Inflation is hot!

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IGM and T Yeomans (2011) arXiv:1102.2833

Hot or cold?

The radiation density redshifts away during inflation,

$$\dot{\rho}_\gamma + 4H\rho_\gamma = 0 \qquad\qquad \rho_\gamma \propto a^{-4}$$

If a rolling inflaton produces particles....

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2 \qquad\qquad \rho_\gamma \to \ \text{const.}$$

Radiation production is determined by a friction coefficient $\Gamma(\phi, T)$, where $T \sim \rho_\gamma^{1/4}$.

Inflaton equation

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V_{,\phi} = 0$$

Potential V friction coefficient Γ

- Warm inflation: $T_{\rm eff} >> H$
- Weak regime: $\Gamma \leq 3H$
- Strong regime: $\Gamma >> 3H$

IGM 1985, Berera 1995

Slow roll parameters

$$\varepsilon = \frac{m_p^2}{16\pi}\left(\frac{V_{,\phi}}{V}\right)^2, \eta = \frac{m_p^2}{8\pi}\left(\frac{V_{,\phi\phi}}{V}\right), \beta = \frac{m_p^2}{16\pi}\left(\frac{V_{,\phi}\Gamma_{,\phi}}{V\Gamma}\right)$$

Inflation occurs when $\eta, \varepsilon, \beta << 1 + \Gamma/3H$

The need for small couplings is reduced.

Temperature dependence can be important:

$$\delta = \frac{TV_{,\phi T}}{V_{,\phi}}, \quad c = \frac{T\Gamma_{,T}}{\Gamma} \qquad \delta << 1, \qquad c < 4$$

Hall, IGM and Berera 2004

How to make it work

Thermal corrections to the potential can easily violate the slow-roll conditions

- Decouple the radiation from the inflaton
- Use SUSY to reduce quantum corrections

$$\underset{\text{inflaton}}{\phi} \to \underset{\text{heavy boson}}{\chi} \to \underset{\text{light fermion}}{\tilde{y}}$$

Beware of early calculations of the friction coefficient

Berera and Ramos 2003

Thermal field primer

We use the closed-time path formalism,

$$Z[J_1, J_2] = \mathrm{tr}\left(\rho T^* \exp(-i\int J_2\hat{\phi}) T \exp(i\int J_1\hat{\phi})\right)$$

The Keldysh variables are

$$\phi_c = \frac{1}{2}(\phi_1 + \phi_2), \quad \phi_\Delta = \phi_1 - \phi_2 \quad (= 0 \text{ on shell})$$

The effective action becomes

$$\Gamma = -\int \mathcal{F}[\phi_c]\phi_\Delta + \frac{1}{2}\int \phi_\Delta i\Sigma_F \phi_\Delta + O(\phi_\Delta^3)$$

Langevin equation

A new way to derive the Langevin equation:

$$W[J^c] = -i \lim_{g\to 0} g \ln \int d\phi_c d\phi_\Delta e^{i(\Gamma + \int J^c \phi_c)/g}$$

W generates connected n-point functions.

Change variable, $\mathcal{F}[\phi_c] = g^{1/2}\xi$

$$W[J^c] = -i \lim_{g\to 0} g \ln \int d\xi D[\phi_c] e^{-\frac{1}{2}\int \xi \Sigma_F^{-1} \xi + i g^{-1} \int J^c \phi_c}$$

Where $D[\phi_c] = |\mathrm{det}\Sigma_F|^{-1/2} |\det \delta\mathcal{F}/\delta\phi_c|^{-1}$

We can generate n-point functions from an ensemble average (in the g=0 limit):

$$e^{iW[J^c]/g} = \langle e^{i\int J^c \phi_c/g} \rangle_\xi$$

with Langevin equation

$$\mathcal{F}[\phi_c] = g^{1/2}\xi$$

Note that we have an expansion parameter g,

$$\phi_c = \phi_0 + g^{1/2}\phi_1 + \ldots$$

Inflaton fluctuations

Density fluctuations originate from thermal fluctuations.

The inflaton can be described by a stochastic equation:

$$\ddot{\phi}(x,t) + (3H+\Gamma)\dot{\phi}(x,t) + \frac{\partial V}{\partial \phi} - \frac{1}{a^2}\nabla^2\phi(x,t) = g^{1/2}\xi(x,t)$$

The correlation function for the noise is

$$\langle \xi(x,t)\xi(x',t')\rangle = a^{-3}(H+\Gamma)T\delta(x-x')\delta(t-t')$$

Power spectrum $P_\zeta(k)$

Density fluctuations are described by $\zeta = H\delta\phi/\dot{\phi}$

We have an expansion, $\delta\phi = g^{1/2}\delta_1\phi + g\,\delta_2\phi + \ldots$

The first order gives:

$$\delta_1\ddot{\phi}(x,t) + \Gamma\delta_1\dot{\phi}(x,t) + \Gamma_{,T}\dot{\phi}\delta_1 T - \frac{1}{a^2}\nabla^2\delta_1\phi = \xi$$

This can be solved using Green functions and we get the power spectrum.

Hall, IGM and Berera 2004, Berera, IGM and Ramos 2009

Power spectrum

- Fluctuations `freeze out' before horizon crossing[1]
- If $\Gamma \equiv \Gamma(\phi)$, the amplitude $\delta\phi^2 = (H\Gamma)^{1/2}T$
- If $\Gamma \equiv \Gamma(\phi, T)$, the amplitude is larger[2].
- The spectral index $n_s \approx 1$

[1] Berera 2000 [2] C. Graham and IGM 2009

Non-gaussianity

Radiation fluid velocity v temperature T,

$$\delta_2\ddot{\phi}(x,t) + \Gamma\delta_2\dot{\phi}(x,t) - \frac{1}{a^2}\nabla^2\delta_2\phi(x,t) =$$

$$-\Gamma\mathbf{v}\cdot\nabla\delta_1\phi(x,t) - \Gamma_{,T}\,\delta_1\dot{\phi}\,\delta_1 T$$

The terms in red are not small in the slow-roll approximation

IGM and Xiong 2007, IGM and Yeomans 2011

Bispectrum

There are two different parts to the bispectrum:

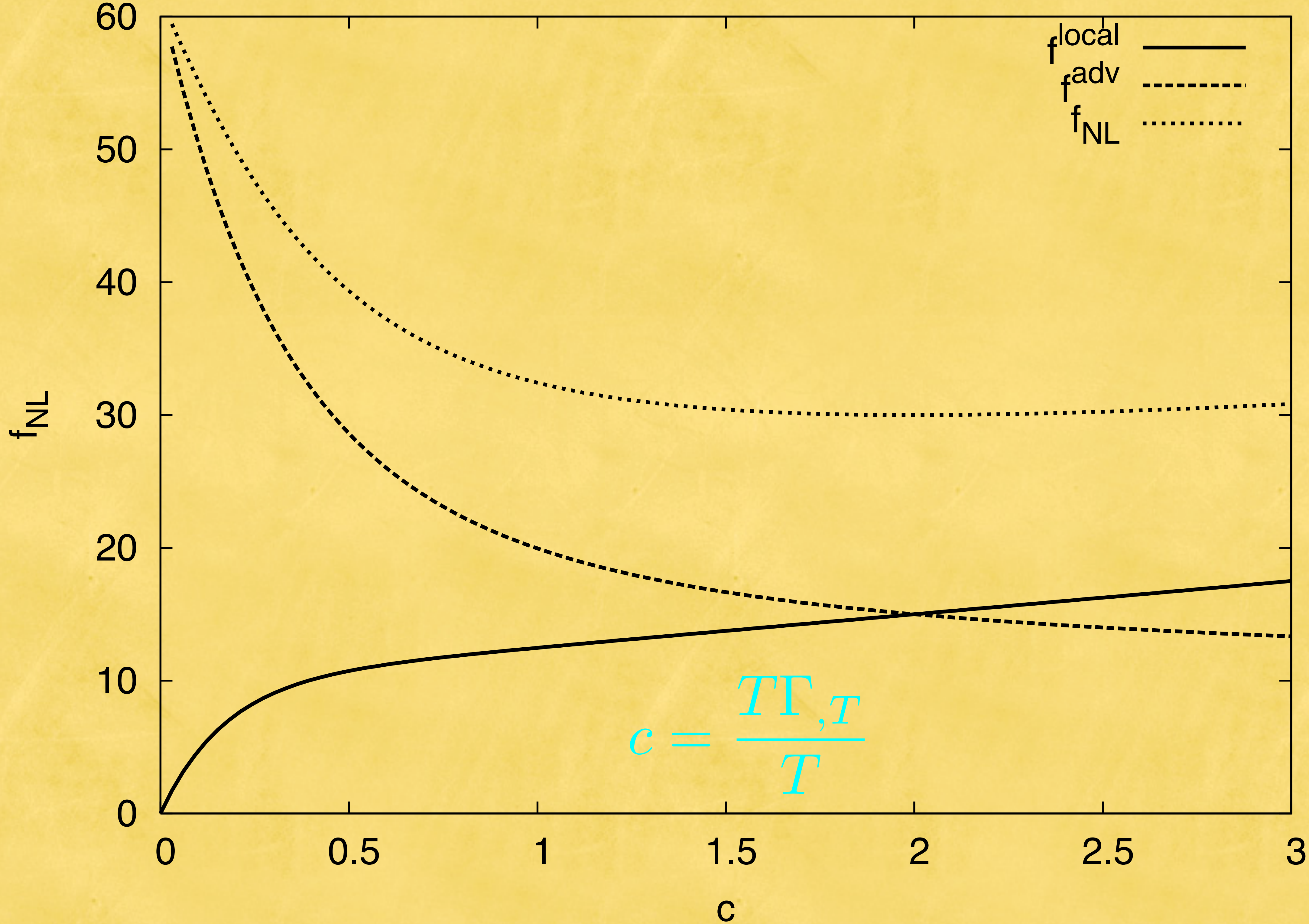
$$B_\zeta(k_1,k_2,k_3) = B_\zeta^{local}(k_1,k_2,k_3) + B_\zeta^{adv}(k_1,k_2,k_3)$$

$$B_\zeta^{local}(k_1,k_2,k_3) = \frac{6}{5} f_{NL}^{local} \sum_{\text{cyclic}} P_\zeta(k_1) P_\zeta(k_2)$$

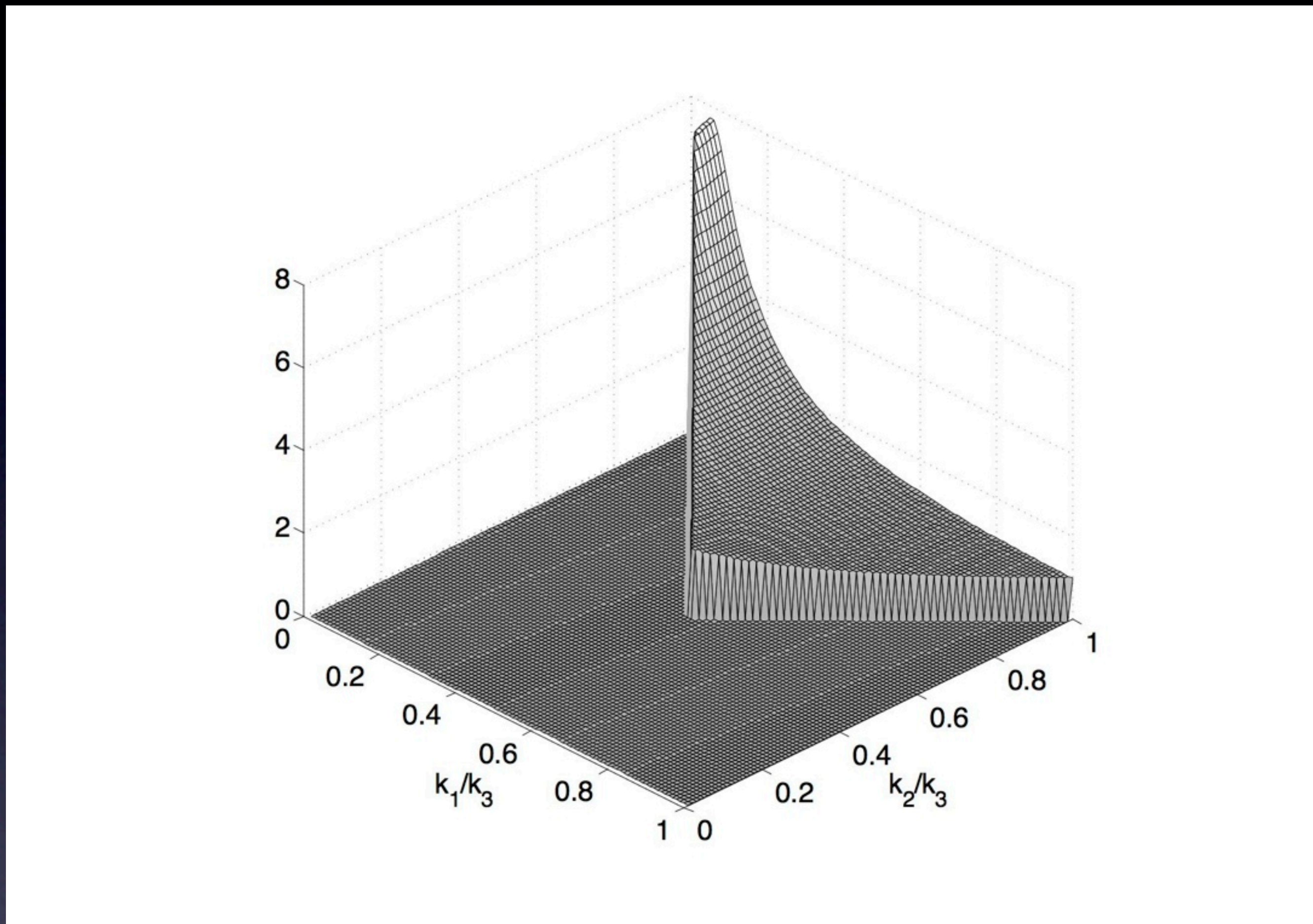
$$B_\zeta^{adv}(k_1,k_2,k_3) = -\frac{6}{5} f_{NL}^{adv} \sum_{\text{cyclic}} (k_1^{-2} + k_2^{-2})\, \mathbf{k_1} \cdot \mathbf{k_2}\, P_\zeta(k_1) P_\zeta(k_2)$$

(Only applies away from the squeezed triangle limit.)

Bispectrum coefficients

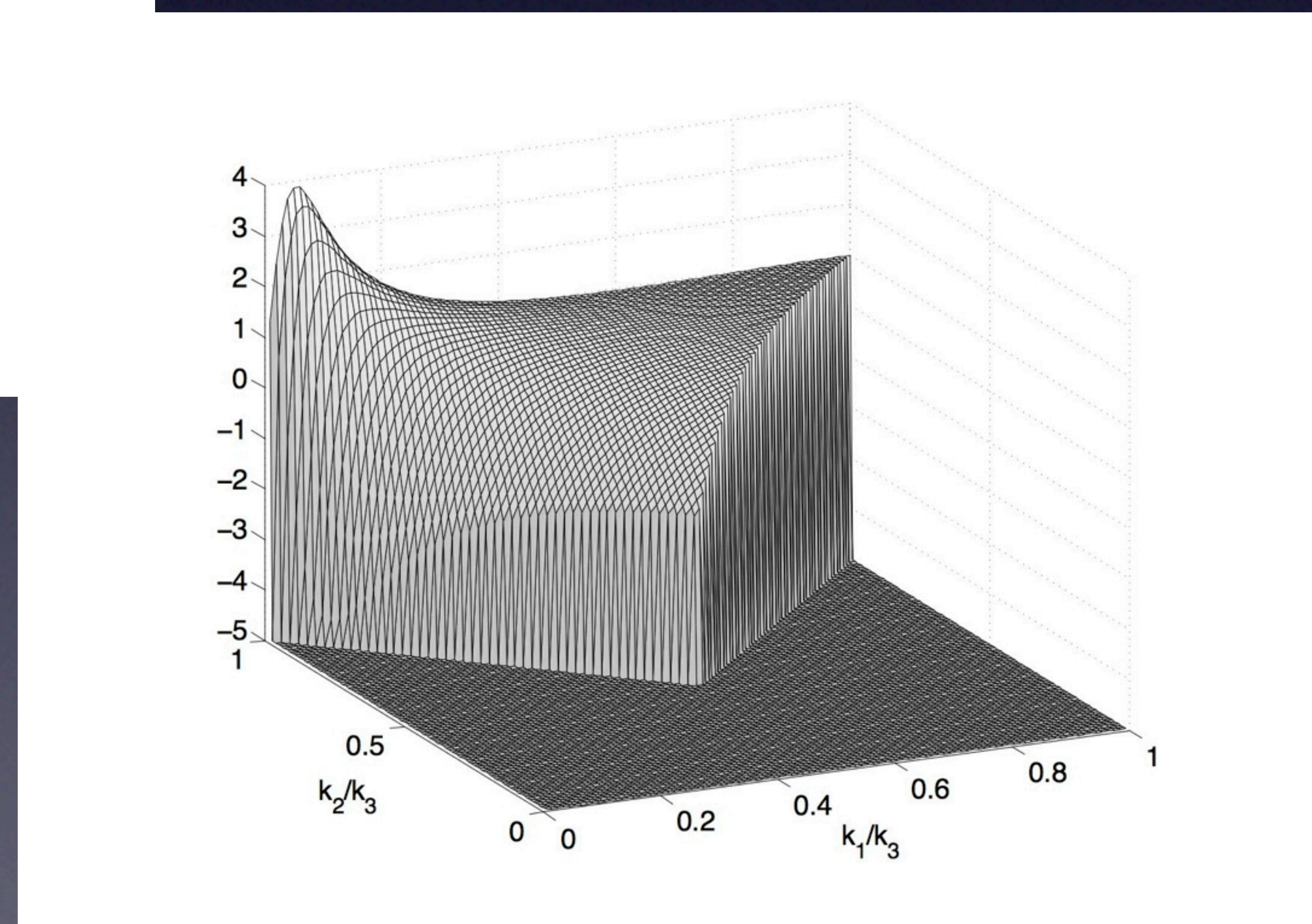


Bispectrum shape



local

adv



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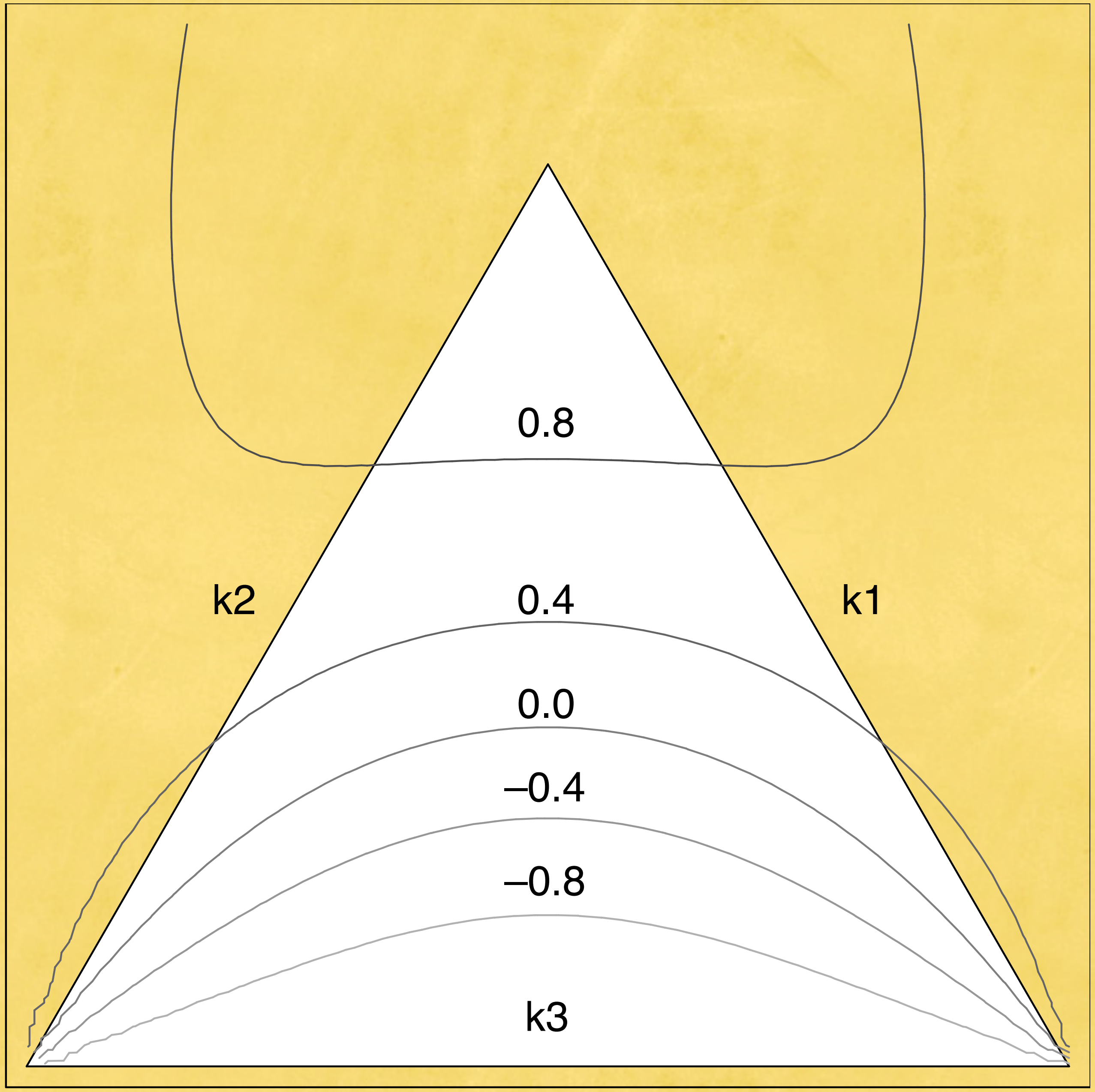

Truncation

approximations fail

$$B_\zeta^{\mathrm{trunc}}(k_1, k_2, k_3) = B(k_1, k_2, k_3) \text{ if } k_i/k_j > \epsilon$$

Examine the covariance of the truncated and the original bispectrum in angular modes.

Very little effect if epsilon<0.1.

Fergusson and Schellard 2009, Tim Yeomans and IGM

Summary

- Perturbative expansion of the LE makes sense.
- Present models require a two-stage decay mechanism.
- When $\Gamma \equiv \Gamma(T)$, you need to use the most recent results for the density fluctuations.
- If strong warm inflation took place, then the bi-spectrum shape is a combination of the local shape and the special warm inflationary shape.
- Further work needs to be done on the non-gaussianity for the weak regime.

References

C Graham and IGM (2007) astro-ph/0707.1647

C Graham and IGM (2008) arXiv:0810.2039

Berera, IGM and Ramos (2009) arXiv:0809.2244

C. Graham and IGM (2009) arXiv:0905.3500

IGM and T Yeomans (2011) arXiv:1102.2833