# Higgsless models of strong EWSB and EWPT
# Technicolor vs Conformal Technicolor

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AO, Slava Rychkov, arXiv:1111.3534v1 [hep-ph]

# Outline

- Basics of EWSB and EW Chiral Lagrangian
  - ▷ Unitarity
  - ▷ ElectroWeak Precison Tests
- Adding a scalar
  - ▷ Unitarity
  - ▷ ElectroWeak Precison Tests
  - ▷ Hierarchy problem
- Adding spin-1 Resonances
  - ▷ QCD and TechniColor
  - ▷ Unitarity
  - ▷ The S parameter
  - ▷ Conformal TechniColor
  - ▷ The T parameter
  - ▷ Results
- Conclusion

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- It allows perturbative computations up to 3 TeV.

- Unitarity

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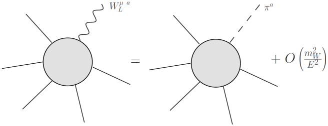

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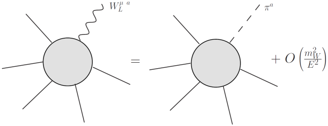


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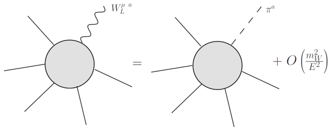


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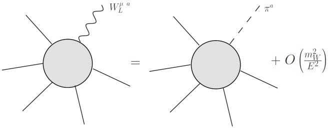


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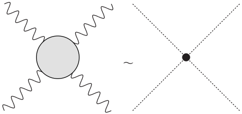

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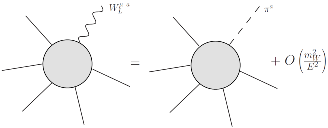


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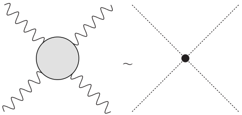


$a_0 = -\frac{s}{16\pi v^2}$
$\left| Re(a_0) \right| < \frac{1}{2} \ \Rightarrow \sqrt{s} \lesssim 1.7\, TeV$
All channels (WW, ZZ , ZW)
$\Rightarrow \sqrt{s} \lesssim 1.2\, TeV$

- Parametrization of Oblique Radiative Corrections:
$\Pi^{ij}_{\mu\nu} = -i g_{\mu\nu} \left[ \Pi_{ij}(0) + q^2 \Pi'_{ij}(0) + \ldots \right] + q_\mu q_\nu$ terms

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- Usefull decomposition for Heavy Models ($M_{newphysics} >> M_Z$):
  $\epsilon_i = e_i + \delta\epsilon_i$
  - ▷ $e_1 = \frac{\Pi_{33}(0) - \Pi_{+-}(0)}{M_W^2}$ (Breaking of custodial symmetry)
  - ▷ $e_2 = \Pi'_{+-}(0) - \Pi'_{33}(0)$ (idem+higher order in derivatives $\rightarrow$ negligible)
  - ▷ $e_3 = \frac{c_w}{s_w}\Pi'_{3B}(0)$

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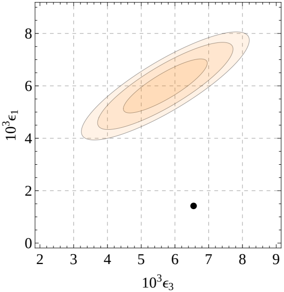
  $\delta\epsilon_i \rightarrow$ higher orders in derivatives (dominated by Z,W loops)
- S and T parameters:
  - ▷ $\hat{S} = \epsilon_3 - \epsilon_3^{SM}(m_h^{ref}) \overset{heavy}{\sim} e_3 - e_3^{SM}(m_h^{ref})$
  - ▷ $\hat{T} = \epsilon_1 - \epsilon_1^{SM}(m_h^{ref}) \overset{heavy}{\sim} e_1 - e_1^{SM}(m_h^{ref})$

68%, 95%, 99% Confidence Level ellipses:




- $\epsilon_1 \sim -\frac{3g'^2}{32\pi^2} \log \frac{\Lambda}{m_Z} + cst$
- $\epsilon_3 \sim \frac{g^2}{96\pi^2} \log \frac{\Lambda}{m_Z} + cst'$

$\Rightarrow$ Bad fit to EWPT...

Unitarity and the EWPT need to be fixed
$\Rightarrow$ New Physics at the TeV scale

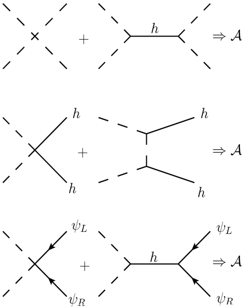
# Minimal modification :add a scalar

Let's add a scalar under $SU(2)_L \times SU(2)_R$ (global) and $SU(2)_L \times U(1)_Y$ (local):

## Effective Lagrangian at $O(p^2)$

$$\mathcal{L}^{(2)} = \frac{v^2}{4} Tr\left(D_\mu U \left(D^\mu U\right)^\dagger\right)\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + ...\right)$$
$$- \frac{v}{\sqrt{2}} \sum_{i,j} \left(\overline{u}_L^{(i)} \overline{d}_L^{(i)}\right) U \begin{pmatrix} \lambda^u_{ij} u_R^{(j)} \\ \lambda^d_{ij} d_R^{(j)} \end{pmatrix} \left(1 + c\frac{h}{v} + ...\right) + h.c.$$
$$+ \frac{1}{2}\left(\partial_\mu h\right)^2 - V(h)$$

WW scattering (elastic + inelastic):




$$\Rightarrow \mathcal{A} \sim (1 - a^2)\frac{s}{v^2} + (s \leftrightarrow t)$$

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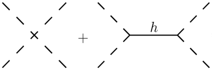


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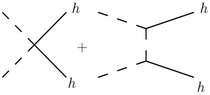


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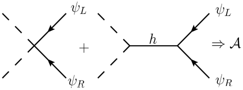


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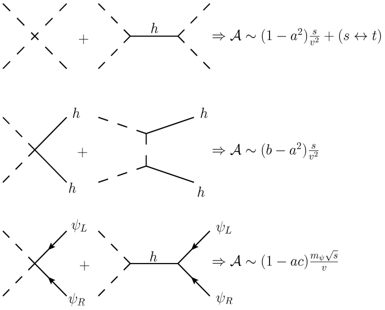


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- $a_0 \overset{s>>m_h^2}{\rightarrow} -\frac{m_h^2}{8\pi v^2}$ Unitarity restored up to arbitrarily high scales as long as $m_h < 700\ GeV$.

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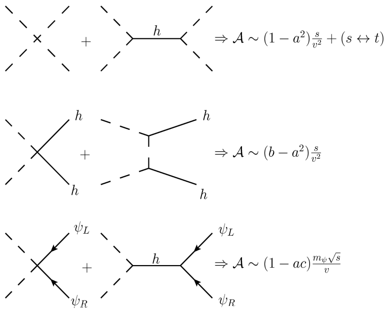


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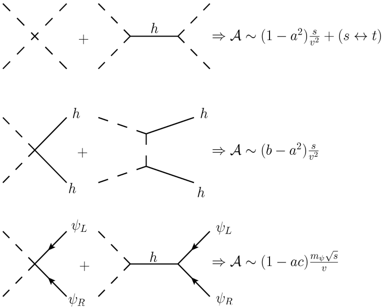


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- For different values for a,b,c ⇒ Composite Higgs, SUSY...

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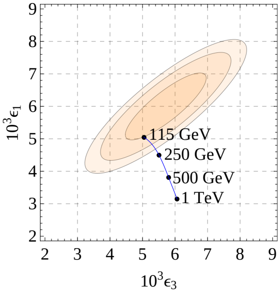


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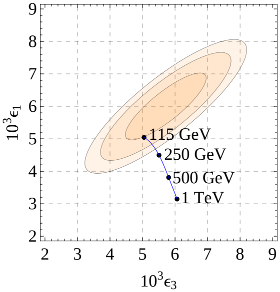


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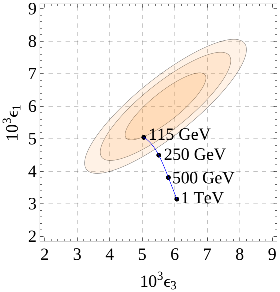


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- For $a \neq 1$: $\Delta\epsilon_{1,3} = \mp \frac{3g'^2}{32\pi^2}(1-a^2)\log\frac{\Lambda'}{m_h}$, $\Lambda' = \frac{4\pi v}{\sqrt{1-a^2}}$
  $m_h = 125\,GeV \Rightarrow\ 0.84 < a^2 < 1.4$
  [Azatov,Contino,Galloway,2012]

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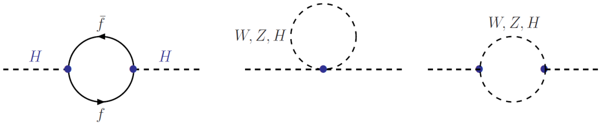

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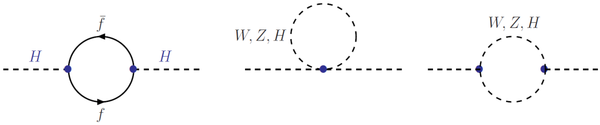


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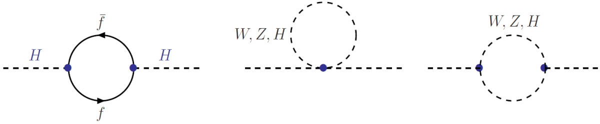


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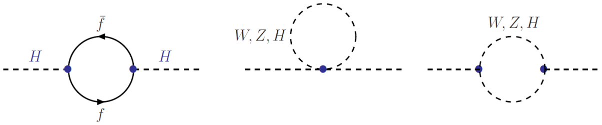


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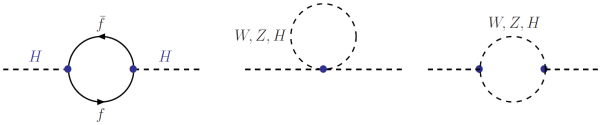


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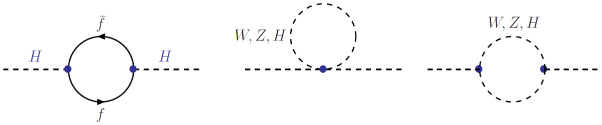


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- I will now focus on the Technicolor scenario.

# Chiral symmetry breaking in QCD

- massless QCD with two flavors $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$:

$$\mathcal{L} = \overline{\psi} i \not{D} \psi = \overline{\psi}_L i \not{D} \psi_L + \overline{\psi}_R i \not{D} \psi_R$$

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- Let's build now an effective Lagrangian for the spin-1 resonances

# Effective Lagrangian for spin-1 resonances

- SB Pattern: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$ (no Higgs)
- Representation for the spin-1 resonances: Antisymmetric tensors transforming in the adjoint of $SU(2)_V$: $R^{\mu\nu} \rightarrow hR^{\mu\nu}h^\dagger$
- We included 2 spin-1 resonances: one axial $A^{\mu\nu}$and one vector$V^{\mu\nu}$ (VMD)

## Effective Lagrangian at $O(p^2)$

$$
\begin{aligned}
\mathcal{L} =& \frac{v^2}{4} Tr\left(D_\mu U \left(D^\mu U\right)^\dagger\right) \\
&+ \mathcal{L}_{kin,mass}(A^{\mu\nu}, V^{\mu\nu}) + \frac{iG_V}{2\sqrt{2}} Tr\left(V^{\mu\nu}\left[u_\mu, u_\nu\right]\right) \\
&+ \frac{F_V}{2\sqrt{2}} Tr\left(V^{\mu\nu}\left[\xi \hat{W}^{\mu\nu}\xi^\dagger + \xi^\dagger \hat{B}^{\mu\nu}\xi\right]\right) \\
&+ \frac{F_A}{2\sqrt{2}} Tr\left(A^{\mu\nu}\left[\xi \hat{W}^{\mu\nu}\xi^\dagger - \xi^\dagger \hat{B}^{\mu\nu}\xi\right]\right)
\end{aligned}
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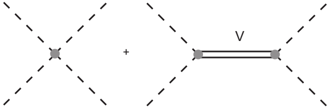
New diagram in WW scattering:



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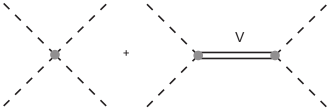


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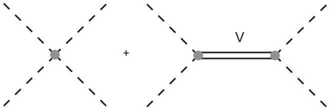


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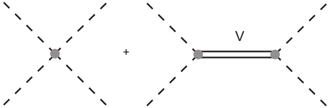


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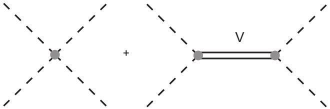


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- But there is still a logarithmic growth in $a_0$ (Better for $G_V \gtrsim \frac{v}{\sqrt{3}}$)
- It's an imperfect unitarization, possible up to a few TeV. At higher energies, inelastic channels opens and heavier resonances can enter the game....

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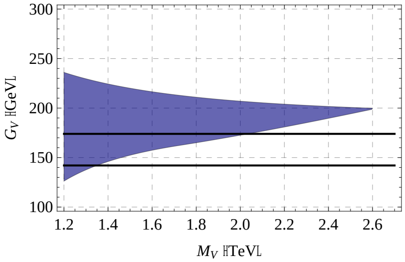

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- Result:

$$\hat{S} = \frac{g^2}{4} \left( \frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right) + \frac{g^2}{96\pi^2} \left( \log \frac{M_V}{m_h^{ref}} + O(1) \right)$$

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- $SU(2)_L$ and $SU(2)_R$ currents OPE: $\left\langle J_L^{a\mu}(q^2) J_R^{b\nu}(0) \right\rangle \equiv \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \Pi_{LR}(q^2) \sim (-q^2)^{1-\Delta\Phi/2} \langle \Phi \rangle + \ldots$

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- Dispersive relation : $\Pi_{LR}(q^2) = v^2 + q^2 \int_0^\infty ds \frac{\rho_V(s) - \rho_A(s)}{s - q^2 - i\epsilon}$ with $\rho_V(s) = \frac{1}{\pi}\mathrm{Im}\tilde{\Pi}_{VV}(s) \quad \rho_A(s) = \frac{1}{\pi}\mathrm{Im}\tilde{\Pi}_{AA}(s)$

**Consequences:Weinberg Sum Rules**

- if $\Delta\Phi > 2$ : $\int_0^\infty ds \left(\rho_V(s) - \rho_A(s)\right) = v^2$ 1st WSR
- if $\Delta\Phi > 4$ : $\int_0^\infty dss \left(\rho_V(s) - \rho_A(s)\right) = 0$ 2nd WSR

# Consequences of the WSR's for Minimal Technicolor

- lowest dimension operator contributing to the OPE:
  $\phi^{ab} = \left(\overline{\psi} P_L T^a \psi\right)\left(\overline{\psi} P_R T^b \psi\right) \Rightarrow \Delta\Phi = 6$
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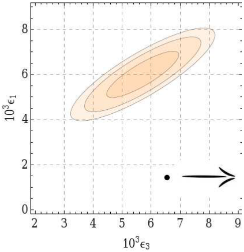
**Constraints on the parameters:**

- $F_V^2 - F_A^2 = v^2$ 1st WSR
- $F_V^2 M_V^2 - F_A^2 M_A^2 = 0$ 2nd WSR

# Minimal Technicolor and S parameter

Problem of the constraints:

This gives $\Delta \hat{S}_{tree} = \frac{g^2}{4}\left(\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right) > 0$ and big...




We recover here the usual problem of minimal Technicolor models: a bad fit to EWPT...

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$\Rightarrow$ Possible cancelation between axial and vector contributions to $\hat{S}$

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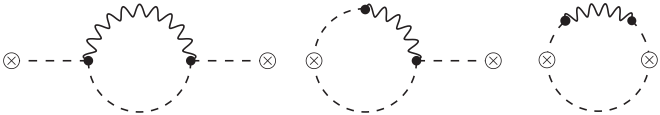
$\Rightarrow$ 3 types of diagrams contributing:



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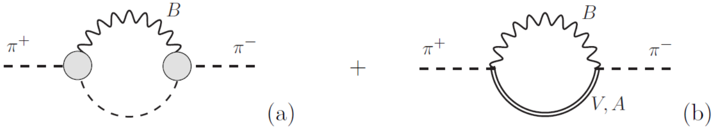

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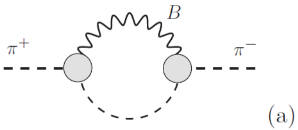


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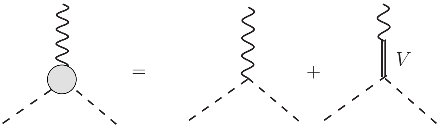

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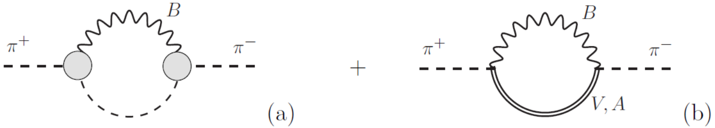


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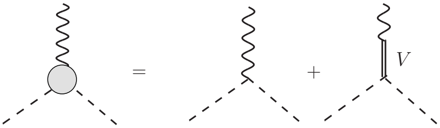


- Good UV behavior $\Rightarrow F_V G_V = v^2$

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- Quadratic divergences: $\Delta\hat{T}_{\Lambda^2} = \frac{3g'^2}{8}\left[\frac{(F_V - 2G_V)^2}{M_V^2} + \frac{F_A^2}{M_A^2}\right]\frac{\Lambda^2}{16\pi v^2}$

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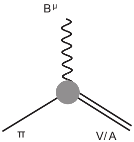

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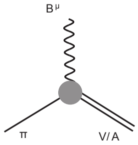


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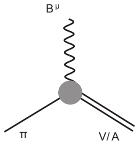


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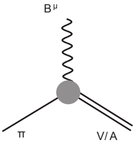


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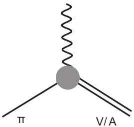


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# T parameter-$\lambda$ coupling

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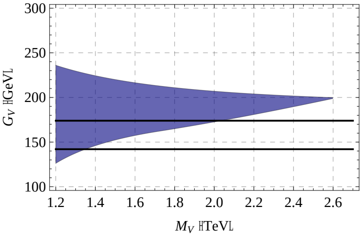
- Interesting for EWPT

# Constraining the space of parameters

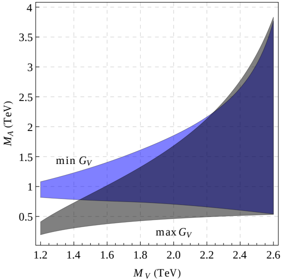
## Sum Rules

- $F_V^2 - F_A^2 = v^2$ (1st WSR), ~~$F_V^2 M_V^2 - F_A^2 M_A^2 = 0$~~ (2nd WSR)
- $F_V G_V = v^2$ ($\pi\pi$ formfactor)
- $F_V - 2G_V = -2\kappa_V F_A$ and $F_A = -2\kappa_A F_V$ ($\pi R$ formfactors)

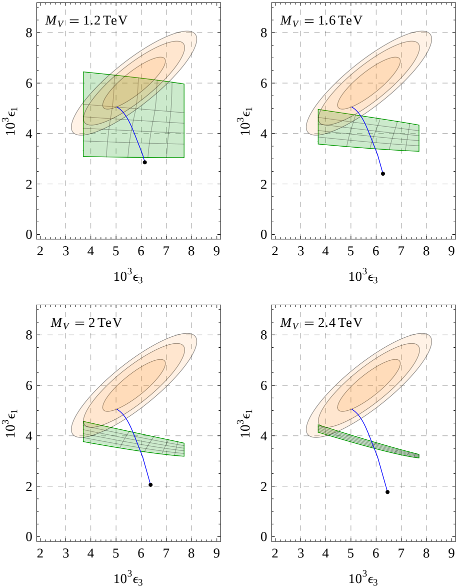
Constraints from Unitarity



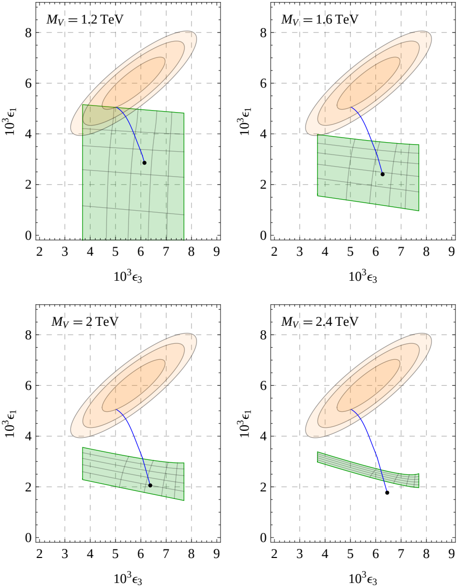

Constraints from $\hat{S}$



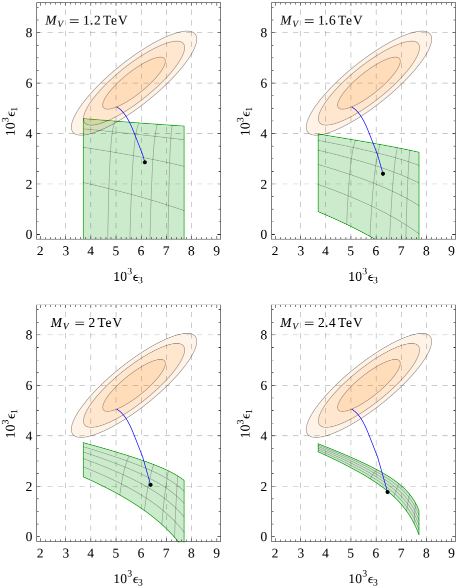
# Results: $\lambda = -0.5$



# Results: $\lambda = 0$



# Results: $\lambda = +0.5$



# Conclusion

- Chiral EW Lagrangian $\Rightarrow$ Unitarity violation and bad fit to EWPT
- Two solutions for unitarity: Higgs,Spin-1 Resonances
- But for EWPT the Higgs seems at first sight to be the only solution (large S in Technicolor)
- Conformal Technicolor provides a way out for the Spin-1 Resonances