Degeneracies between canonical and non-canonical inflation

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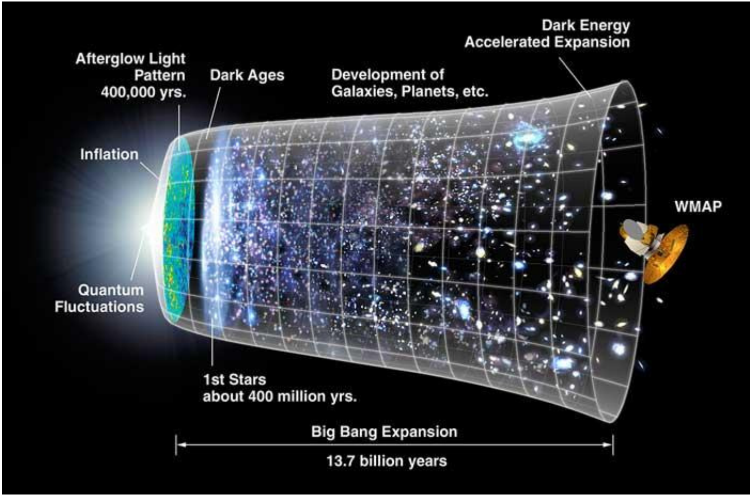
June 5, 2013

[1211.0070] Gwyn, Rummel and Westphal, “Resonant non-Gaussianity with equilateral properties,”

[1212.4135] Gwyn, Rummel and Westphal, “Relations between canonical and non-canonical inflation,”

Outline

Physics in the early universe



Degeneracies

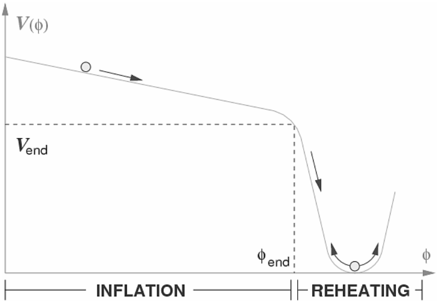
- Ideally, would like to use data to rule out models or even confirm specific inflationary model
- However, this goal is probably out of reach, because of
  1. limitations in observation
  2. degeneracy between predictions

- But we have no choice: to connect data with theory, must take potential degeneracies into account!

Inflation: Slow Roll

Vanilla inflation: single field theory with canonical kinetic term

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$$



Get inflation when the potential is flat enough for a large enough field range...

Inflation: Usual approach

For FRW metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2,$$

we get exponential expansion $a(t) \sim e^{Ht}$ for slow-roll:

$$H = \frac{\dot{a}}{a}; \quad \epsilon = -\frac{\dot{H}}{H^2} \ll 1; \quad \eta = \frac{\dot{\epsilon}}{H\epsilon} \ll 1.$$

$$\epsilon = \epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta = 4\epsilon_V - 2\eta_V; \quad \eta_V = \frac{V''}{V}$$

UV sensitivity of inflation

The UV-complete theory in which inflation operates is unknown, so we take an EFT approach:

- Effective field theory: corrections from higher-dimensional operators should be suppressed by the cut-off Λ:

$$\mathcal{L}_{eff} = \mathcal{L}_{relevant} + \sum_n c_n \frac{\mathcal{O}_n}{\Lambda^{n-4}}$$

However, the EFT can be sensitive to the UV physics:

- Eta problem: Mass dimension 6 corrections can spoil the flatness of the potential: $\frac{\mathcal{O}_6}{M_p^2} \to \frac{\mathcal{O}_4}{M_p^2}\phi^2$

$$V_{eff} = V_0 + \frac{1}{2} m_0^2 \phi^2 + \frac{\mathcal{O}_4}{M_p^2}\phi^2$$

$$< \mathcal{O}_4 > \sim V_0 \Rightarrow \eta_V = M_p^2 \frac{V''}{V} \sim \mathcal{O}(1).$$

UV sensitivity of kinetic terms

In particular, non-canonical kinetic terms arise when massive degrees of freedom are integrated out:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\rho)^2 + \frac{\rho}{M}(\partial\phi)^2 - \frac{1}{2}M^2\rho^2 \\ \Rightarrow \mathcal{L}_{eff} &= \frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^4}{M^4} \quad \text{for } H \ll M \end{aligned}$$

at energy scales $H << M$. E.g., the DBI action [Silverstein and Tong, 0310221]

$$\begin{aligned} \mathcal{L}_{DBI} &= -\Lambda^4\left[\sqrt{1-\frac{(\partial\phi)^2}{\Lambda^4}} - 1\right] - V(\phi) \\ &\approx \frac{1}{2}(\partial\phi)^2 + \frac{1}{8}\frac{(\partial\phi)^4}{\Lambda^4} + \ldots - V(\phi) \end{aligned}$$

Non-canonical Lagrangian

A single scalar field coupled minimally to gravity ($X = \frac{1}{2}\dot{\phi}^2$):

$$S = \int d^4x\sqrt{-g_4}\left[\frac{M_p^2}{2}\mathcal{R}_4 + p(X,\phi)\right]$$

For example,

$$\begin{aligned}
p_{can} &= X - V(\phi) \\
p_{DBI} &= -\frac{1}{f(\phi)}\left(\sqrt{1-2f(\phi)X} - 1\right) - V(\phi) \\
p_{Tach} &= -V(\phi)\sqrt{1-2\frac{X}{\Lambda^4}} \\
p_K &= K(\phi)X + \frac{X^2}{\Lambda^4}
\end{aligned}$$

Non-canonical Inflation

Take separable action

$$p(X,\phi) = \Lambda^4 S(X) - V(\phi).$$

The inflationary solution is given by $X_{inf}(A)$ satisfying

$$\sqrt{\frac{2X}{\Lambda^4}}\frac{dp}{dX} = A = \frac{V'}{3H\Lambda^2}$$

where A is the noncanonicalness parameter.

- $A << 1 \Rightarrow \epsilon_V \ll 1$ i.e. canonical regime
- NCI is attractive (small perturbations driven to zero)
- overshoot/ICFTP reduced when the NC regime is relevant

[Franche, RG, Underwood and Wissanji: 0912.1857 & 1002.2639]

Observational signatures of NCI: Nongaussianity

The power spectrum and spectral index are given by the two-point function of scalar perturbations:

$$
\begin{aligned}
< \mathcal{R}_k \mathcal{R}_{k'} > &= (2\pi)^3 \delta(k + k') \mathcal{P}_{\mathcal{R}}(k) \\
\Delta_s^2 &= \frac{k^3}{2\pi^2} \mathcal{P}_{\mathcal{R}}(k) \\
n_s - 1 &= \frac{d \ln \Delta_s^2}{d \ln k}
\end{aligned}
$$

For Gaussian fields even higher point functions are determined by this and odd higher point functions are zero (order SR parameters). For Nongaussian fields, get nonnegligible contribution to bispectrum (three point function) etc $< \mathcal{R}_{k1} \mathcal{R}_{k2} \mathcal{R}_{k3} >$

Observational signatures of NCI:

NCI models $P(X, \phi)$ can lead to an observable amount of nongaussianity, of the **equilateral** type: $k_1 \approx k_2 \approx k_3$ [Chen, Huang, Kachru, Shiu: 0605045]

$$f_{NL}^{equil} \sim c_s^{-2}$$

where

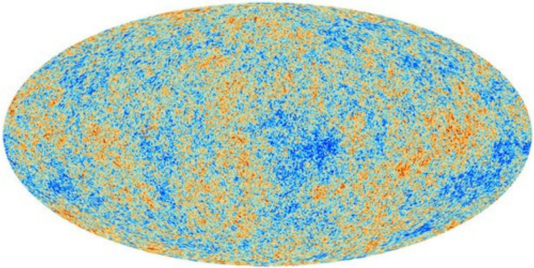
$$c_s^2 = \left(1 + 2X\frac{p_{XX}}{p_X}\right)^{-1}$$

- Potentially clear observational signature of NCI!
- Not yet ruled out by data....

Constraints on NCI

New Planck bounds: [1303.5082 etc]

$$
\begin{aligned}
f_{NL}^{local} &= 2.7 \pm 5.8 \\
f_{NL}^{equil} &= -42 \pm 75 \\
f_{NL}^{ortho} &= -25 \pm 39 \\
c_s &\geq 0.02
\end{aligned}
$$



Outline

1. Planck: local f_{NL} severely constrained, putting pressure on multifield models
2. f_{NL}^{equil} (NC kinetic terms, varying c_s) relatively unconstrained
3. NC kinetic terms are also fairly generic in string theory models of inflation
4. However, there is degeneracy between canonical and noncanonical models *even* at the 3pt function level (Non gaussianities)
5. in [1211.0070] and [1212.4135] we try to understand this degeneracy better...

Field redefinitions

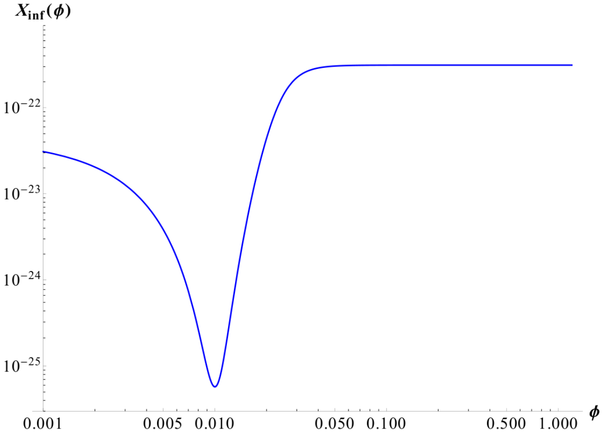
- for simple Lagrangians $p(X, \phi)$ can transform to a canonical action via a field redef eg $p(X, \phi) = -\frac{1}{2\phi^2}(\partial_\mu \phi)^2 - V(\phi)$ using $\psi = \ln \phi$.
- for more general $p(X, \phi)$ can always transform a canonical theory to a noncanonical one via canonical transformations in $0 + 1$D [Bean et al, 0801.0742]:

$$p = \frac{\partial F}{\partial \phi}, \ \tilde{p} = -\frac{\partial F}{\partial \tilde{\phi}}$$

  for a generating functional $F(\phi, \tilde{\phi})$.
- However only separable NC theories with quadratic potentials can be transformed to canonical theories this way (AFAIK...). [RG, Rummel and Westphal, 1212.4135]

Onshell transformation

Can we construct a potential $V_{can}(\phi)$ which gives rise (in a canonical theory) to the same trajectory $X_{inf}(\phi)$ as in the noncanonical theory?



Onshell transformation

Noncanonical theory:

$$\Pi_{inf}(\phi) \approx \frac{\partial p}{\partial \phi}\frac{1}{3H}$$

$$\Pi = -\sqrt{2X}\frac{\partial p}{\partial X}$$

$$H^2 = \frac{\rho}{3M_p^2} = \frac{2Xp_X - p}{3M_p^2}$$

Canonical theory:

$$\dot{\phi} = -\frac{V'_{can}(\phi)}{3H(\phi)}$$

$$H^2(\phi) = \frac{V_{can}(\phi)}{3}$$

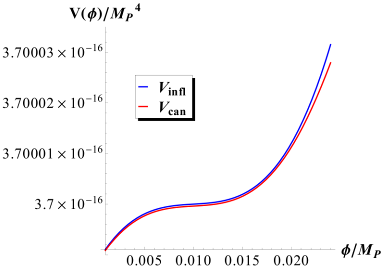
Given some $X(\phi) = X_{inf}$, integrate

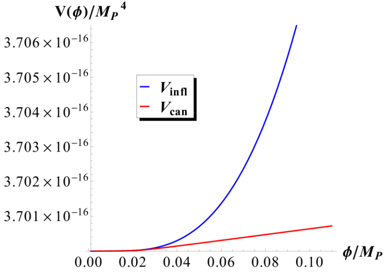
$$\sqrt{6X}d\phi = \frac{dV_{can}}{\sqrt{V_{can}}}$$

$$\Rightarrow V_{can}(\phi) = \left(\sqrt{V_{can}} + \int_{\phi_0}^{\phi} d\phi' \sqrt{\frac{3}{2}X_{inf}(\phi')}\right)^2$$

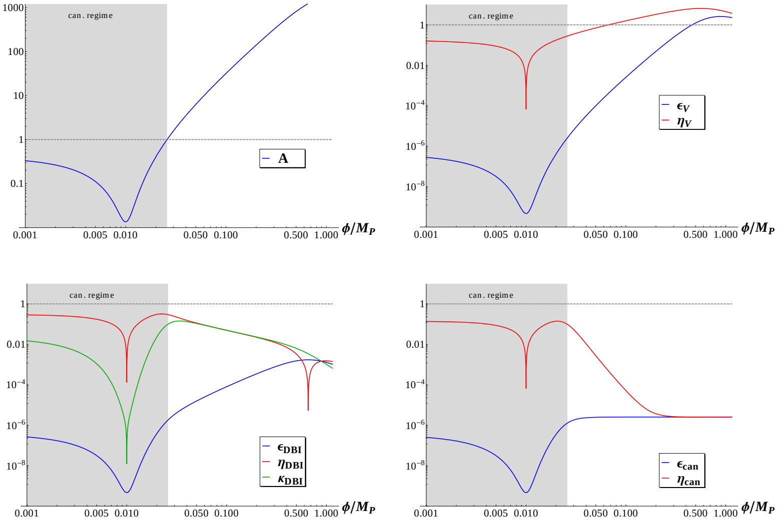
DBI + Inflection point potential

$$V_{inf}(\phi) = V_0 + \lambda(\phi = \phi_0) + \beta(\phi - \phi_0)^3$$





Canon vs Noncanon



Observables

(Non)Canonical theory:

$$\Delta_s^2(k) = \frac{1}{8\pi^2}\frac{H^2}{M_p^2}\frac{1}{c_s\epsilon}\bigg|_{c_s k=aH}$$

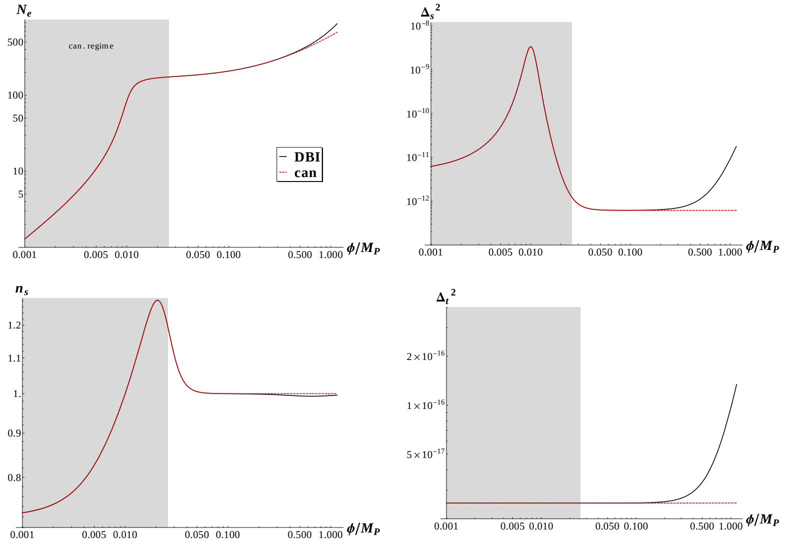
$$\Delta_t^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_p^2}\bigg|_{k=aH}$$

$$n_s(k)-1 = -2\epsilon-\eta\big|_{c_s k=aH}$$

$$n_t(k) = -2\,\epsilon\big|_{k=aH}$$

- $\epsilon=-\frac{\dot H}{H^2};\ \eta=\frac{\dot\epsilon}{H\epsilon}$ so in the canonical limit $\epsilon\to\epsilon_V;\ \eta\to 4\epsilon_V-2\eta_V$
- Recall that $c_s^{-2}=1+2X\frac{p_{XX}}{p_X}$
- note that time of horizon crossing is different for scalar modes in NCI

Comparison of Observables



Why is DBI special?

For theories with (1) a canonical limit where $V = V_{can}$ and (2) a speed limit st $X_{inf} = \Lambda^4 R$ when A is large (from finite convergence radius), $\Delta_s^2(k), \Delta_t^2(k), N_e$ match when

$$V_{can} \approx V \; ; \; c_s = \frac{\sqrt{2R}}{A} \text{ for } A \gg 1$$

.

- can have $V \approx V_{can}$ and $V' >> V''_{can}$ in some intermediate regime for A
- $c_s^2(A) = \frac{A\frac{\partial X_{inf}}{\partial A}}{2X_{inf}} \approx \frac{1}{A^n}$ for $X_{inf} = X_{inf}(A^n)$. Then we get the matching condition for DBI:

$$X_{inf}^{DBI} = \frac{\Lambda^4}{2}\frac{A^2}{1+A^2}$$

No other working examples.... DBI special?

Two-point function degeneracy

- in all 2 point observables
- over a large range of efolds
- for a large range of field values well outside the canonical regime of DBI
- (Recall that the field range in DBI is limited by the phase space bounds so that one cannot access the very large A regime)

This degeneracy is not resolved by observation and

- may not see a measurement of eq type NG for many years, even if it is large and present
- Even should non-negligible eq NG be observed, it is possible that this could arise from a canonical SF model.
- Only sufficiently precise measurement of r/n_T could break the degeneracy.

What about nongaussianities??

Resonant NG

Axionic shift symmetry will receive small periodic modulations from NP effects [Chen, Easther, Lim 0801.3295 & Flauger and Pajer 1002.0833]

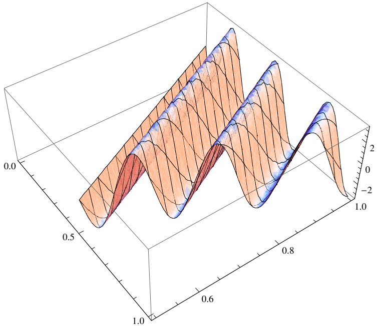
$$\begin{aligned} V(\phi) &= V_0(\phi) + \Lambda^4 \cos(\frac{\phi}{f}) \\ \Rightarrow \frac{\mathcal{G}(k_1, k_2.k_3)}{k_1 k_2 k_3} &= f^{res} \left[\sin(\frac{\sqrt{2\epsilon_\star}}{f} \ln \frac{K}{k_\star}) + \sum \cos() + \ldots \right] \end{aligned}$$

where

$$\begin{aligned} f^{res} &= \frac{3 b_\star \sqrt{2\pi}}{8} (\frac{\sqrt{2\epsilon_\star}}{f})^{3/2} \\ b_\star &= \frac{\Lambda^4}{V_0'(\phi_\star) f} \\ K &= k_1 + k_2 + k_3. \end{aligned}$$

NG comes from $\dot{\delta}$ where $\delta = \frac{\ddot{H}}{2H\dot{H}}$ in interaction term. f is the axion decay constant.

Resonant NG



Less than 10 % overlap with the other shapes (local, equilateral, orthogonal)

Multiple sources

$$
\begin{aligned}
V(\phi) &= V_0(\phi) + \sum_i A_i \cos\left(\frac{\phi + c_i}{f_i}\right) \\
\frac{\dot{\delta}}{H} &= \sum_i \frac{\sqrt{2\epsilon_\star}}{f_i} 3 b_i^\star \cos\left(\frac{\phi_0 + c_i}{f_i}\right) \\
\frac{\mathcal{G}(k_1, k_2, k_3)}{k_1 k_2 k_3} &= \sum_i \frac{3\sqrt{2\pi} b_i^\star}{8} \left(\frac{\sqrt{2\epsilon_\star}}{f_i}\right)^{3/2} \sin\left(\frac{\sqrt{2\epsilon_\star}}{f_i} \ln \frac{K}{k_\star} + \frac{c_i}{f_i}\right)
\end{aligned}
$$

Can choose $b_i^\star, f_i, c_i$ to get an overlap with a periodic equilateral shape for $N = \mathcal{O}(10)$ terms!

One-dimensional limit

$$x_2 = \frac{k_2}{k_1}, \quad x_3 = \frac{k_3}{k_1}, \quad x_\pm = x_2 \pm x_3$$

Resonant NG is to 1st order in $\frac{f_i}{\sqrt{2\epsilon_\star}}$ a fn of x_+, k_1 but not x_-:

$$\sin\left(\frac{\sqrt{2\epsilon_\star}}{f_i}\ln\frac{K}{k_\star}\right) = \sin\left(\frac{\sqrt{2\epsilon_\star}}{f_i}(\ln(1+x_+) + \ln\frac{k_1}{k_\star})\right)$$

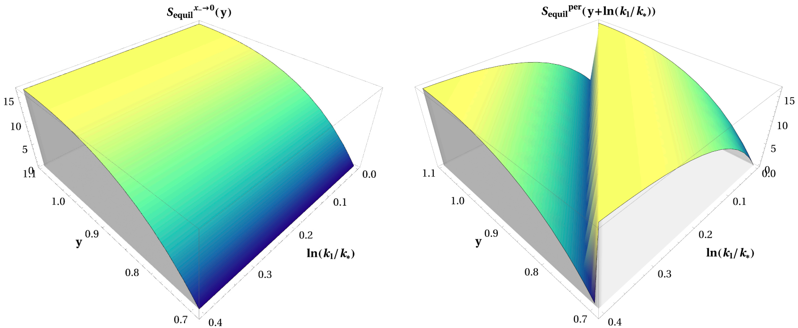
Can only reproduce NG which is predominantly a function of x_+, such as equil:

$$S_{eq}(k_1, k_2, k_3) = \frac{(k_1+k_2-k_3)(k_1+k_3-k_2)(k_3+k_2-k_1)}{k_1 k_2 k_3}$$

$$S_{eq}^{x_- \to 0}(x_+) = \frac{4(x_+ - 1)}{x_+^2}$$

$$C(S_{eq}, S_{eq}^{x_- \to 0}) = 0.93$$

Periodic approximation

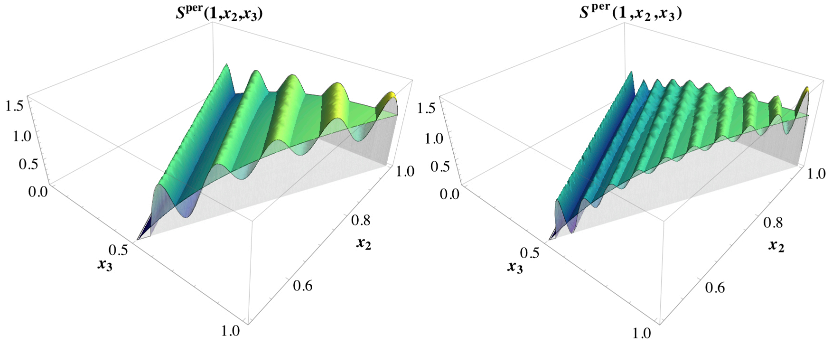


Need to approximate a scale-inv shape by a scale-dep shape: make a periodic generalisation of $S_{equil}^{x_-\to 0}$. The overlap is still considerable:

$$C(S_{equil}, S_{equil}^{per}) = 0.83$$

Can now fourier synthesize.

Fourier series



Fourier expansion for $N = 5$(left) and $N = 10$(right)

NG for a single field model!

- We can get equilateral-type nongaussianity from summing oscillating potential terms in a single field theory with canonical kinetic term.
- Equilateral type NG can also arise from inflaton fluctuations sourced by gauge quanta via $\phi\tilde{F}F$ [Barnaby and Peloso, 1011.1500; Barnaby, Pajer and Peloso, 1110.3327]
- May not be possible to distinguish canonical and noncanonical theories.

Observational constraints

$$f_i < 1; \quad b_i^\star < 1; \quad b_i^\star f_i < \frac{10^{-5}}{\sqrt{2\epsilon_\star}}$$

$$\Rightarrow \epsilon_\star f_{NL}^{eq} < 10^{-2}.$$

- as is, the power spectrum constraint implies a resonantly generated $f_{NL}^{equil} \leq \mathcal{O}(1)$
- if shift symmetry is collectively broken [Behbahani & Green, 1207.2779] (i.e. scale invariance is protected by several independent symmetries), N pt functions are no longer hierarchically suppressed with N.
- Then can have f_{NL} up to 140 without implying a large oscillation in the power spectrum
- for small field models expect no NG (f too large to have fourier sum)

Conclusions/Future Work

Canonical/Noncanonical

- We suspect the description in terms of a canonical theory may be special to the DBI case
- We don't know why this works (asking for fluctuations around the bg to match...)
- Might be able to match 3pt observables

Resonant NG

- Possible string theory/axion monodromy embedding? (series of instanton corrections expected)
- Applications elsewhere?