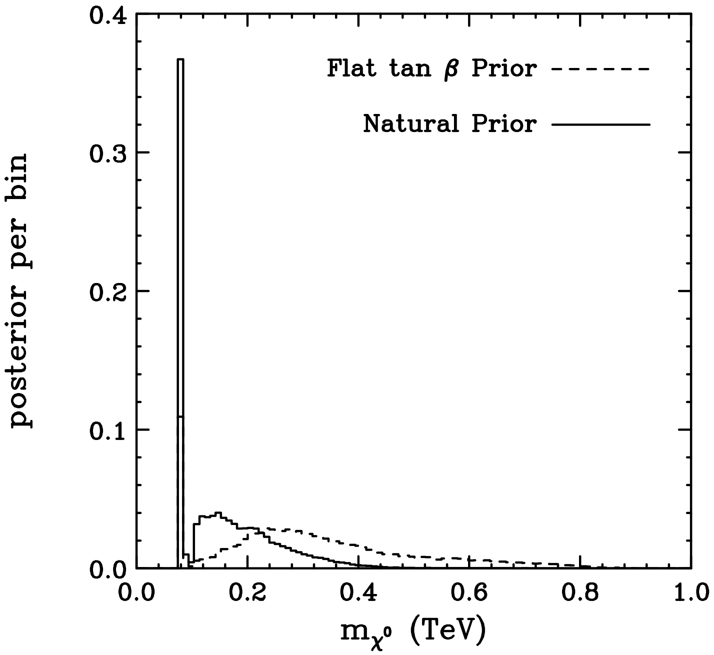



# Killer Inference for Susy METeorology

BCA, Cranmer, Weber, Lester, arXiv:0705.0487

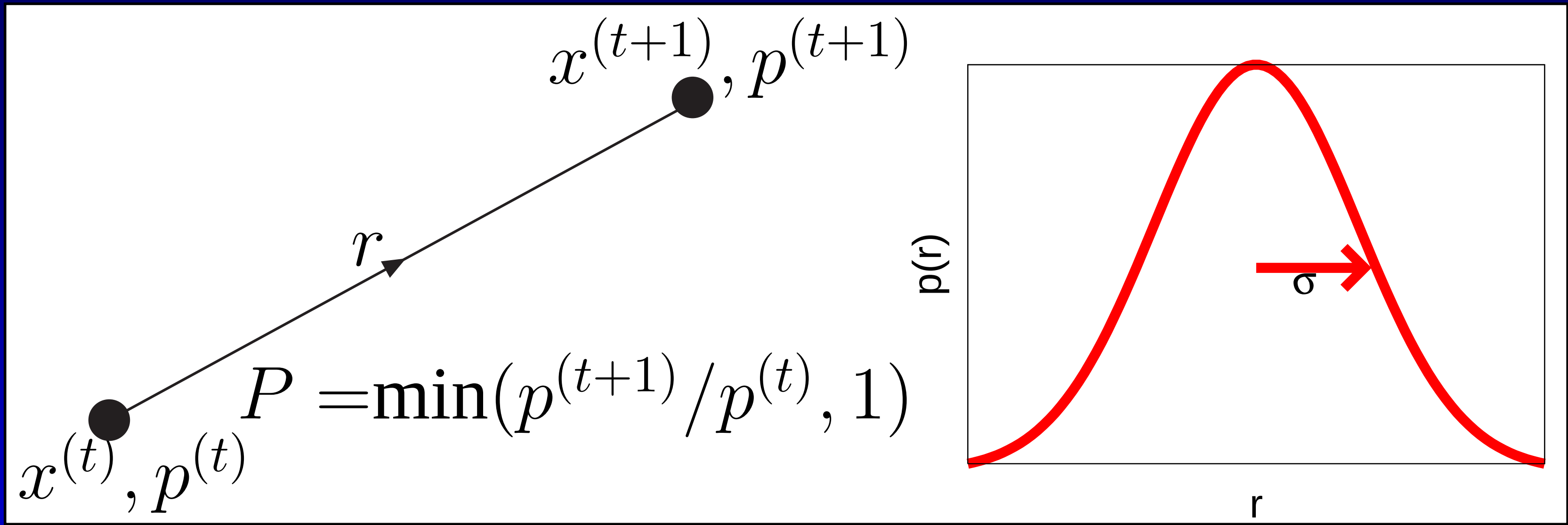




# Markov-Chain Monte Carlo

Metropolis-Hastings Markov chain sampling consists of list of parameter points $x^{(t)}$ and associated posterior probabilities $p^{(t)}$.



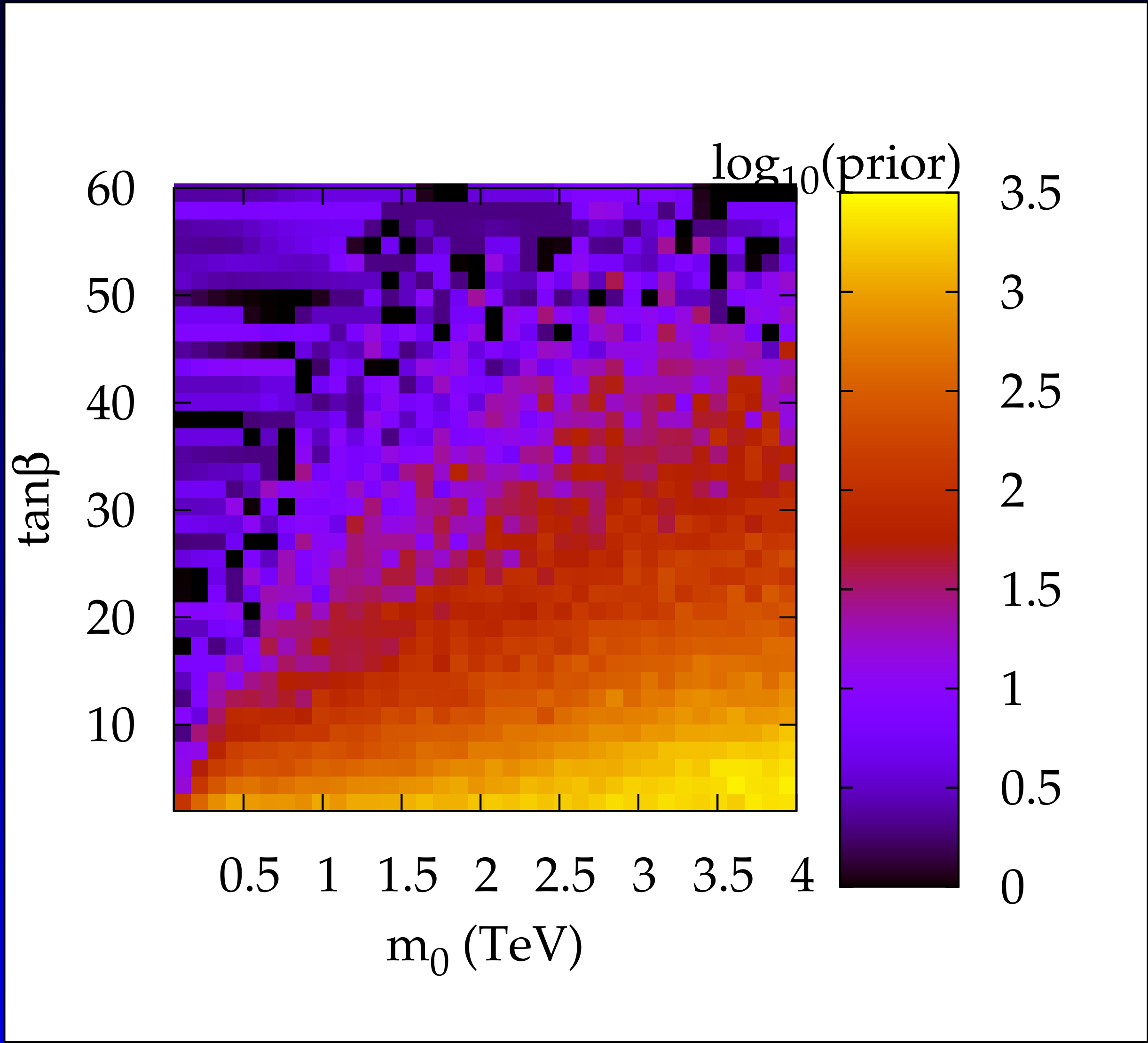

Final density of $x$ points $\propto p$. Required number of points goes *linearly* with number of dimensions.

# Ice Cube

Neutralinos can become trapped in the sun $\tilde{h}^0 - Z$ coupling $\sigma_{\chi^0 \mathrm{p,SD}} \propto [|N_{1d}|^2 - |N_{1u}|^2]^2$ dominates.
$A^\odot \equiv \sigma v / V$:

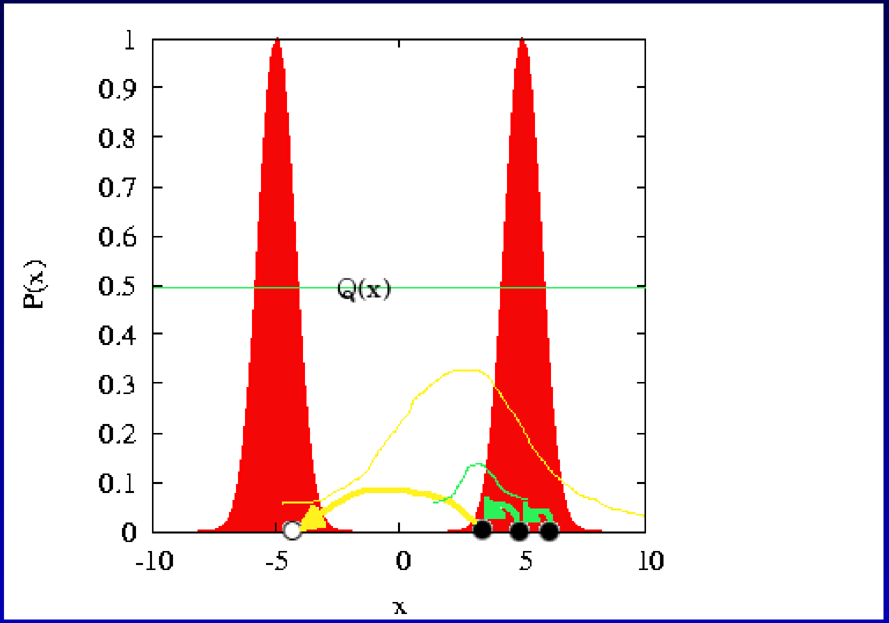
$$\begin{aligned} \dot{N} &= C^\odot - A^\odot N^2, \\ \Gamma &= \frac{1}{2} A^\odot N^2 = \frac{1}{2} C^\odot \tanh^2 \left( \sqrt{C^\odot A^\odot}\, t_\odot \right) \\ \frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} &= \frac{C_\odot F_{\mathrm{Eq}}}{4\pi D_{\mathrm{ES}}^2} \left( \frac{dN_\nu}{dE_\nu} \right)^{\mathrm{Inj}} \\ N_{\mathrm{ev}} &\approx \int \int \frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} \frac{d\sigma_\nu}{dy} R_\mu((1-y)\, E_\nu)\, A_{\mathrm{eff}}\, dE_{\nu_\mu}\, dy \end{aligned}$$

# Naturalness priors



# Potential Problem

Often, people use a flat $Q(x)$. The trouble with this *"blind drunk"* sampling is the following situation:




Either large or small proposal widths $\sigma$ lead to low efficiencies of sampling. Our proposal is to determine a $Q(x)$ closer to $P(x)$ *semi-automatically*.

# Bank Sampling

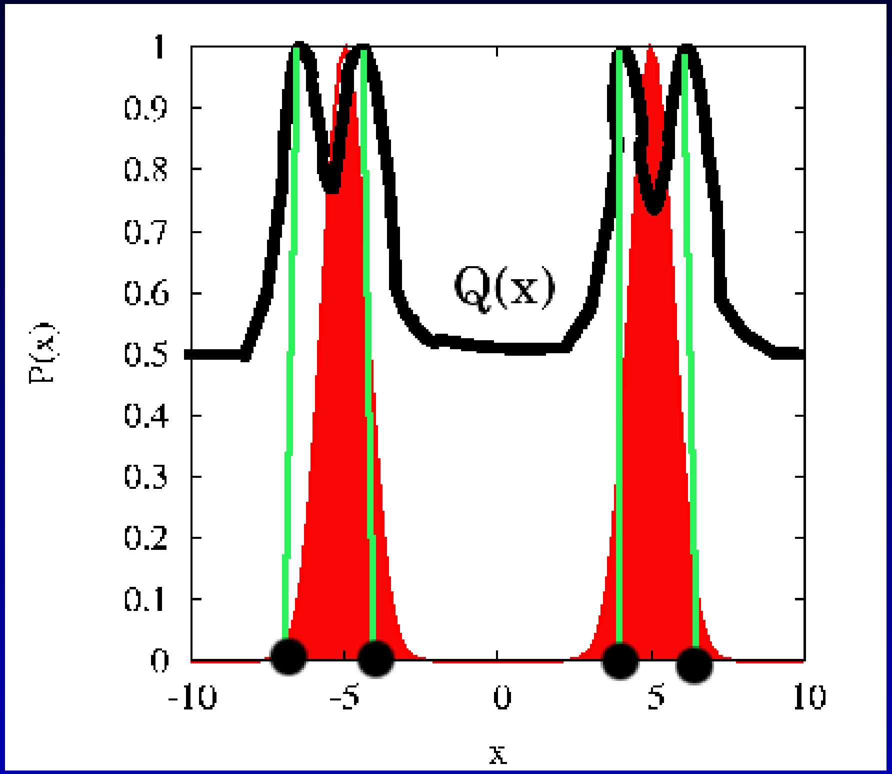



Figure 5: Bank points determined from previous runs: want to have at least one point in each maximum. *Knowledgeable drunk*

# Proposal Distribution

$$Q_{bank}(\mathbf{x};\mathbf{x}^{(t)}) = (1-\lambda)K(\mathbf{x};\mathbf{x}^{(t)}) + \lambda \sum_{i=1}^{N} w_i K(\mathbf{x};\mathbf{y}^{(i)})$$

$w_i$ are a set of $N$ weights: $\sum_{i=1}^{N} w_i = 1$, $0 < \lambda < 1$, while $K$ is the proposal distribution.

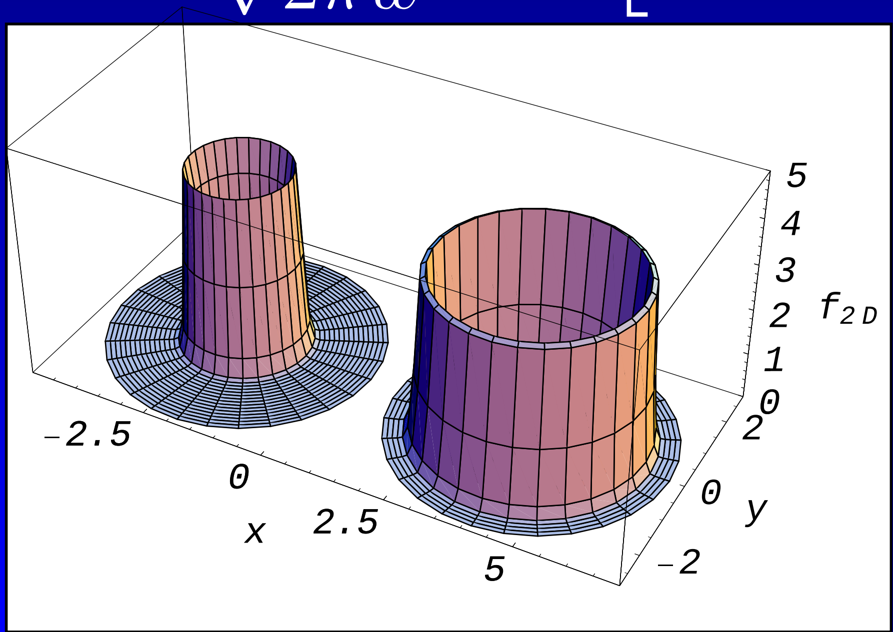
With probability $(1-\lambda)$ propose a local Metropolis step of the usual kind, i.e. "close" to the last point in the chain. With probability $\lambda$, teleport to the vicinity of one of the number of "banked" points, chosen with weight $w_i$ from within the bank.

# Example Distribution

$$f_{2D}(\mathbf{x}) = \mathrm{circ}(\mathbf{x}; c_1, r_1, w_1) + \mathrm{circ}(\mathbf{x}; c_2, r_2, w_2)$$

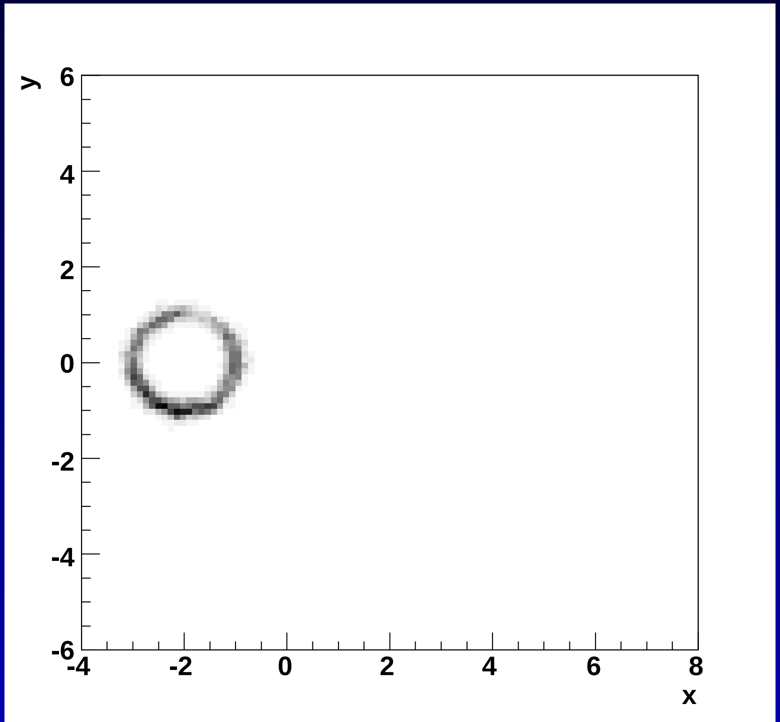
where $c_1 = (-2, 0)$, $r_1 = 1$, $w_1 = 0.1$, $c_2(+4, 0)$, $r_2 = 2$, $w_2 = 0.1$ and

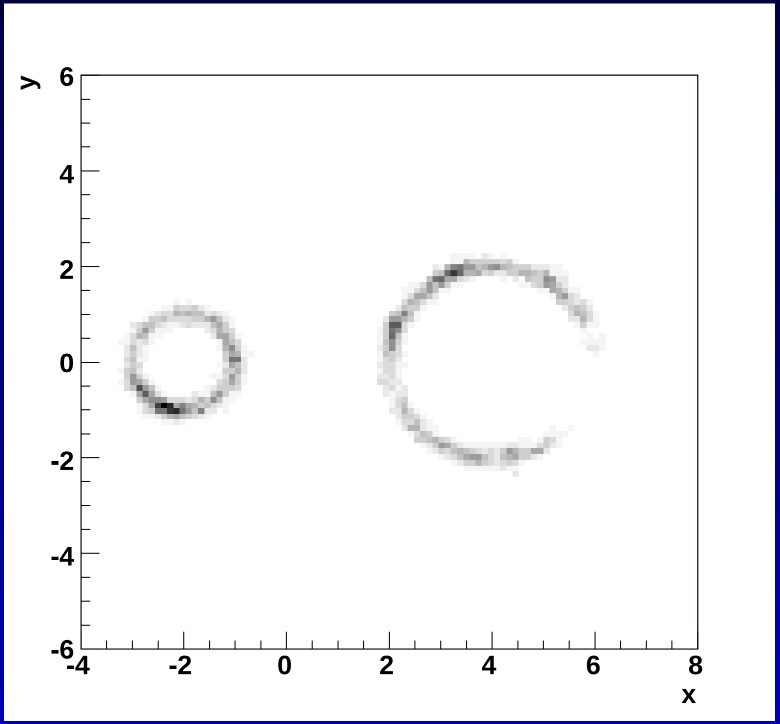
$$\mathrm{circ}(\mathbf{x}; \mathbf{c}, r, w) = \frac{1}{\sqrt{2\pi w^2}} \exp\left[-\frac{(|\mathbf{x} - \mathbf{c}| - r)^2}{2w^2}\right].$$



# Bank vs Metropolis

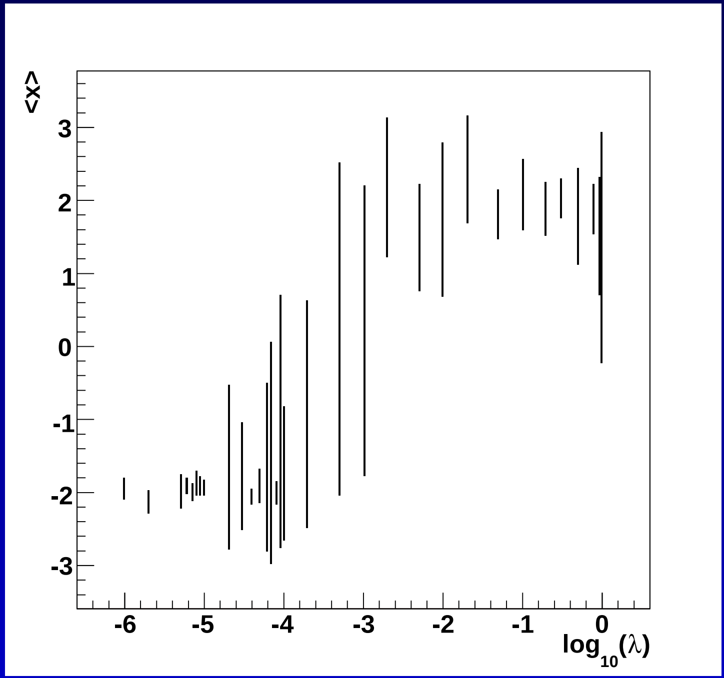
10 000 samples for MCMC and bank sampling:

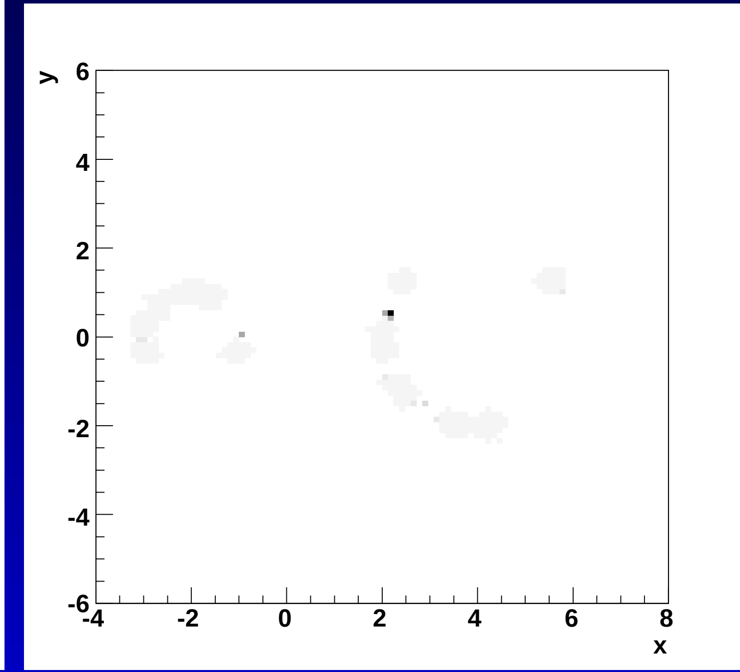


# Safety with respect to $\lambda$

10 bank samplers, with 10 bank points generated in each circle: 10 000 samples. All started from $x = -2$. Correct $\langle x \rangle = 2$. $\lambda \approx 1$ is importance sampling limit.

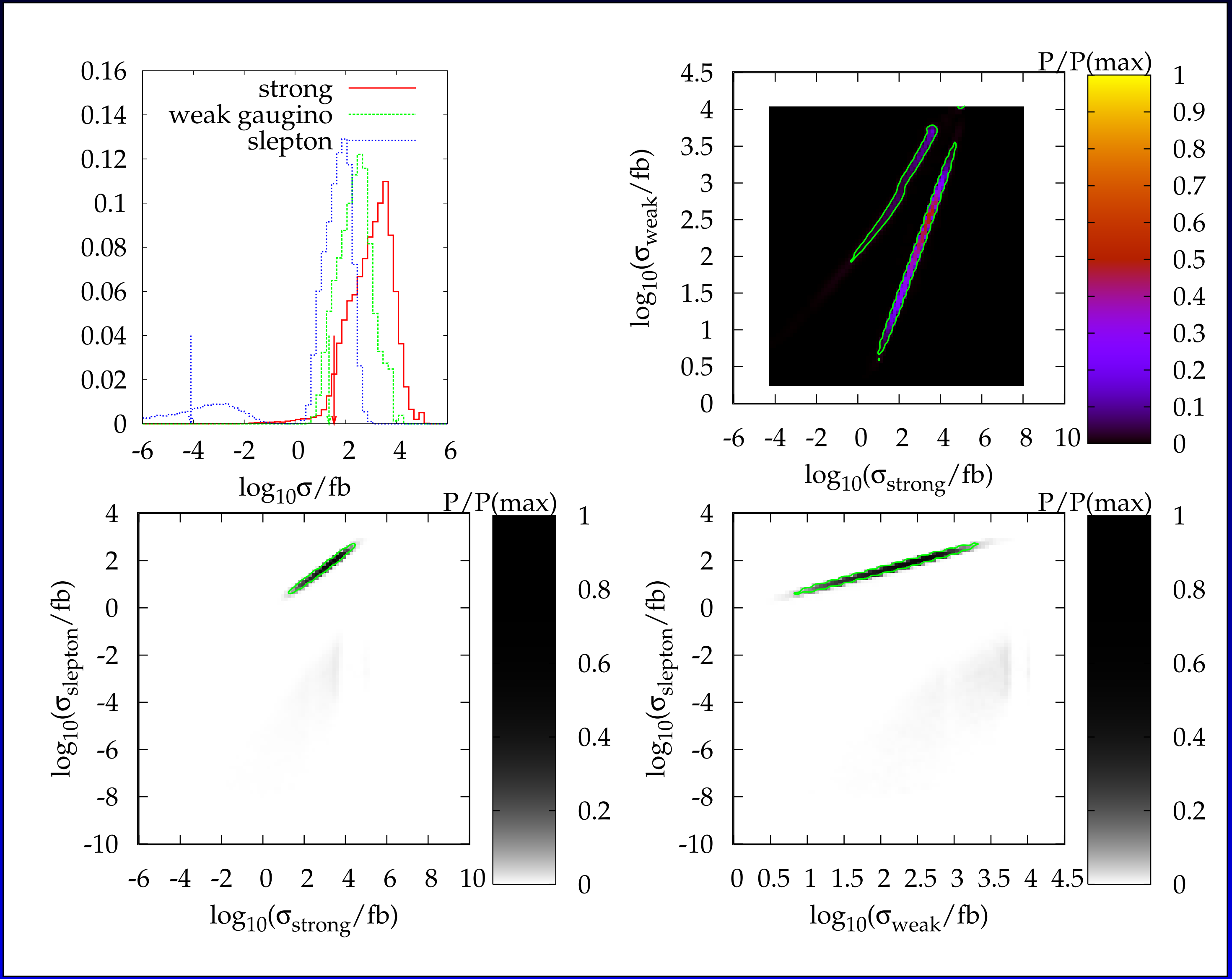



*Q*: What values of $\lambda$ are "safe"?

*A*: [0.001, 0.9]

# LHC Cross-sections



# Collider Check

Need corroboration with *direct detection*.

If we can pin particle physics down, a comparison between the predicted relic density and that observed is a test of the cosmological assumptions used in the prediction.[a]

Thus, if it doesn't fit, you change the cosmology until it does.

[a] BCA, G. Belanger, F. Boudjema, A. Pukhov, JHEP 0412 (2004) 020.; M. Nojiri, D. Tovey, JHEP 0603 (2006) 063

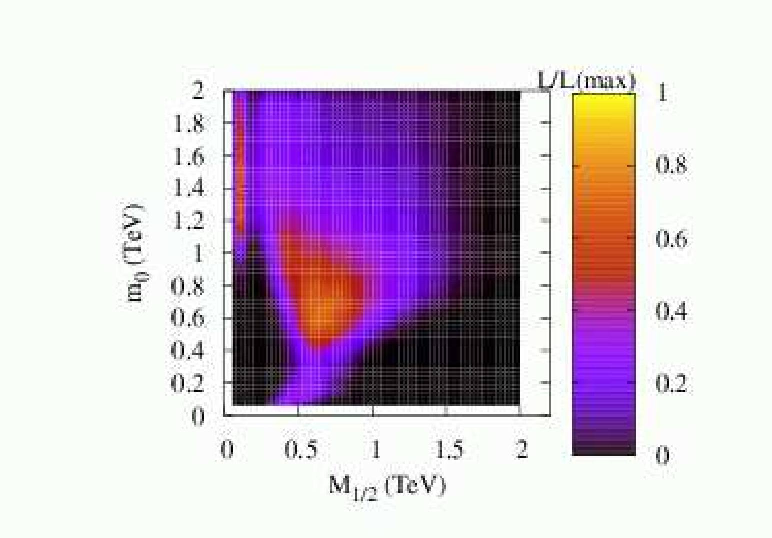
# CMSSM Regions

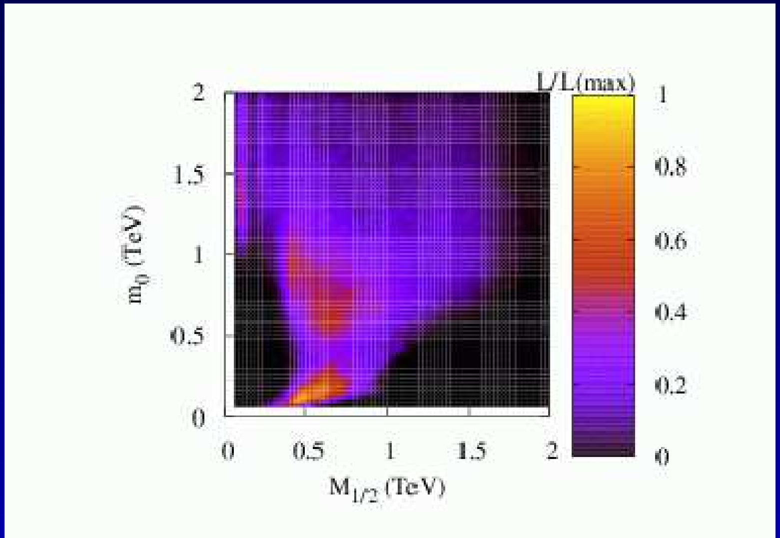
After WMAP+LEP2, bulk region diminished. Need specific mechanism to reduce overabundance:

- $\tilde{\tau}$ coannihilation: small $m_0$, $m_{\tilde{\tau}_1} \approx m_{\chi_1^0}$. Boltzmann factor $\exp(-\Delta M/T_f)$ controls ratio of species. $\tilde{\tau}_1 \chi_1^0 \to \tau\gamma$, $\tilde{\tau}_1 \tilde{\tau}_1 \to \tau\bar{\tau}$.
- Higgs Funnel: $\chi_1^0 \chi_1^0 \to A \to b\bar{b}/\tau\bar{\tau}$ at large $\tan\beta$. Also via[a] $h$ at large $m_0$ small $M_{1/2}$.
- Focus region: Higgsino LSP at large $m_0$: $\chi_1^0 \chi_1^0 \to WW/ZZ/Zh/t\bar{t}$.
- $\tilde{t}$ coannihilation: high $-A_0$, $m_{\tilde{t}_1} \approx m_{\chi_1^0}$. $\tilde{t}_1 \chi_1^0 \to gt$, $\tilde{t}\tilde{t} \to tt$

[a] Datta, Djouadi, Drees, hep-ph/0504090

# Comparison



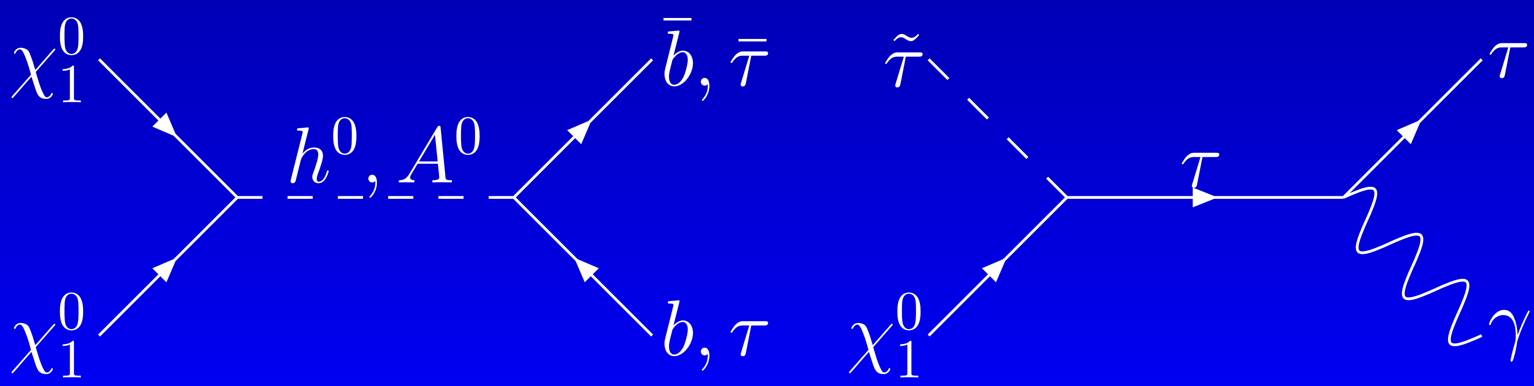




- LHS: allowing non thermal-$\chi_1^0$ contribution
- RHS: only $\chi_1^0$ dark matter
- *(flat priors)*

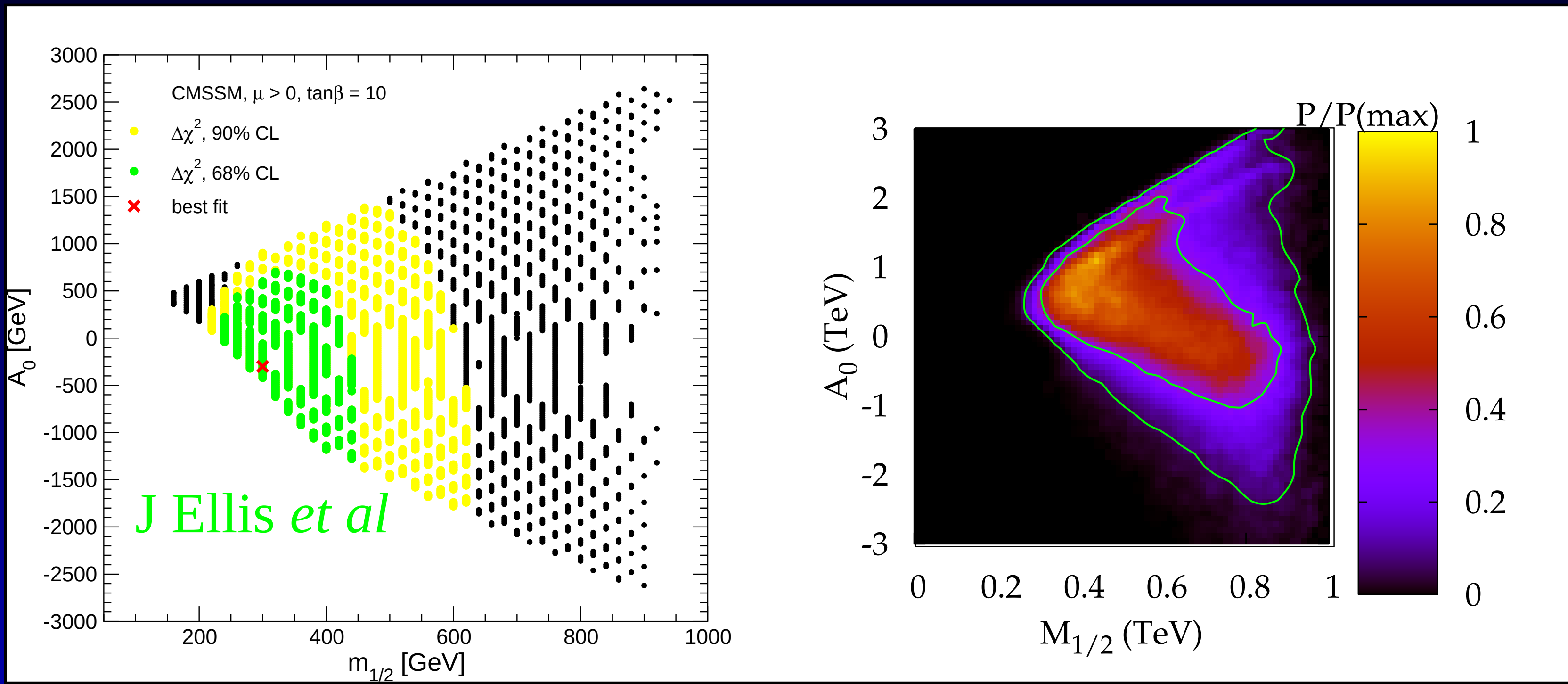
# Annihilation Mechanism

Define stau co-annihilation when $m_{\tilde{\tau}}$ is within 10% of $m_{\chi_1^0}$ and Higgs pole when $m_{h,A}$ is within 10% of $2m_{\chi_1^0}$.

| mechanism | flat prior | natural prior |
|---|---|---|
| $h^0-$pole | 0.025 | 0.07 |
| $A^0-$pole | 0.41 | 0.14 |
| $\tilde{\tau}-$co-annihilation | 0.26 | 0.18 |
| rest | 0.31 | 0.61 |

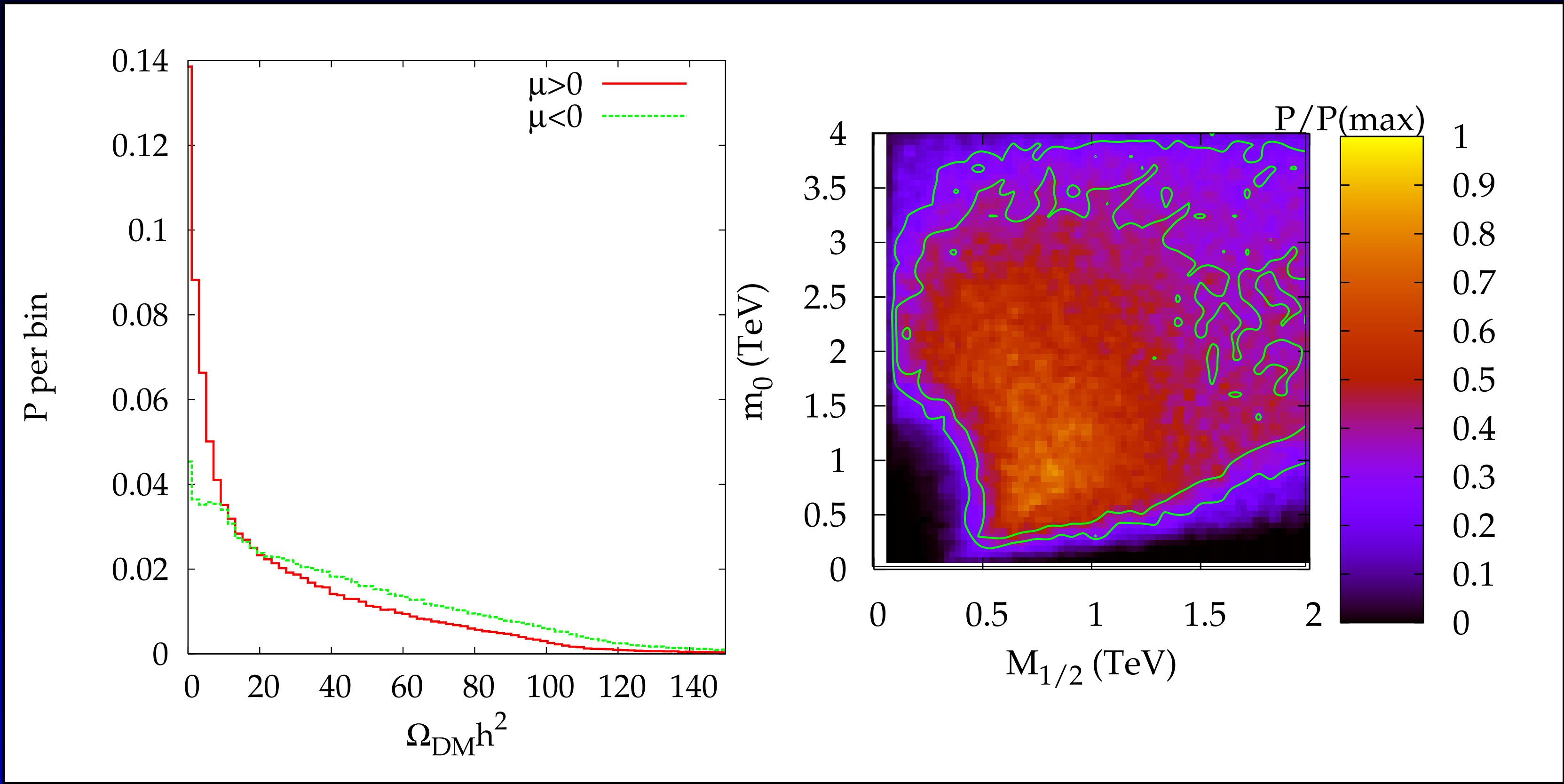


# Comparison




- Fix $\tan\beta = 10$ and all SM inputs
- Restrict $m_0, M_{1/2} < 1$ TeV.
- *Same* fits!

# No Dark Matter Fits




Huge $\chi^2$ from the dark matter relic density.

# Volume Effects

*Can't rely on a good $\chi^2$ in non-Gaussian situation*

# Likelihood and Posterior

$Q$: What's the chance of observing someone to be pregnant, given that they are female?

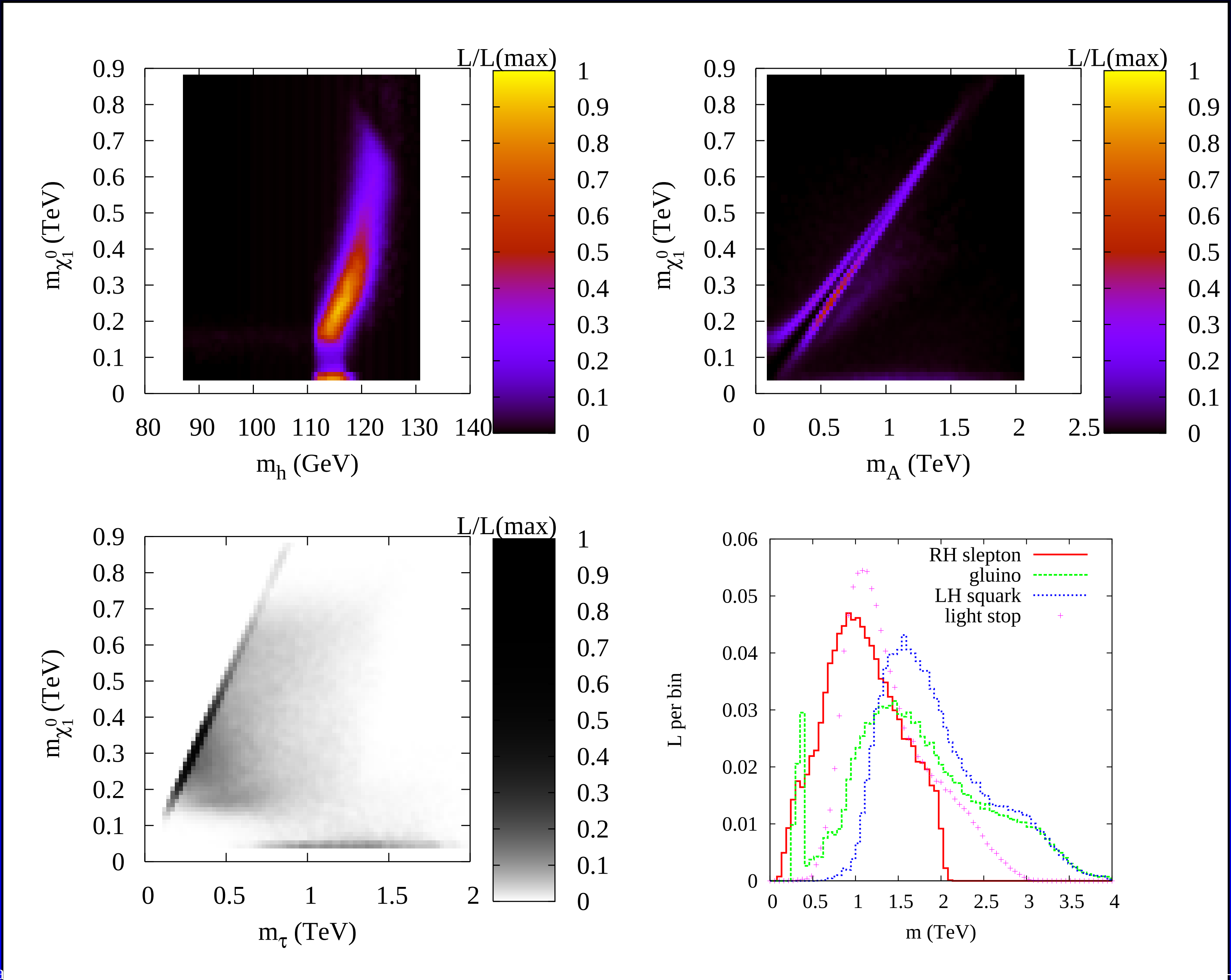
Likelihood

$p(\text{pregnant} \mid \text{female, human}) = 0.01$

Posterior

$p(\text{female} \mid \text{pregnant, human}) = 1.00$

# Sanity Check



# LHC vs LC in SUSY Measurement

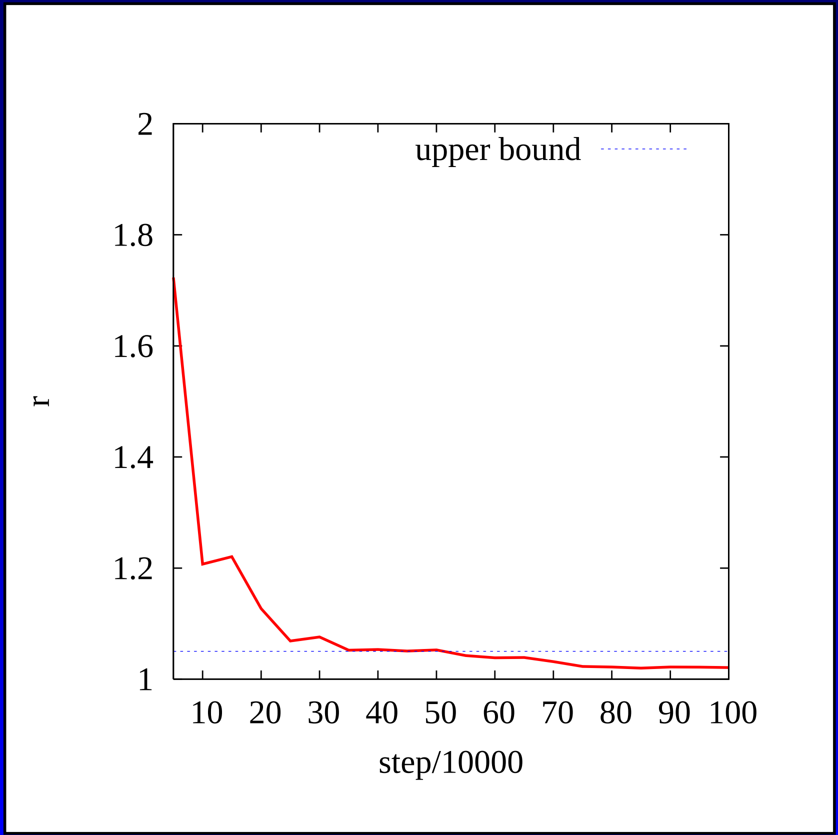
- LHC (start date 2007) produces strongly interacting particles up to a few TeV. Precision measurements of mass *differences* possible if the decay chains exist: possibly per mille for leptons, several percent for jets.
- ILC has several energy options: 500-1000 GeV, CLIC up to 3 TeV. Linear colliders produce less strong particles but much easier to make precision measurements of masses/couplings.

$\mathcal{Q}$: What energy for LC?
$\mathcal{Q}$: What do we get from LHC[a]?

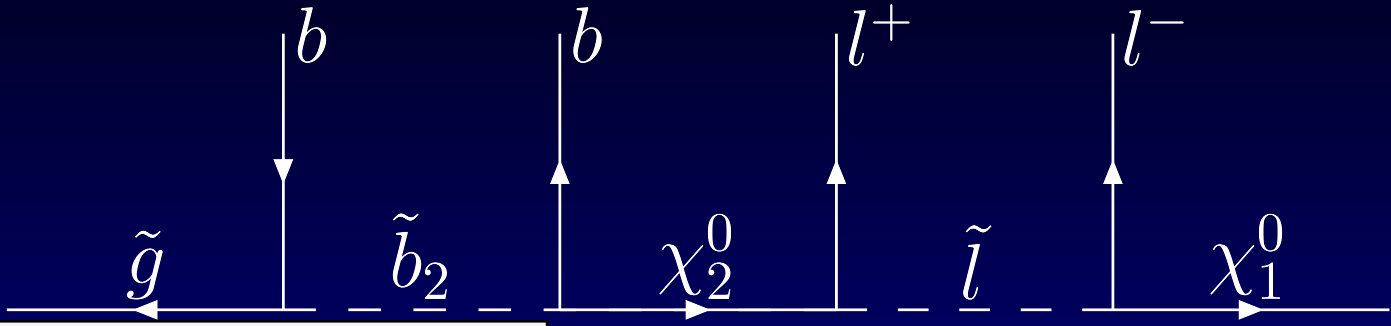
[a] LHC/ILC Working Group Report: `hep-ph/0410364`

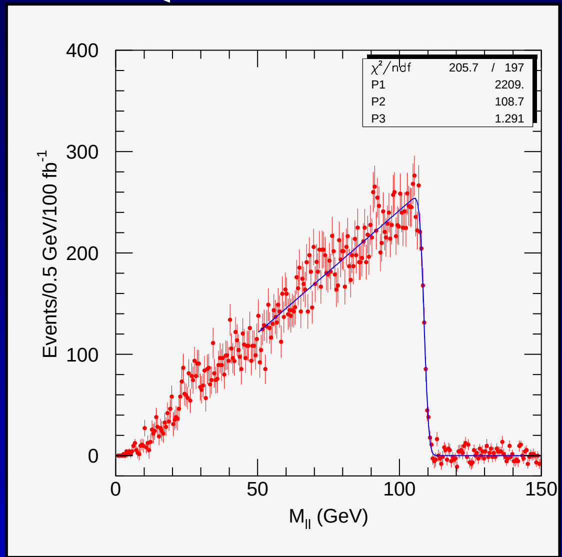
# Convergence

We run 9×1 000 000 points. By comparing the 9 independent chains with random starting points, we can provide a statistical measure of convergence: an upper bound $r$ on the excepted variance decrease for infinite statistics.



# LHC SUSY Measurements



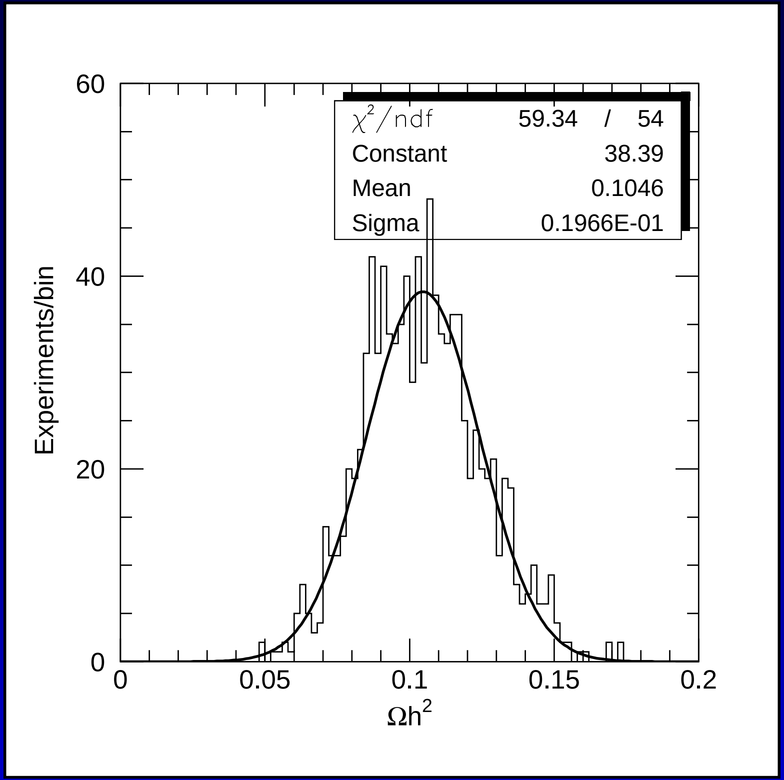

$$m_{ll}^2 = \frac{(m^2_{\chi_2^0} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\chi_1^0})}{m^2_{\tilde{l}}}$$

*Q*: Can we measure enough of these to pin SUSY[a] down?

[a] BCA, Lester, Parker, Webber, JHEP 0009 (2000) 004

# Predicting $\Omega h^2$

Not much left that's allowed but edge measurements allow reasonable $\Omega h^2$ error[a] for 300 $\mathrm{fb}^{-1}$.




$\mathcal{Q}$: What about other bits of parameter space?

[a] M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`.

# Bulk Region

M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, `hep-ph/0512204`. for 300 fb$^{-1}$. SPA point $m_0 = 70$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -300$ GeV, $\tan\beta = 10$, $\mu > 0$: $\Omega h^2 = 0.108$. Put in $m_{ll}^{max}$, $m_{llq}^{max}$, $m_{lq}^{low}$, $m_{lq}^{high}$, $m_{llq}^{min}$, $m_{l_L} - m_{\chi_1^0}$, $m_{ll}^{max}(\chi_4^0)$, $m_{\tau\tau}^{max}$, $m_h$.

| | |
|---|---|
| $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to \ell^+\ell^-$ | 40% |
| $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to \tau^+\tau^-$ | 28% |
| $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to \nu\bar{\nu}$ | 3% |
| $\tilde{\chi}_1^0\tilde{\tau}_1 \to Z\tau$ | 4% |
| $\tilde{\chi}_1^0\tilde{\tau}_1 \to A\tau$ | 18% |
| $\tilde{\tau}_1\tilde{\tau}_1 \to \tau\tau$ | 2% |

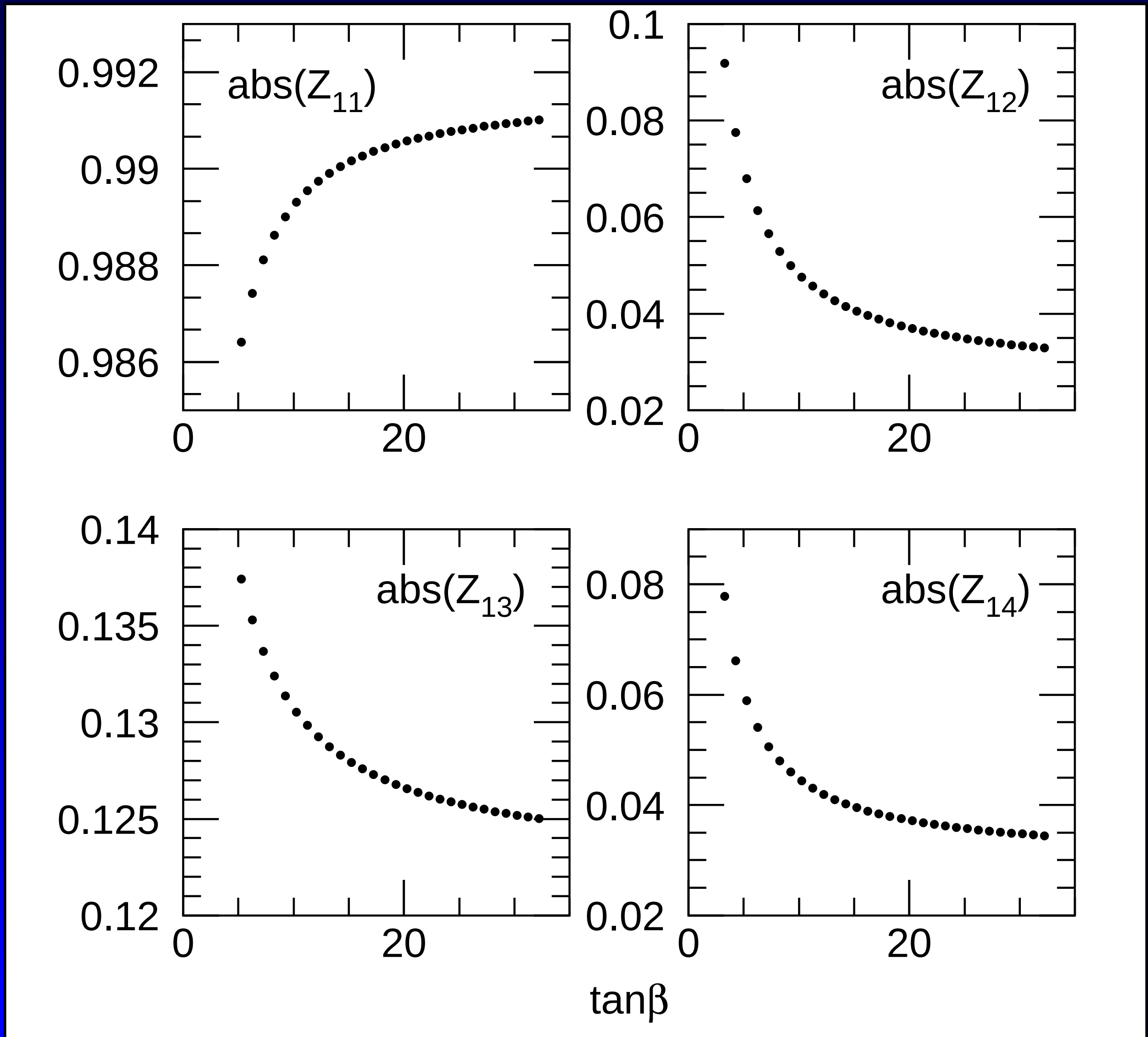
# Neutralino mass matrix

Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.

$$\left[\begin{array}{cccc} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{array}\right] \tag{1}$$

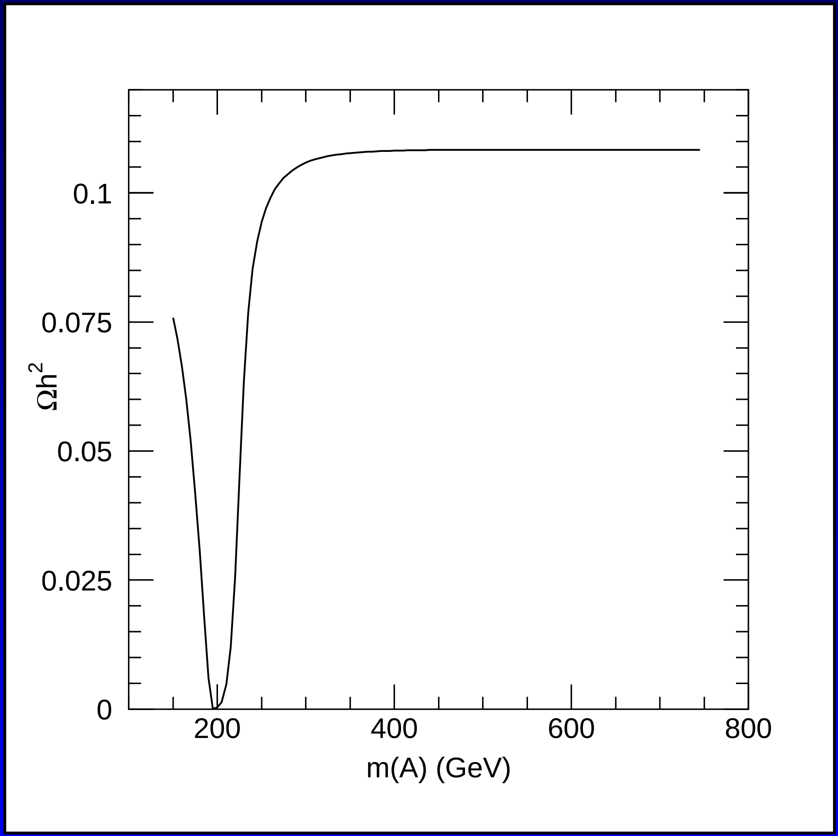
# Neutralino mass matrix

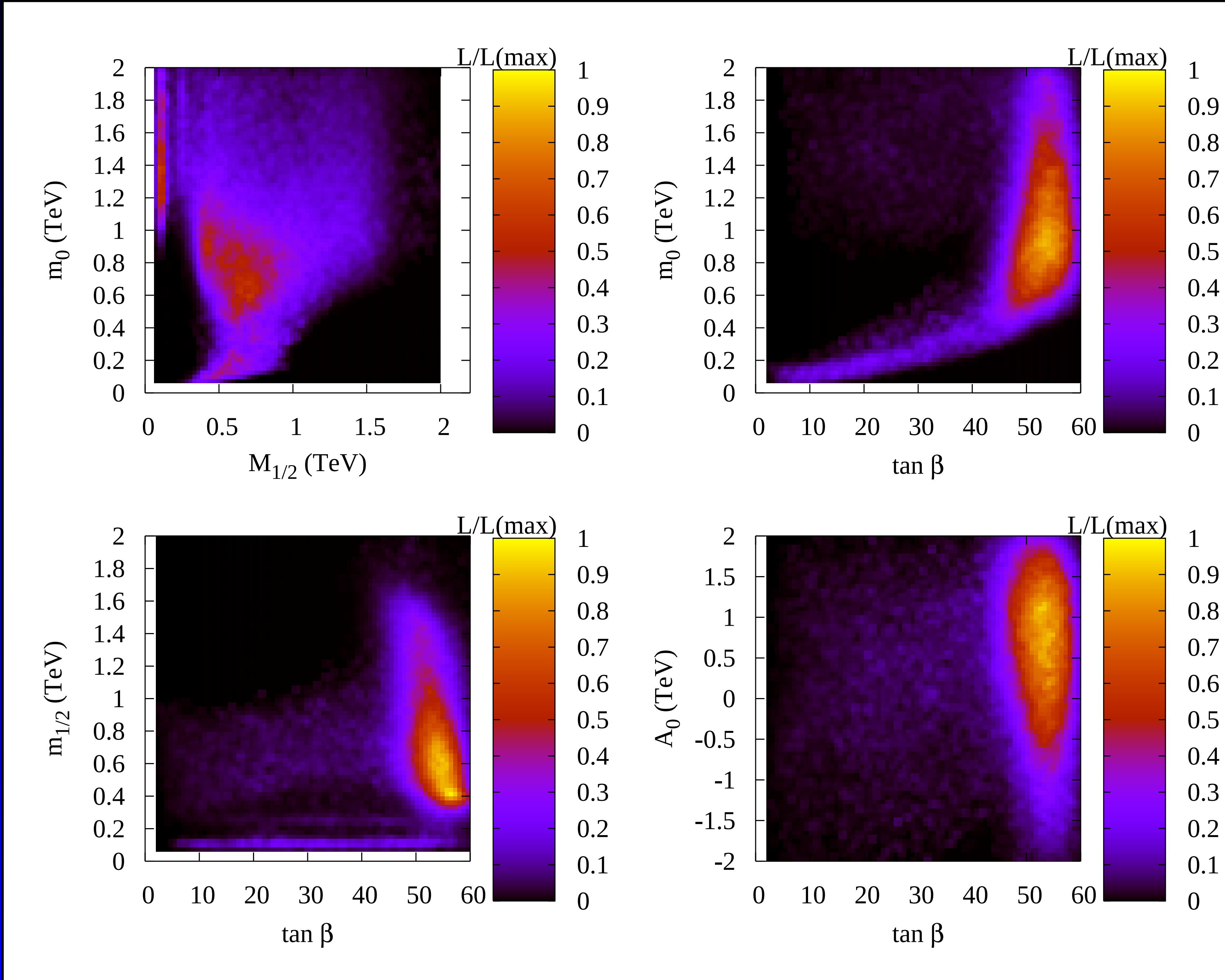
Neutralino masses measured: $\chi^0_{1,2,4}$ but need mixing matrix to determine couplings. Left with $\tan\beta$.



# Slepton/$A^0$ Higgs

$\Gamma(\chi_2^0 \rightarrow \tilde{l}_R l)/\Gamma(\chi_2^0 \rightarrow \tilde{\tau}_1 \tau)$ then helps determine $\theta_\tau$ for a given $\tan\beta$. Exclusion of $A^0$ helps you to exclude $A^0$ appearing in cascade decays. Meaurement of $m_h$ provides constraints in $m_A - \tan\beta$ plane.




L/L(max)
m_0 (TeV)
M_1/2 (TeV)
tan β
m_1/2 (TeV)
A_0 (TeV)

v

# Uncertainties in Relic Density

Bulk region: $\tilde{B}\tilde{B} \to Z, h \to l\bar{l}$. Coannihilation: $\tilde{\tau}\chi_1^0 \to \tau + X$

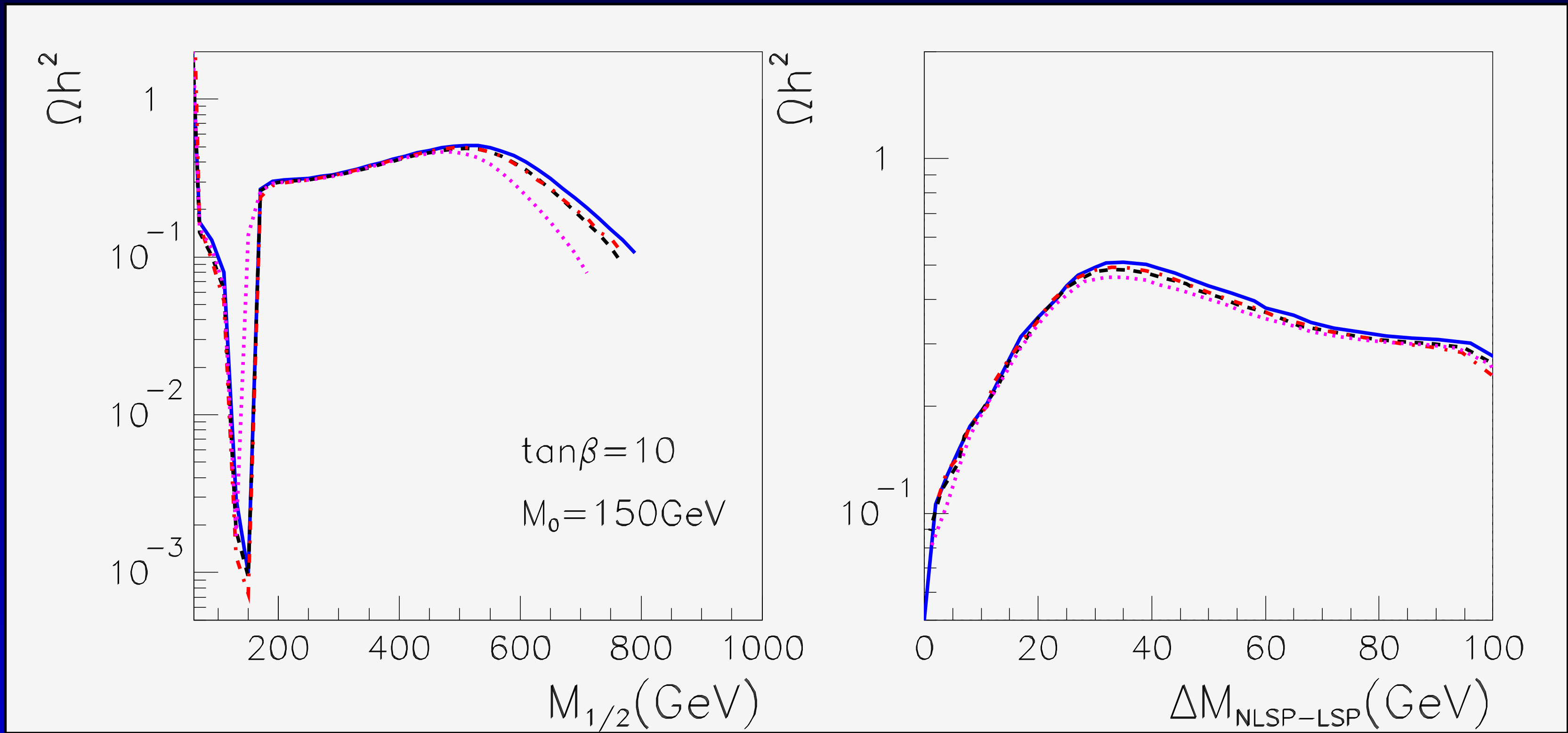



Figure 7: Bulk/coannihilation region. Full: `SoftSusy`, dotted: `SPheno`.

# Focus Point

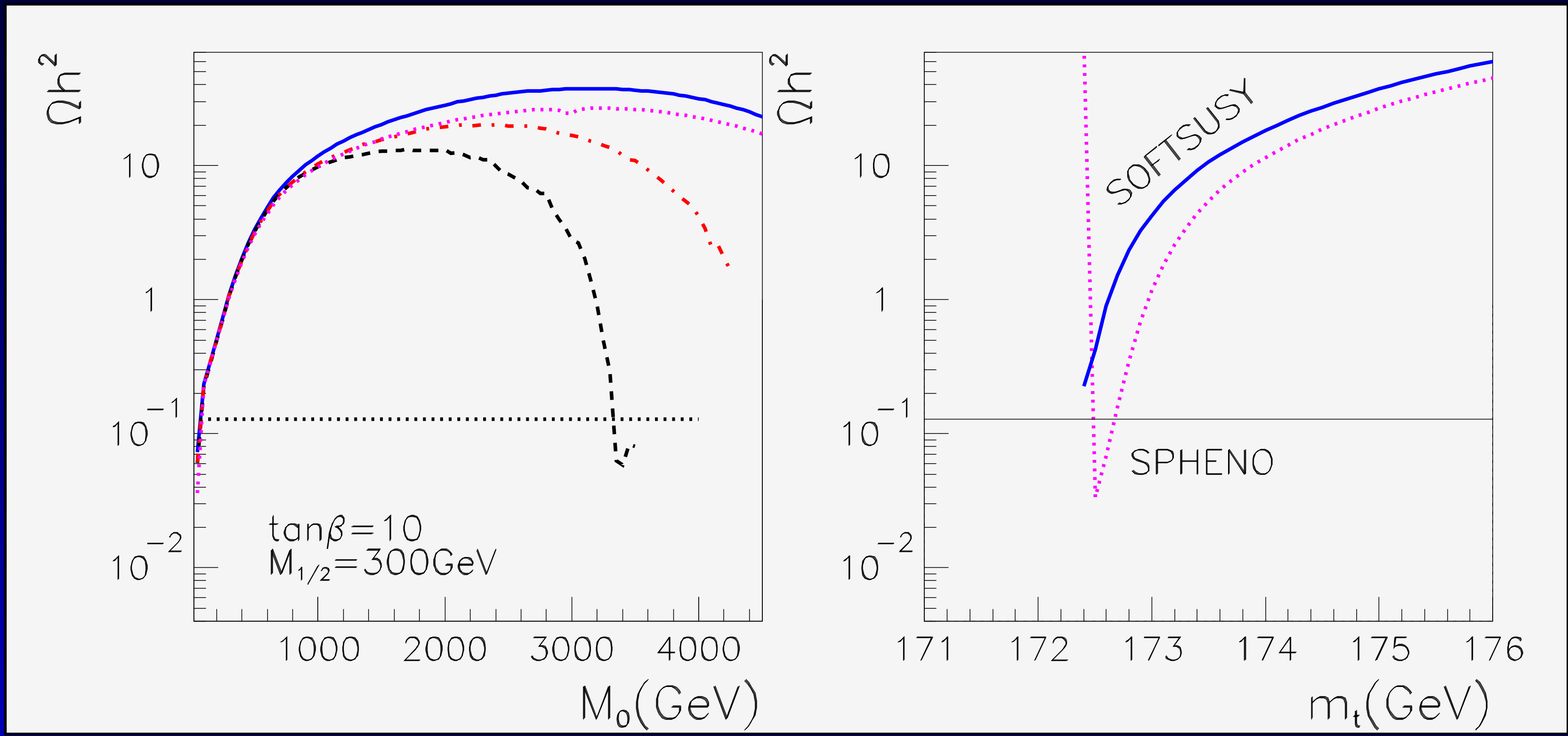



Figure 8: Focus point region. Full: `SoftSusy`, dotted: `SPheno`, dashed: `SuSpect`. Higgsino LSP annihilates into $ZZ/WW$

# High $\tan\beta$

BCA, Belanger, Boudjema, Pukhov, Porod, hep-ph/0402161. Baer *et al*

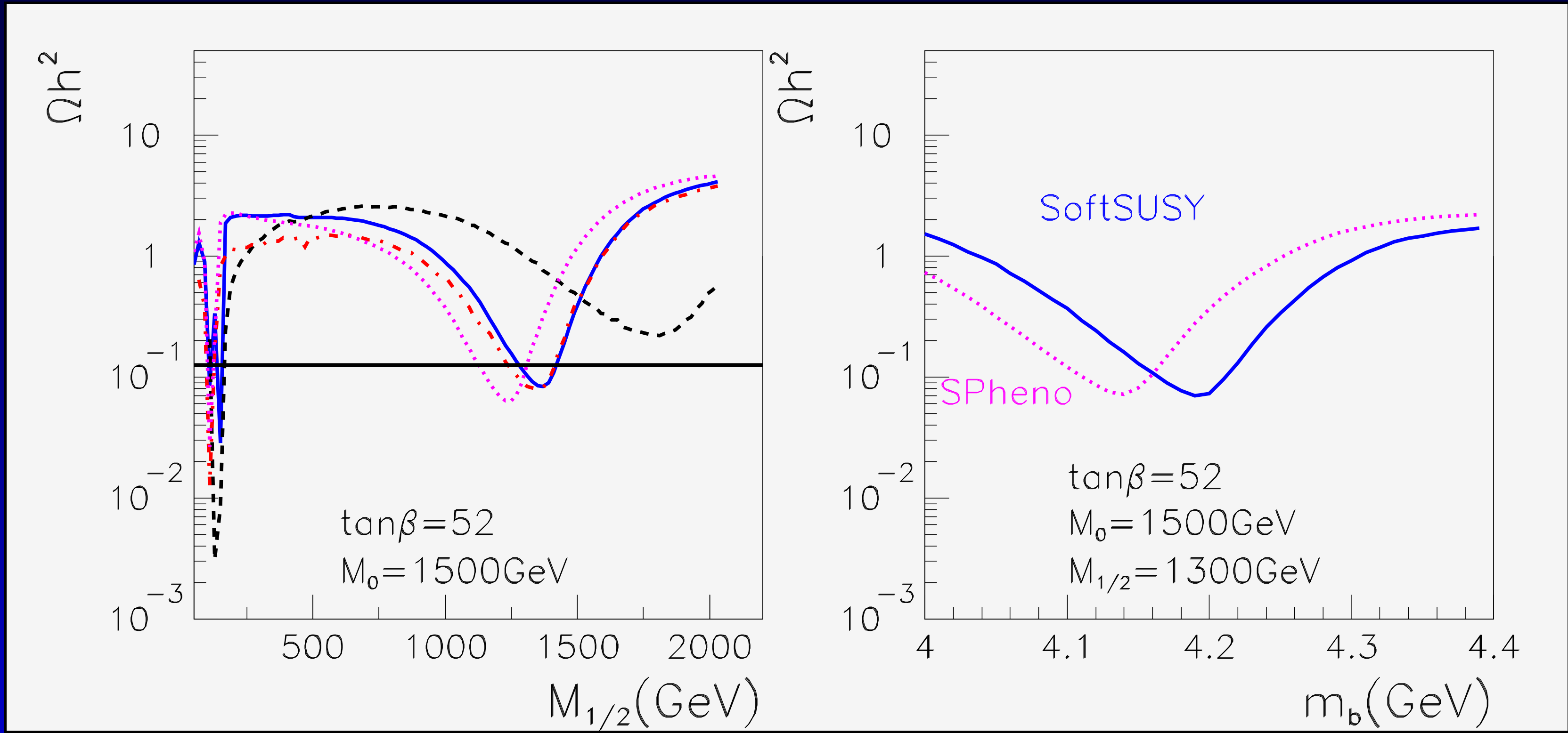
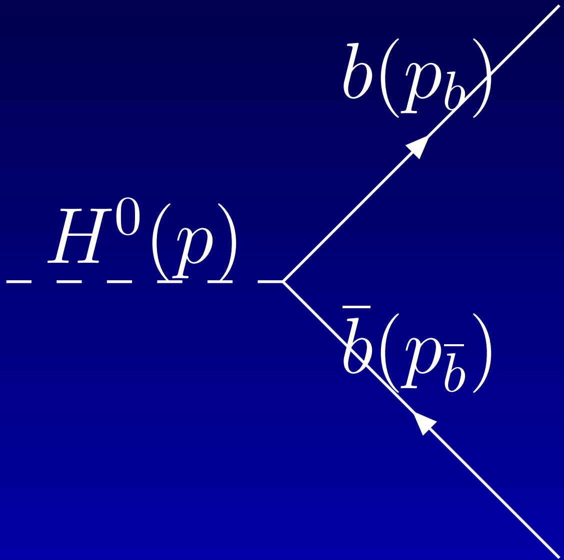



Figure 9: High $\tan\beta$ region. Full: `SoftSusy`, dotted: `SPheno`, dashed: `SuSpect`. Get annihilation into $A$.

# SUSY Kinematics: a Reminder

Take a particle decaying into 2 particles, eg $H^0 \to b\bar{b}$.
We define the invariant mass of the $b\bar{b}$ pair such that:



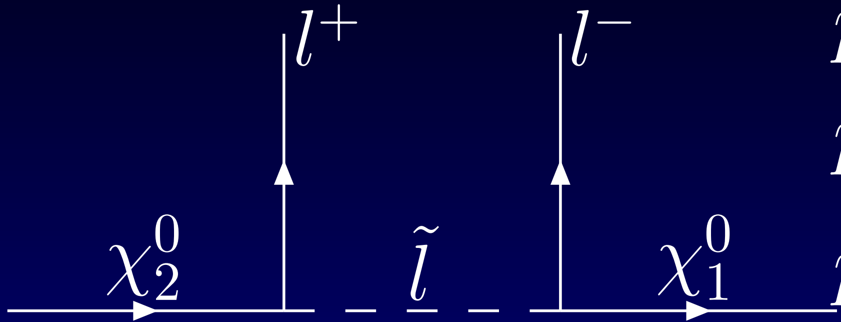

$$p^\mu = (\sqrt{m_H^2 + p^2}, \underline{p}) = p_b^\mu + p_{\bar{b}}^\mu$$

$$\Rightarrow p^2 = m_H^2 = (p_b + p_{\bar{b}})^2$$

Is *invariant* in boosted frames

*Question*: What happens to invariant mass in SUSY cascade decays, where we miss the final particle?

# Cascade Decay




$$p^\mu_{\tilde{l}} = (m_{\tilde{l}}, \underline{0})$$
$$p^\mu_{l^\pm} = (|\underline{p}_{l^\pm}|, \underline{p}_{l^\pm})$$
$$p^\mu_{\chi^0_{1,2}} = \left(\sqrt{{m_{\chi^0_{1,2}}}^2 + |\underline{p}_{\chi^0_{1,2}}|^2}, \underline{p}_{\chi^0_{1,2}}\right)$$

The invariant mass of the $l^+l^-$ pair is

$$m^2_{ll} = (p_{l^+} + p_{l^-})^\mu (p_{l^+} + p_{l^-})_\mu = p^2_{l^+} + p^2_{l^-} + 2 p_{l^+} \cdot p_{l^-}$$
$$= 2|\underline{p}_{l^+}||\underline{p}_{l^-}|(1 - \cos\theta) \leq 4|\underline{p}_{l^+}||\underline{p}_{l^-}|.$$

Momentum conservation:

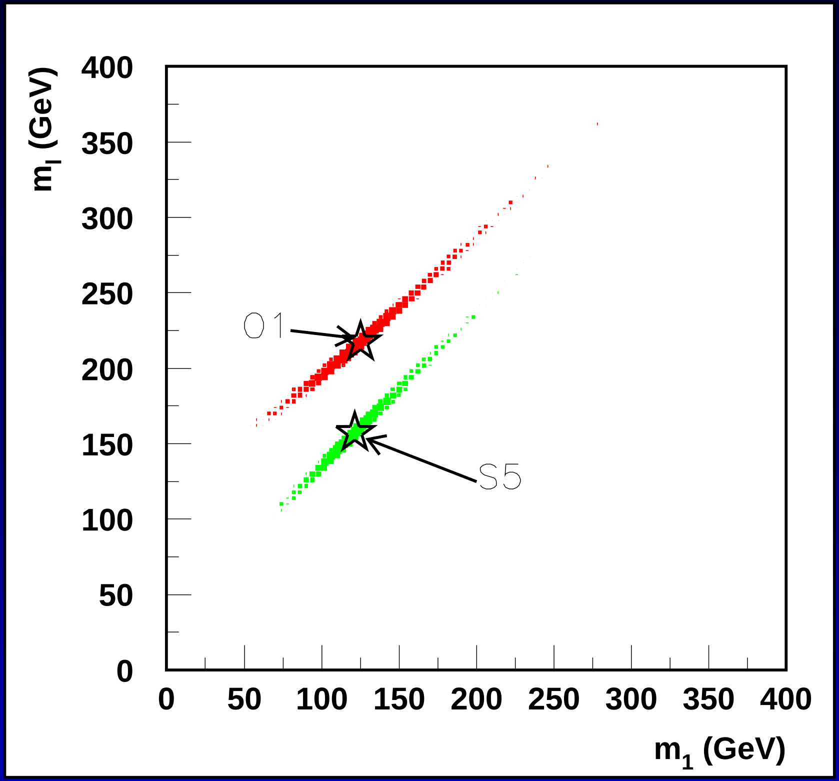
$$\Rightarrow \underline{p}_{\chi^0_2} + \underline{p}_{l^+} = \underline{0}, \qquad \underline{p}_{l^-} + \underline{p}_{\chi^0_1} = \underline{0}.$$

Energy conservation: $\sqrt{{m_{\chi^0_2}}^2 + |\underline{p}_{l^+}|^2} = m_{\tilde{l}} + |\underline{p}_{l^+}|$,

$$\Rightarrow |\underline{p}_{l^+}| = \frac{m^2_{\chi^0_2} - m^2_{\tilde{l}}}{2 m_{\tilde{l}}}. \text{ Similarly } |\underline{p}_{l^-}| = \frac{m^2_{\tilde{l}} - m^2_{\chi^0_1}}{2 m_{\tilde{l}}}.$$

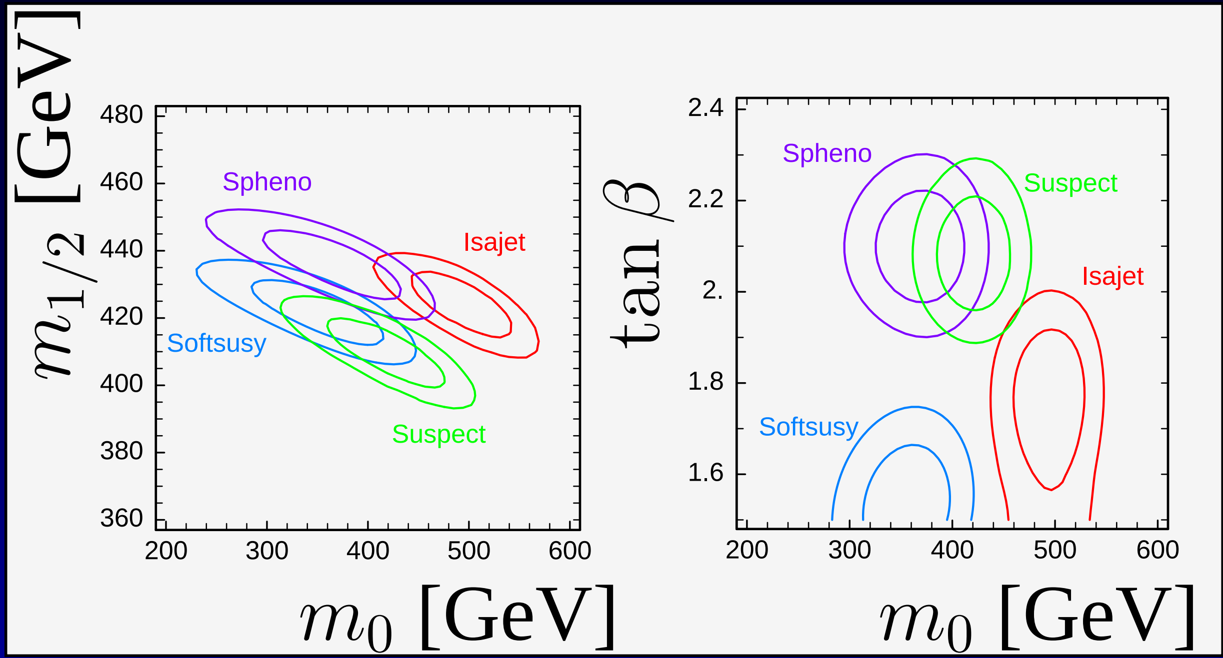
# Edge to Mass Measurements

| | width S5 | width O1 |
|---|---|---|
| $\chi_1^0$ | 17 | 22 |
| $\tilde{l}_R$ | 17 | 20 |
| $\chi_2^0$ | 17 | 20 |
| $\tilde{q}$ | 22 | 20 |




*Mass differences well constrained, but overall mass scale not so well constrained by LHC*

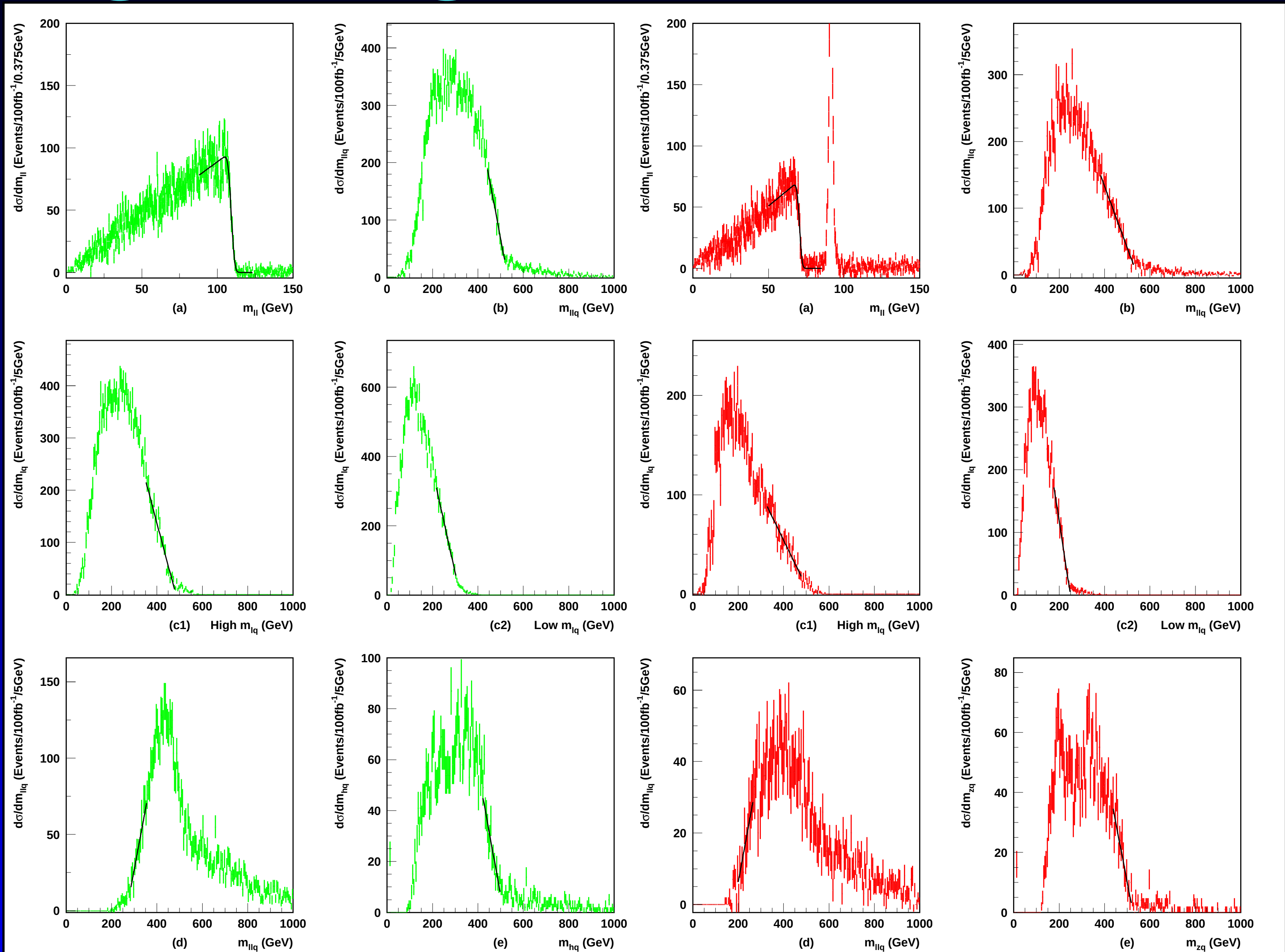
# Fitting to SUSY Breaking Model




- Experimenters pick a SUSY breaking point
- They derive observables and errors after detector simulation
- We fit[a] this "data" with our codes

[a] BCA, S Kraml, W Porod, JHEP 0303 (2003) 016

# Edge Fitting at S5 and O1

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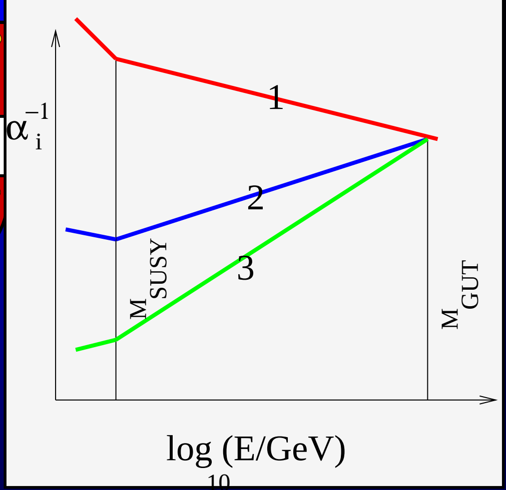
# Edge Positions

| endpoint | S5 fit | O1 fit |
|---|---|---|
| $m_{ll}$ | 109.10±0.13 | 70.47±0.15 |
| $m_{llq}$ edge | 532.1±3.2 | 544.1±4.0 |
| $lq$ high | 483.5±1.8 | 515.8±7.0 |
| $lq$ low | 321.5±2.3 | 249.8±1.5 |
| $llq$ thresh | 266.0±6.4 | 182.2±13.5 |

*Best case lepton mass measurements can be as accurate as 1 per mille, but jets are a few percent*

# SOFTSUSY




Get $g_i(M_Z)$, $h_{t,b,\tau}(M_Z)$.

$\downarrow$

Run to $M_S$.

$\downarrow$

REWSB, iterative solution of $\mu$

$\downarrow$

$M_X$. Soft SUSY breaking BC.

$\downarrow$

Run to $M_S$. Calculate[a] sparticle pole masses.

$\downarrow$

Run to $M_Z$

---

[a]BCA, Comp. Phys. Comm. 143 (2002) 305.

# Other Observables

Often more complicated, eg $m_{llq}$ edge:

$$\max\left[\frac{(m_{\tilde{q}}^2 - m_{\chi_2^0}^2)(m_{\chi_2^0}^2 - m_{\chi_1^0}^2)}{m_{\chi_2^0}^2}, \frac{(m_{\tilde{q}}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{\chi_1^0}^2)}{m_{\tilde{l}}^2}, \right.$$

$$\left. \frac{(m_{\tilde{q}} m_{\tilde{l}} - m_{\chi_2^0} m_{\chi_1^0})(m_{\chi_2^0}^2 - m_{\tilde{l}}^2)}{m_{\chi_2^0} m_{\tilde{l}}}\right]$$

Also $m_{lq}^{high}$, $m_{lq}^{low}$, $llq$ *threshold* [a], $M_{T_2}^2(m) =$

$$\min_{\not{p}_1 + \not{p}_2 = \not{p}_T}\left[\max\left\{m_T^2(p_T^{l_1}, \not{p}_1, m), m_T^2(p_T^{l_2}, \not{p}_2, m)\right\}\right],$$

$\max[M_{T_2}(m_{\chi_1^0})] = m_{\tilde{l}}]$ for dislepton production.

# Same order prior

We wish to encode the idea that "SUSY breaking terms should be of the same order of magnitude"

$$p(m_0|M_S) = \frac{1}{\sqrt{2\pi w^2} m_0} \exp\left(-\frac{1}{2w^2}\log^2(\frac{m_0}{M_S})\right),$$

$$p(A_0|M_S) = \frac{1}{\sqrt{2\pi e^{2w}} M_S} \exp\left(-\frac{1}{2e^{2w}}\frac{A_0^2}{M_S^2}\right),$$

We don't know SUSY breaking scale $M_S$:

$$p(m_0, M_{1/2}, A_0, \mu, B) = \int_0^\infty dM_S \, p(m_0, M_{1/2}, A_0, \mu, B|M_S) \, p(M_S)$$

# Naturalness

$$M_Z^2 = \tan 2\beta \left[m_{H_2}^2 \tan\beta - m_{H_1}^2 \cot\beta\right] - 2\mu^2$$

Cancellation implied by sparticle mass bounds. Quantify by

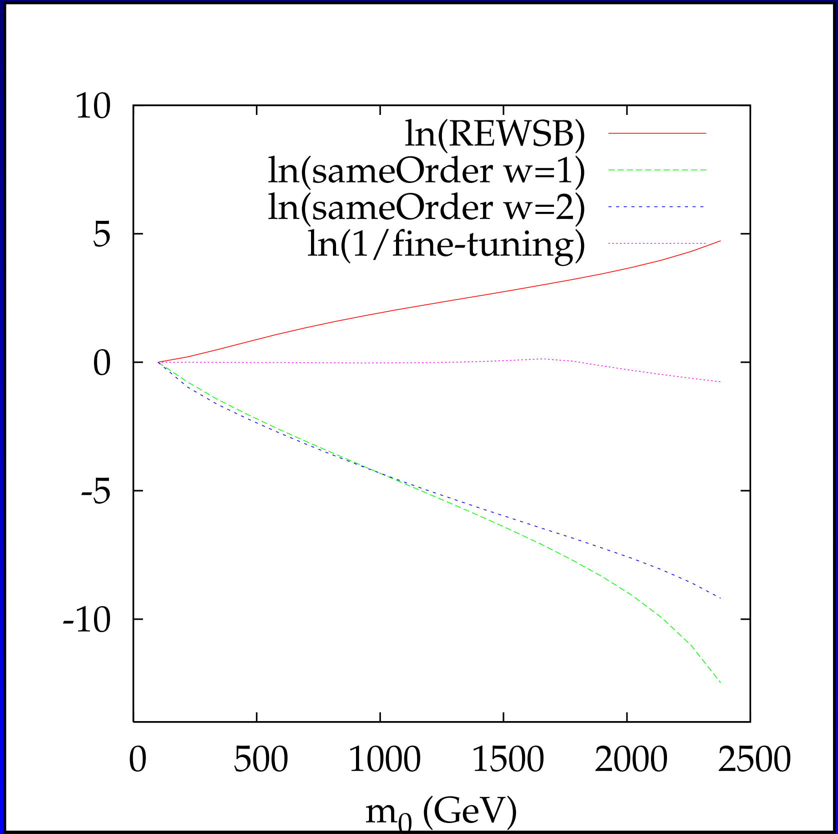
$$f = \max_x \{ \| \frac{d \ln M_Z^2}{d \ln x} \| \}$$

where $x \in \{M_{1/2}, m_0, A_0, \mu, B\}$. We will choose the prior to be $1/f$.

# Fine Tuning

Compare with usual definition of *fine-tuning*:

$$f = \max_p \frac{d \ln M_Z}{d \ln p}$$




SPS1a Point

$M_{1/2} = 250$ GeV

$\tan\beta = 10$ GeV

$A_0 = -100$ GeV