Asymptotic energy dependence of hadronic total cross sections and lattice QCD

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Edinburgh, 12th December 2012

Based on work in collaboration with
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Outline

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Soft High-Energy Scattering in Strong Interactions

Soft high-energy hadron-hadron scattering ($s \to \infty\,,\ |t| \le 1\mathrm{GeV}^2$) and high-energy behaviour of total cross sections: one of the oldest unsolved problems of strong interactions

Related through the optical theorem: $\sigma_{\rm tot} \underset{s\to\infty}{\simeq} \frac{1}{s}\,\mathrm{Im}\,\mathcal{M}(s,t=0)$

$\sim$1955–1973:

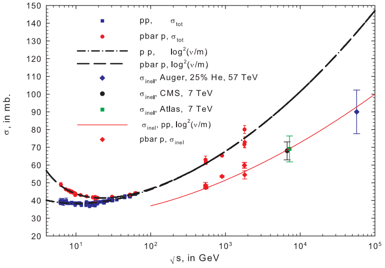
- Fundamental theory not known, expected not to be a QFT
- Phenomenological models, general properties of the S–matrix (Pomeranchuk, Regge, Yang)
- Constant cross section $\sim$ Regge trajectory with intercept 1 (*Pomeron*)

↙ 1973 ↘

Discovery of QCD
(a QFT, after all. . .)

Rising total cross sections
[Amaldi *et al.* (1973), Amendolia *et al.* (1973)]

Rising Total Cross Sections



(figure taken from [Block, Halzen (2011)])

Experimental data support

$$\sigma_{tot}(s) \sim B \log^2 s$$

with **universal** $B \simeq 0.3\,\mathrm{mb}$, independent of the colliding hadrons [up to $\sqrt{s} = 7\,\mathrm{TeV}$ (TOTEM)]
[COMPETE coll. (2002)]

Consistent with Froissart bound [Froissart (1961)] (unitarity + mass gap)

$$\sigma_{tot}^{(hh)}(s) \leq \frac{\pi}{m_\pi^2} \log^2 \left(\frac{s}{s_0} \right)$$

Arguments supporting universality of B (eikonal unitarisation, Color Glass Condensates) but situation still unsettled

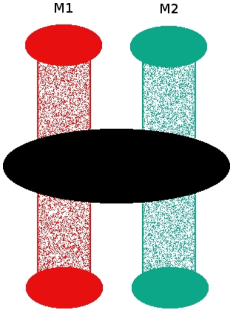
Soft High-Energy Scattering and QCD

1973–2012: QCD has acquired the status of fundamental theory of strong interactions, and it should explain the rise of total cross sections

- Early attempts in PT: Low-Nussinov Pomeron (two-gluon exchange), BFKL Pomeron (*hard* Pomeron)
- Two different energy scales, $\sqrt{s} \to \infty$ and $\sqrt{|t|} \lesssim 1\mathrm{GeV}$: PT not reliable $\to$ a nonperturbative approach is needed [Nachtmann (1991)]
- Need for analytic continuation into Euclidean space to apply usual NP techniques [Meggiolaro (1997)]

Outline

Nachtmann's Nonperturbative Approach



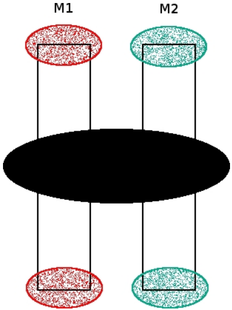
1. Partonic description of hadrons over a small time–window ($\sim 2\text{fm}$)
2. Partons do not split or annihilate, treated as in/out states of a scattering process
3. Lightlike trajectories approximately unchanged in the process
4. Hadronic amplitude after folding with hadronic wave function

Partonic scattering amplitudes from the correlation function of infinite lightlike Wilson lines [Nachtmann (1991)]

Partonic amplitudes suffer from IR divergences $\rightarrow$ hadronic amplitudes: mesons as wave packets of transverse colourless dipoles [Dosch *et al.* (1996)]

Extends to baryon-baryon scattering adopting a quark-diquark description

Nachtmann's Nonperturbative Approach



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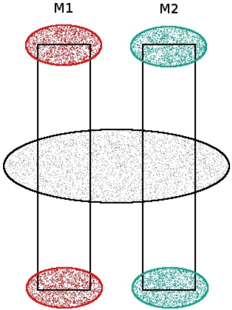

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Meson–Meson (Dipole–Dipole) Scattering

Elastic meson-meson from dipole-dipole scattering [Dosch *et al.* (1996)]

$$\mathcal{M}_{(12\to 12)}(s,t) = \langle\langle \mathcal{M}^{(dd)}(s,t;\vec{R}_{1\perp},\vec{R}_{2\perp})\rangle\rangle$$

$$\langle\langle f\rangle\rangle = \int d^2\vec{R}_{1\perp}|\psi_1(\vec{R}_{1\perp})|^2 \int d^2\vec{R}_{2\perp}|\psi_2(\vec{R}_{2\perp})|^2 f(\vec{R}_{1\perp},\vec{R}_{2\perp})$$

$$\mathcal{M}^{(dd)}(s,t;\vec{R}_{1\perp},\vec{R}_{2\perp}) = \lim_{\chi\to\infty} -i\ 2s\int d^2\vec{b}_\perp e^{i\vec{q}_\perp\cdot\vec{b}_\perp}\mathcal{C}_M(\chi;\vec{b}_\perp,\vec{R}_{1\perp},\vec{R}_{2\perp})$$

$$\chi \underset{s\to\infty}{\simeq} \log\frac{s}{m^2},\ t = -\vec{q}_\perp^{\,2}$$

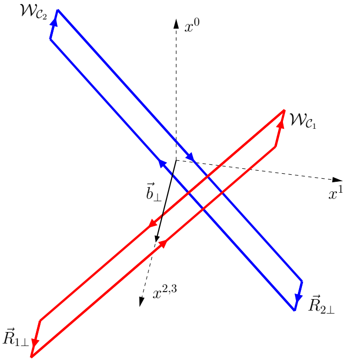
Wilson-loop correlation function

$$\mathcal{G}_M(\chi;T;\vec{b}_\perp,\vec{R}_{1\perp},\vec{R}_{2\perp}) \equiv \frac{\langle\mathcal{W}_{\mathcal{C}_1}\mathcal{W}_{\mathcal{C}_2}\rangle}{\langle\mathcal{W}_{\mathcal{C}_1}\rangle\langle\mathcal{W}_{\mathcal{C}_2}\rangle} - 1\,, \qquad \mathcal{C}_M \equiv \lim_{T\to\infty}\mathcal{G}_M$$

Finite hyperbolic angle χ and length $2T$ as IR regularisation [Verlinde, Verlinde (1993)]

Physical amplitude free of IR divergences → finite limit for $T\to\infty$ expected

Minkowskian Wilson Loops



$$u_1 \cdot u_2 = \cosh\chi$$

$$\chi \underset{s\to\infty}{\simeq} \log\frac{s}{m^2}$$

$$\mathcal{G}_M \equiv \frac{\langle \mathcal{W}_{\mathcal{C}_1}\mathcal{W}_{\mathcal{C}_2}\rangle}{\langle \mathcal{W}_{\mathcal{C}_1}\rangle\langle \mathcal{W}_{\mathcal{C}_2}\rangle} - 1$$

- Longitudinal sides $\Rightarrow q-\bar{q}$ trajectories ($\tau \in [-T, T]$)

$$\mathcal{C}_1 \to X^{(\pm 1)}(\tau) = b + u_1\tau \pm \frac{R_1}{2}$$

$$\mathcal{C}_2 \to X^{(\pm 2)}(\tau) = u_2\tau \pm \frac{R_2}{2}$$

$$u_{1,2} = \left(\cosh\frac{\chi}{2}, \pm\sinh\frac{\chi}{2}, \vec{0}_\perp\right)$$

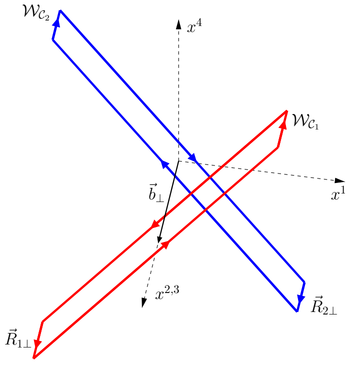
- Closed at $\tau = \pm T$ by straight "links" in the transverse plane to ensure gauge invariance

$$R_i = (0, 0, \vec{R}_{i\perp}) \quad b = (0, 0, \vec{b}_\perp)$$

Euclidean Correlation Functions

Nonperturbative techniques available in Euclidean space ⇒
Euclidean formulation of soft high-energy scattering

Minkowskian correlators reconstructed from Euclidean correlators



$$u_1 \cdot u_2 = \cos\theta$$

$$\mathcal{G}_E = \frac{\langle \mathcal{W}_{\mathcal{C}_1} \mathcal{W}_{\mathcal{C}_2} \rangle}{\langle \mathcal{W}_{\mathcal{C}_1} \rangle \langle \mathcal{W}_{\mathcal{C}_2} \rangle} - 1$$

$$\mathcal{G}_E = \mathcal{G}_E(\theta; T; \vec{b}_\perp, \vec{R}_{1\perp}, \vec{R}_{2\perp})$$

$$\mathcal{C}_E \equiv \lim_{T\to\infty} \mathcal{G}_E$$

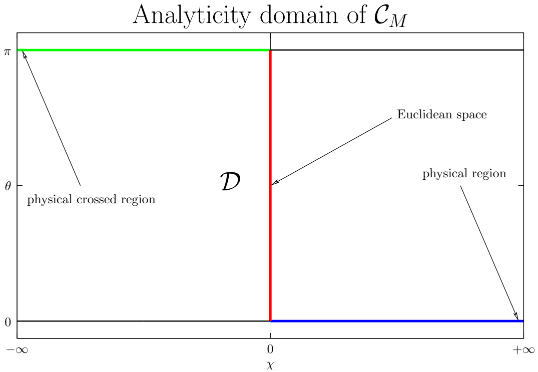
$$\mathcal{C}_1 : X^{(\pm 1)}(\tau) = u_1 \tau \pm \frac{R_1}{2} + b$$

$$\mathcal{C}_2 : X^{(\pm 2)}(\tau) = u_2 \tau \pm \frac{R_2}{2}$$

$$u_{1,2} = \left(\pm \sin\frac{\theta}{2}, \vec{0}_\perp, \cos\frac{\theta}{2} \right)$$

$$R_i = (0, \vec{R}_{i\perp}, 0) \quad b = (0, \vec{b}_\perp, 0)$$

Analytic Continuation to Euclidean Space



Analytic continuation relations [Meggiolaro (2005), MG, Meggiolaro (2009)]

$$\mathcal{G}_M(\chi; T) = \mathcal{G}_E(\theta \to -i\chi; T \to iT)\,, \quad \mathcal{C}_M(\chi) = \mathcal{C}_E(\theta \to -i\chi)$$

AC + Euclidean symmetries ⇒ crossing relations [MG, Meggiolaro (2006)]

$$\mathcal{G}_M(i\pi - \chi; \vec{R}_{1\perp}, \vec{R}_{2\perp}) = \mathcal{G}_M(\chi; \vec{R}_{1\perp}, -\vec{R}_{2\perp}) = \mathcal{G}_M(\chi; -\vec{R}_{1\perp}, \vec{R}_{2\perp})$$

Outline

Wilson Loop Correlator on the Lattice

Euclidean formulation opens the way to NP techniques:

- Stochastic Vacuum Model [Berger, Nachtmann (1999), Shoshi *et al.* (2003)]
- Instanton Liquid Model [Shuryak, Zahed (2000), MG, Meggiolaro (2010)]
- AdS/CFT Correspondence [Janik, Peschanski (2000a,b), MG, Peschanski (2010)]
- Lattice Gauge Theory [MG, Meggiolaro (2008), MG, Meggiolaro (2010)]

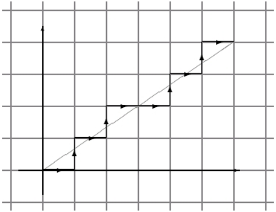
Lattice calculation of the correlator gives first-principles "true" prediction of QCD (within errors) $\Rightarrow$ analytic NP calculations have to be compared to lattice results, in order to test the goodness of the approximations involved

- Compare data with numerical predictions of the various models
- Fit lattice data with model functions

Lattice data can also be fitted with more general functions, but care is needed. . .

Loop Construction

Rotation invariance breaking → approximation for tilted Wilson loops



Bresenham prescription: lattice path that minimizes the distance from the continuum path

$\mathcal{W}_L(\vec{l}_{\|}, \vec{r}_{\perp}; n)$: center in n, sides l ($\|$ plane) and r ($\perp$ plane)

Lattice Wilson-loop correlators

$$\mathcal{G}_L(\vec{l}_{1\|}, \vec{l}_{2\|}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) = \frac{\langle \mathcal{W}_L(\vec{l}_{1\|}, \vec{r}_{1\perp}; d)\mathcal{W}_L(\vec{l}_{2\|}, \vec{r}_{2\perp}; 0)\rangle}{\langle \mathcal{W}_L(\vec{l}_{1\|}, \vec{r}_{1\perp}; d)\rangle\langle \mathcal{W}_L(\vec{l}_{2\|}, \vec{r}_{2\perp}; 0)\rangle} - 1$$

$d = (0, 0, \vec{d}_{\perp})$: transverse distance

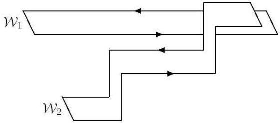
Rotation invariance restored in the continuum limit

$$\mathcal{G}_L(\vec{l}_{1\|}, \vec{l}_{2\|}; \vec{d}_{\perp}, \vec{r}_{1\perp}, \vec{r}_{2\perp}) \underset{a\to 0}{\simeq} \mathcal{G}_E(\theta; aL_1, aL_2; a\vec{d}_{\perp}, a\vec{r}_{1\perp}, a\vec{r}_{2\perp}) + \mathcal{O}(a)$$

$2L_i = |\vec{l}_{i\|}|$: length

Loop Construction

Rotation invariance breaking → approximation for tilted Wilson loops



Bresenham prescription: lattice path that minimizes the distance from the continuum path

$\mathcal{W}_L(\vec{l}_\parallel, \vec{r}_\perp; n)$: center in n, sides l ($\parallel$ plane) and r ($\perp$ plane)

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$$\mathcal{G}_L(\vec{l}_{1\parallel}, \vec{l}_{2\parallel}; \vec{d}_\perp, \vec{r}_{1\perp}, \vec{r}_{2\perp}) = \frac{\langle \mathcal{W}_L(\vec{l}_{1\parallel}, \vec{r}_{1\perp}; d)\mathcal{W}_L(\vec{l}_{2\parallel}, \vec{r}_{2\perp}; 0)\rangle}{\langle \mathcal{W}_L(\vec{l}_{1\parallel}, \vec{r}_{1\perp}; d)\rangle\langle \mathcal{W}_L(\vec{l}_{2\parallel}, \vec{r}_{2\perp}; 0)\rangle} - 1$$

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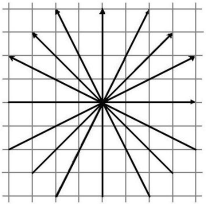
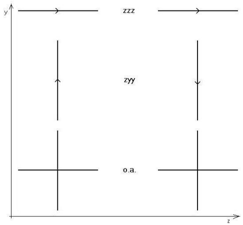
Setup

- longitudinal plane

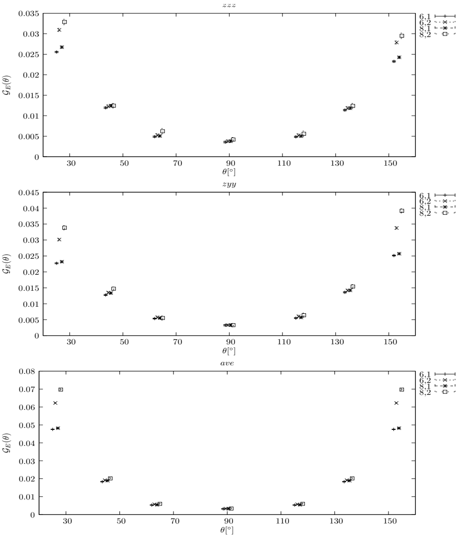


- transverse plane



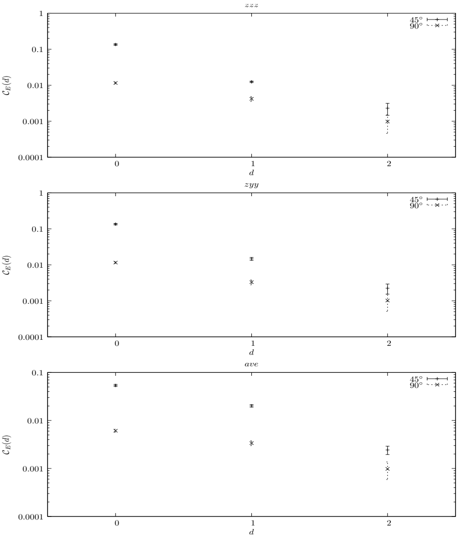
- Wilson action for $SU(3)$ pure gauge theory (*quenched* QCD)
  - 16^4 hypercubic lattice, periodic bc
  - $\beta = 6.0 \rightarrow a \simeq 0.1\,\mathrm{fm}$
  - 30000 measurements
- Parameters of Wilson loop correlators
  - angles: $\cot\theta = 0, \pm 1/2, \pm 1, \pm 2$
  - transverse size $= 1a$
  - transverse distance $= 0, 1, 2a$
- Configurations in the transverse plane:
  - "zzz": $\vec{d}_\perp \parallel \vec{r}_{1\perp} \parallel \vec{r}_{2\perp}$
  - "zyy": $\vec{d}_\perp \perp \vec{r}_{1\perp} \parallel \vec{r}_{2\perp}$
  - "ave": average over orientations (relevant to meson-meson scattering)
- Analytic calculations for $T \rightarrow \infty$ $\rightarrow$ longest available loops ($L \simeq 8$)

Angular Dependence



- $\mathcal{G}_L$ against $\theta[°]$ for various L_1, L_2 and for the various configurations at $d = 1$
- Stable vs. loop lengths, stabilisation slower near $\theta = 0, \pi$ due to the relation with the static $d - d$ potential [Appelquist, Fischler (1978)]
- $\mathcal{G}_L^{ave}$ symmetric with respect to $\pi/2 \to$ sensitive only to C–even contributions
- C-odd component (Odderon) in "zzz" / "zyy" (possibly relevant to baryon-baryon scattering)

Distance Dependence



- $\mathcal{C}_L$ against d (lattice units) for $\theta = 45^\circ, 90^\circ$ for the various configurations (logarithmic scale)
- $\mathcal{C}_L$: correlator for the largest loops available ($\approx \lim_{L_{1,2}\to\infty} \mathcal{G}_L$)
- Rapid (exponential) decrease with distance
- Errors become large at $d = 2$ $\to$ "brute force" approach not viable at larger distances

Outline

Stochastic Vacuum Model

- Basic assumption: contributions of high and low frequency modes to QCD (Euclidean) functional integral accounted for separately
  - perturbation theory for the high frequency modes
  - low frequency ones in terms of a Gaussian stochastic process for the gauge–invariant two–point correlation function of the field strength

[Dosch (1987)]

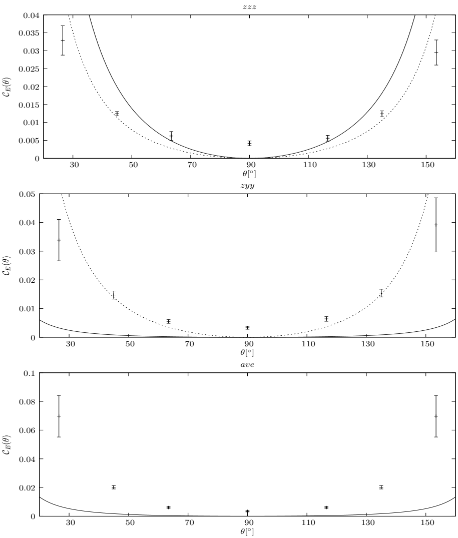
- Wilson-loop correlator in the SVM

$$\mathcal{C}_E^{\mathrm{SVM}}(\theta) = \frac{2}{3} e^{-\frac{1}{3}\cot\theta\, K_{\mathrm{SVM}}} + \frac{1}{3} e^{\frac{2}{3}\cot\theta\, K_{\mathrm{SVM}}} - 1$$

[Berger, Nachtmann (1999), Shoshi *et al.* (2003)]

- K_{SVM} function of the impact parameter and of the size of dipoles, dependent on the details of the model

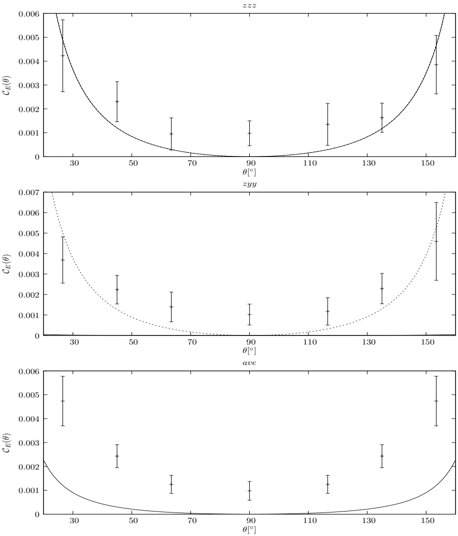
SVM vs. Lattice



$$\mathcal{C}_E^{\rm SVM}(\theta) = \frac{2}{3} e^{-\frac{1}{3}\cot\theta K_{\rm SVM}} + \frac{1}{3} e^{\frac{2}{3}\cot\theta K_{\rm SVM}} - 1$$

- SVM prediction at $d = 1$ (solid)
- $K_{\rm SVM}$ from [Shoshi *et al.* (2003)]
- "Reasonable" for *zzz* but not for *zyy* and *ave*
- Fit with SVM functional form (dashed) for the *zzz* and *zyy* cases

SVM vs. Lattice

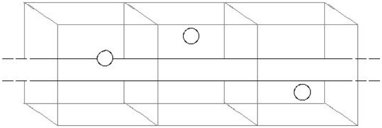


$$\mathcal{C}_E^{\rm SVM}(\theta) = \frac{2}{3} e^{-\frac{1}{3}\cot\theta K_{\rm SVM}} + \frac{1}{3} e^{\frac{2}{3}\cot\theta K_{\rm SVM}} - 1$$

- SVM prediction at $d = 2$ (solid)
- $K_{\rm SVM}$ from [Shoshi *et al.* (2003)]
- "Reasonable" for *zzz* but not for *zyy* and *ave*
- Fit with SVM functional form (dashed) for the *zzz* and *zyy* cases

Outline

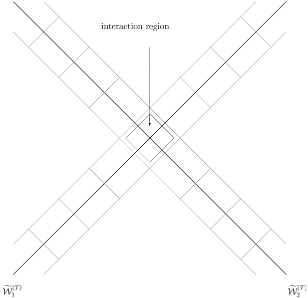
Wilson Loop in the Instanton Liquid



- Instanton Liquid Model [Shuryak (1982)]: QCD vacuum described as a liquid of instantons and anti–instantons with equal densities $n_{\mathrm{I}} = n_{\bar{\mathrm{I}}} = n/2$ and with fixed radius ρ
- Diluteness of the medium + short-range nature of instanton effects $\to$ subdivide the loop in sections, each affected by a single instanton
- Instanton effects on Wilson-loop correlators evaluated in [Shuryak, Zahed (2000)], but the calculation contains a divergent integral

Correlation Function of Wilson Loops at an Angle

- Finite result obtained in [MG, Meggiolaro (2010)], properly taking into account the non-local nature of the Wilson loop
- One–instanton approximation: the two loops interact through a single instanton (reasonable for θ not too near $0, \pi$)



$$\mathcal{C}_E^{(\mathrm{ILM})}(\theta; \vec{b}_\perp; \vec{R}_{1\perp}, \vec{R}_{2\perp}) = n \left(\frac{2}{N_c}\right)^2 \frac{1}{\sin\theta} F(\vec{b}_\perp; \vec{R}_{1\perp}, \vec{R}_{2\perp})$$

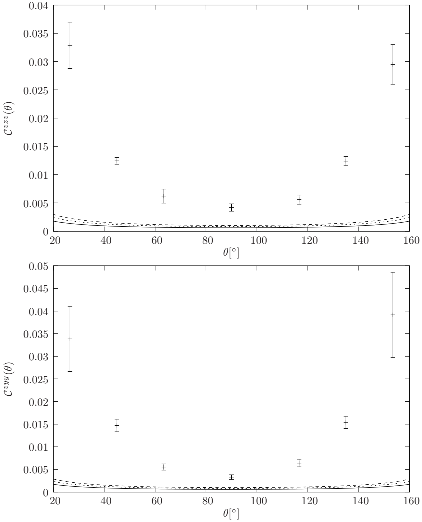
$$F(\vec{b}_\perp; \vec{R}_{1\perp}, \vec{R}_{2\perp}) = \int d^4x\, \Delta_1(x)\Delta_2(x)$$

$$\Delta_i(x) = 1 - \cos\alpha_{i+} \cos\alpha_{i-} - \hat{n}_{i+} \cdot \hat{n}_{i-} \sin\alpha_{i+} \sin\alpha_{i-}$$

$$\alpha_{i\pm} = \pi \| n_{i\pm} \| (\| n_{i\pm} \|^2 + \rho^2)^{-\frac{1}{2}}$$

$$n^a_{1\pm} = \eta_{a0\nu}(b \pm \tfrac{R_1}{2} - x)_\nu , \quad n^a_{2\pm} = \eta_{a1\nu}(\pm \tfrac{R_2}{2} - x)_\nu$$

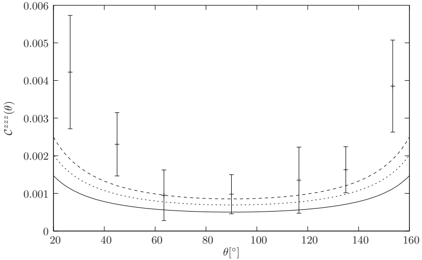
ILM vs. Lattice

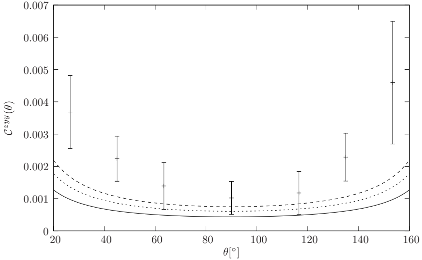


$$\mathcal{C}_E^{(\mathrm{ILM})} = n_q \left(\frac{2}{N_c}\right)^2 \frac{1}{\sin\theta} F$$

- ILM prediction at $d = 1$
- Model parameters measured on the lattice for quenched configurations [Chu *et al.* (1994)]: $n_q = 1.33\,\mathrm{fm}^{-4}$ (dotted), $n_q = 1.64\,\mathrm{fm}^{-4}$ (dashed), with $\rho_q = 0.35\,\mathrm{fm}$
- Phenomenological values $n = 1\,\mathrm{fm}^{-4}$, $\rho = 1/3\,\mathrm{fm}$ (solid)
- Overestimation of the correlation length (confirmed by *dd* potential calculations [MG, Meggiolaro (2010)])

ILM vs. Lattice





$$\mathcal{C}_E^{(\mathrm{ILM})} = n_q \left(\frac{2}{N_c}\right)^2 \frac{1}{\sin\theta} F$$

- ILM prediction at $d = 2$
- Model parameters measured on the lattice for quenched configurations [Chu *et al.* (1994)]: $n_q = 1.33\,\mathrm{fm}^{-4}$ (dotted), $n_q = 1.64\,\mathrm{fm}^{-4}$ (dashed), with $\rho_q = 0.35\,\mathrm{fm}$
- Phenomenological values $n = 1\,\mathrm{fm}^{-4}$, $\rho = 1/3\,\mathrm{fm}$ (solid)
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Outline

Lattice Results and Rising Total Cross Sections

Are the analytic NP calculations compatible with the lattice results?

- Comparison of SVM and ILM to lattice data is not satisfactory (both direct numerical comparison and fit with model functions)
- SVM, ILM do not lead to rising total cross sections: $\sigma_{tot} \underset{s\to\infty}{\to} \mathrm{const.}$
- "Empirical" combination of perturbative and ILM expression into

$$\mathcal{C}_E^{(\mathrm{ILMp})} = K_{\mathrm{ILMp}} \frac{1}{\sin\theta} + K'_{\mathrm{ILMp}} (\cot\theta)^2$$

  gives largely improved best fits, but still does not give a rising σ_{tot}

Are the lattice results compatible with rising total cross sections?

- Fits to more general functions can be performed, but care is needed because of the analytic continuation
- Admissible fitting functions are constrained by physical requirements (unitarity, crossing symmetry,. . .)

Unitarity Constraint

We look for a parameterisation of the lattice data that

1. fits well the numerical results
2. satisfies unitarity (after analytic continuation)
3. leads to rising total cross section at high energy

Unitarity constraint rather restrictive: factorised $\mathcal{C}_M = v(\chi)w(\vec{b}_\perp)$ cannot satisfy 2-3, constraints on the fitting parameters...

$$\mathcal{M}_{(12\to 12)}(s,t) = -i\ 2s \int d^2\vec{b}_\perp e^{i\vec{q}_\perp\cdot\vec{b}_\perp} A(s,|\vec{b}_\perp|)\,,$$

$$A(s,|\vec{b}_\perp|) = \langle\!\langle \mathcal{C}_M(\chi;\vec{b}_\perp,\vec{R}_{1\perp},\vec{R}_{2\perp})\rangle\!\rangle$$

$$\langle\!\langle f\rangle\!\rangle = \int d^2\vec{R}_{1\perp}|\psi_1(\vec{R}_{1\perp})|^2 \int d^2\vec{R}_{2\perp}|\psi_2(\vec{R}_{2\perp})|^2 f(\vec{R}_{1\perp},\vec{R}_{2\perp})$$

Unitarity constraint: $|A(s,|\vec{b}_\perp|)+1| \le 1$

Sufficient condition: $|\mathcal{C}_M(\chi;\vec{b}_\perp,\vec{R}_{1\perp},\vec{R}_{2\perp})+1| \le 1 \qquad \forall \vec{b}_\perp,\vec{R}_{1\perp},\vec{R}_{2\perp}$

A Nontrivial Example: Onium Scattering in $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM: replace mesons with "onia", wave packets of colourless dipoles, and describe "onium-onium" scattering in terms of dipoles

Large N_c, strong coupling: AdS/CFT correspondence [Maldacena (1998)]

$\mathcal{C}_E$ at large $b = |\vec{b}_\perp|$ from a supergravity calculation [Janik, Peschanski (2000a)]

$$\mathcal{C}_E^{(\mathrm{AdS/CFT})} = \exp\left\{ [K_S + K_D] \frac{1}{\sin\theta} + K_B \cot\theta + K_G \frac{(\cos\theta)^2}{\sin\theta} \right\} - 1$$

$K_X = K_X(b) \sim$ exchange of supergravity field X between the string worldsheets

At large b, $K_G(b) \sim \frac{\bar{K}_G}{b^6}$; after $\theta \to -i\chi$, $\chi \to \infty$,

$$\mathcal{C}_M^{(\mathrm{AdS/CFT})} \sim \exp\left\{ \frac{i\bar{K}_G}{b^6} \frac{e^\chi}{2} \right\} - 1 \to \sigma_{tot} \propto s^{\frac{1}{3}}$$

Rising total cross section for "onium-onium" scattering [MG, Peschanski (2010)]

Outline

Exponential Form of the Correlator

Assumption:

$$\mathcal{C}_E = \exp\{K_E\} - 1$$

with K_E real (as $\mathcal{C}_E$ is real)

Well justified assumption:

- large-N_c, $\mathcal{C}_E \sim \mathcal{O}(1/N_c^2)$ so $\mathcal{C}_E + 1 \geq 0$ certainly true at large-N_c
- satisfied by all the known models (SVM, ILM, AdS/CFT)
- $\mathcal{C}_E + 1 \to 1$ at large $|\vec{b}_\perp|$, so certainly true at large impact parameter
- confirmed by lattice data

Minkowskian correlator after analytic continuation

$$\mathcal{C}_M = \exp\{K_M\} - 1$$

Unitarity condition

$$\mathrm{Re}\, K_M \leq 0 \quad \forall \vec{b}_\perp, \vec{R}_{1\perp}, \vec{R}_{2\perp}$$

Large Distance Behaviour of K_E

In a confining theory such as QCD, one expects at large $|\vec{b}_\perp|$

$$\mathcal{C}_E \sim e^{-\mu|\vec{b}_\perp|}$$

for some mass scale μ. Various "natural" possibilities:

- mass of the lightest glueball ($M_G \simeq 1.5\mathrm{GeV}$)
- inverse vacuum correlation length λ_{vac} (e.g., $\mu = 2/\lambda_{\mathrm{vac}}$ in the SVM), measured on the lattice: $\lambda_{\mathrm{vac}}^{\mathrm{quenched}} \simeq 0.22\mathrm{fm}$, $\lambda_{\mathrm{vac}}^{\mathrm{full}} \simeq 0.30\mathrm{fm}$

  [Di Giacomo, Panagopoulos (1992)]
- . . .

Therefore, one expects

$$K_E \sim e^{-\mu|\vec{b}_\perp|}$$

More generally, one can expect a sum of such terms, with different mass scales, and possibly power-like prefactors

How a Froissart-like Total Cross Section Can Be Obtained

1. Assume that after AC the leading term of the Minkowskian correlator is

$$\mathcal{C}_M = \exp\{K_M\} - 1 \sim \exp\left(i\,\beta\, e^{\eta}\, e^{-\mu|\vec{b}_\perp|}\right) - 1 \qquad (\star)$$

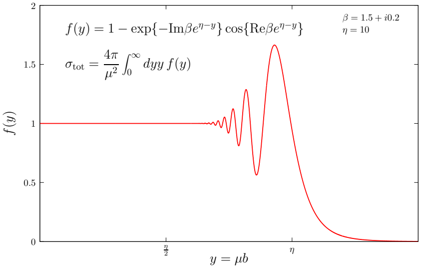
with $\beta = \beta(\vec{R}_{1\perp}, \vec{R}_{2\perp})$ and $\eta(\chi)$ *real* function, $\eta(\chi) \to +\infty$ for $\chi \to +\infty$

2. Optical theorem

$$\sigma_{\rm tot}^{(hh)} \sim \frac{4\pi}{\mu^2}{\rm Re}\langle\langle J(\eta,\beta)\rangle\rangle$$

$$J(\eta,\beta) \equiv \int_0^\infty dy\, y[1 - e^{i\beta e^{\eta - y}}]$$

Expect $J \simeq \int_0^\eta dy\, y \propto \eta^2$



Expression $(\star)$ holds only for large $|\vec{b}_\perp| \gtrsim b_0$, but can be extended to $|\vec{b}_\perp| = 0$, the difference being a constant in χ due to the unitarity bound

How a Froissart-like Total Cross Section Can Be Obtained

3. Setting $z = -i\beta e^{\eta}$

$$\frac{\partial J(\eta,\beta)}{\partial\eta} = -\sum_{n=1}^{\infty}\frac{(-z)^n}{n!n} = E_1(z) + \log(z) + \gamma\,, \quad \text{for } |\arg(z)| < \pi$$

$E_1(z)$: Schlömilch exponential integral, γ: Euler-Mascheroni constant
$E_1(z) \sim e^{-z}/z$ at large $|z|$, for $\mathrm{Re}\, z \geq 0 \Leftrightarrow \mathrm{Im}\beta \geq 0$

In the large-χ limit, we have

$$\sigma_{\rm tot}^{(hh)} \sim \frac{4\pi}{\mu^2}\langle\!\langle \frac{1}{2}\eta^2 + \eta(\log|\beta| + \gamma) + \ldots\rangle\!\rangle$$

4. Choosing $e^{\eta} = \chi^p e^{n\chi} \sim (\log s)^p s^n$

$$\sigma_{\rm tot}^{(hh)} \sim B\log^2 s\,, \qquad B = \frac{2\pi n^2}{\mu^2}$$

MG, Meggiolaro, Moretti, *JHEP* **1209** (2012) 031

Comments

Froissart-like behaviour

$$\sigma_{\rm tot}^{(hh)} \sim B \log^2 s \,, \quad B = \frac{2\pi n^2}{\mu^2} \qquad \text{if} \quad K_M \sim i\,\beta\,\chi^p e^{n\chi}\, e^{-\mu|\vec{b}_\perp|}$$

- B is **universal**, independent of the wave functions and of the masses of the mesons
- B is unaffected by the small-$|\vec{b}_\perp|$ behaviour

Extensions (work in progress):

- leading term in $\sigma_{\rm tot}^{(hh)}$ unaffected by $K_M \to |\vec{b}_\perp|^\alpha K_M$
- for $K_M = i \sum_k \beta_k \chi^{p_k} e^{n_k\chi}\, e^{-\mu_k|\vec{b}_\perp|}$, same behaviour with

$$B \to \max_k \frac{2\pi n_k^2}{\mu_k^2}$$

Outline

New Analysis of the Lattice Data

Try to fit the data with functional forms that satisfy unitarity after analytic continuation and that lead to rising total cross sections

- Use averaged correlators, that are "closer" to the meson-meson amplitude in impact-parameter space

$$\mathcal{C}^{ave}_{E,M} = < \mathcal{C}_{E,M} > = < \exp\{K_{E,M}\} > \; -1 = \exp\{K^{ave}_{E,M}\} - 1$$

$< \bullet > = \int d^2\hat{R}_{1\perp} \int d^2\hat{R}_{2\perp} \bullet$ is a positive and normalised measure

- K^{ave}_M and K^{ave}_E related by AC: $K^{ave}_M(\chi) = K^{ave}_E(\theta = -i\chi)$
- Unitarity constraint: $\mathrm{Re}\, K^{ave}_M \leq 0$
- By construction $\mathcal{C}^{ave}_E(\pi - \theta) = \mathcal{C}^{ave}_E(\theta)$: only C-even (Pomeron) contribution

Parameterisation I

First strategy: combine known QCD results and variations thereof

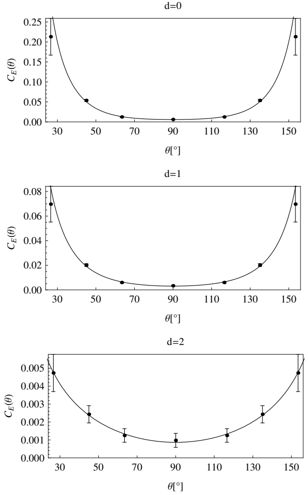
Example: exponentiate two-gluon exchange and one-instanton contribution, plus a term that can yield a rising $\sigma_{\rm tot}$

$$K_E = \frac{K_1}{\sin\theta} + K_2 \cot^2\theta + K_3 \cos\theta \cot\theta$$

$$K_M = i\frac{K_1}{\sinh\chi} + iK_3 \cosh\chi \coth\chi - K_2 \coth^2\chi$$

Unitarity condition: $K_2 \geq 0$ (satisfied within errors)

Leading term: $K_3 \cos\theta \cot\theta \to iK_3 \frac{e^\chi}{2}$



fit parameters

Parameterisation II

Second strategy: adapt to QCD results obtained in related models

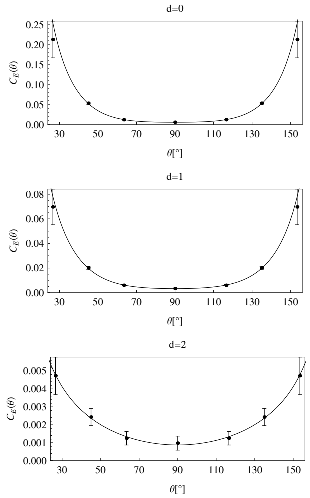
Example: AdS/CFT expression, plus $\theta \cot\theta$ term to make the expression crossing-even

$$K_E = \frac{K_1}{\sin\theta} + K_2(\tfrac{\pi}{2} - \theta)\cot\theta + K_3 \cos\theta \cot\theta$$

$$K_M = i\frac{K_1}{\sinh\chi} + iK_2 \tfrac{\pi}{2} \coth\chi + iK_3 \cosh\chi \coth\chi - \chi K_2 \coth\chi$$

Unitarity condition: $K_2 \geq 0$ (satisfied within errors)

Leading term: $K_3 \cos\theta \cot\theta \to iK_3 \frac{e^{\chi}}{2}$



fit parameters

Parameterisation III

Our best parameterisation:

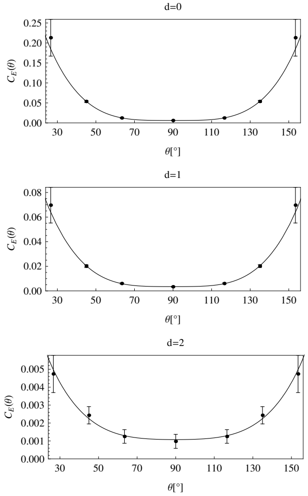
Exponentiate one-instanton contribution, plus a term that can yield a rising cross section

$$K_E = \frac{K_1}{\sin\theta} + K_2(\tfrac{\pi}{2} - \theta)^3 \cos\theta$$

$$K_M = i\frac{K_1}{\sinh\chi} + iK_2 \cosh\chi(\tfrac{3}{4}\pi^2\chi - \chi^3) + K_2 \cosh\chi(\tfrac{\pi^3}{8} - \tfrac{3}{2}\pi\chi^2)$$

Unitarity condition: $K_2 \geq 0$ (satisfied within errors)

Leading term: $K_2(\frac{\pi}{2} - \theta)^3 \cos\theta \to -iK_2\chi^3 \frac{e^\chi}{2}$



▸ fit parameters

Remarks

- **Universal** total cross section $\sigma_{\text{tot}}^{(hh)} \sim B \log^2 s$ in the three cases
- Estimate of B through a fit of the coefficient of the leading term with an exponential: fair agreement with experimental value $B_{\text{exp}} \simeq 0.3\text{mb}$

	μ (GeV)	$\lambda = \frac{1}{\mu}$ (fm)	$B = \frac{2\pi}{\mu^2}$ (mb)
Corr 1	4.64(2.38)	$0.042^{+0.045}_{-0.014}$	$0.113^{+0.364}_{-0.037}$
Corr 2	3.79(1.46)	$0.052^{+0.032}_{-0.014}$	$0.170^{+0.277}_{-0.081}$
Corr 3	3.18(98)	$0.062^{+0.028}_{-0.015}$	$0.245^{+0.263}_{-0.100}$

- Result applies directly to meson-meson scattering, experimental data mainly available for baryon-baryon and meson-baryon
  - prediction of universality for meson-meson scattering
  - Wilson-loop approach can be extended to baryons by adopting a quark-diquark picture: analysis carries over unchanged
- "Quenched" data, but $\sigma_{\text{tot}}^{(hh)}$ expected to depend on bosonic sector of QCD $\rightarrow$ result should not change much with dynamical fermions

Conclusions and Outlook

- We have provided a framework to investigate the issue of total cross sections on the lattice by means of numerical simulations
- We have found parameterisations of the lattice data that yield a good fit and at the same time a rising total cross section
- The comparison of our results with experiments is rather good, even if errors are quite large

Open issues:

- Inclusion of fermion effects
- Larger distances
- More angles, possibly by using anisotropic lattices
- Investigation of other theoretical models (e.g. holographic QCD)

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Fit Parameters

back 1 back 2 back 3

Corr 1	$d=0$	$d=1$	$d=2$
K_1	$5.85(42)\cdot 10^{-3}$	$3.07(37)\cdot 10^{-3}$	$8.7(3.1)\cdot 10^{-4}$
K_2	$9.60(98)\cdot 10^{-2}$	$2.44(49)\cdot 10^{-2}$	$-5.3(84.5)\cdot 10^{-5}$
K_3	$-7.8(1.3)\cdot 10^{-2}$	$-1.37(72)\cdot 10^{-2}$	$1.7(1.9)\cdot 10^{-3}$
$\chi^2_{\mathrm{d.o.f.}}$	2.81	1.25	0.05
Corr 2	$d=0$	$d=1$	$d=2$
K_1	$6.03(42)\cdot 10^{-3}$	$3.26(38)\cdot 10^{-3}$	$8.7(3.2)\cdot 10^{-4}$
K_2	$4.63(46)\cdot 10^{-1}$	$1.33(25)\cdot 10^{-1}$	$-1.2(54.2)\cdot 10^{-4}$
K_3	$-4.54(50)\cdot 10^{-1}$	$-1.26(28)\cdot 10^{-1}$	$1.7(6.7)\cdot 10^{-3}$
$\chi^2_{\mathrm{d.o.f.}}$	0.55	0.31	0.05
Corr 3	$d=0$	$d=1$	$d=2$
K_1	$6.02(36)\cdot 10^{-3}$	$3.46(29)\cdot 10^{-3}$	$1.07(20)\cdot 10^{-3}$
K_2	$1.29(5)\cdot 10^{-1}$	$4.47(27)\cdot 10^{-2}$	$2.11(73)\cdot 10^{-3}$
$\chi^2_{\mathrm{d.o.f.}}$	0.17	0.11	0.10

Table: Parameters (with their errors) for the Correlators 1, 2, and 3, obtained from best fits to the averaged lattice data, and the corresponding $\chi^2_{\mathrm{d.o.f.}}$, for the transverse distances $d = 0, 1, 2$.