

# **Determination of the top mass at the LHC with emphasis on the theoretical uncertainties**

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Based on:  
Juste, Mantry, Mitov, Penin, Skands, Varnes, Vos, Wimpenny '13  
Frederix, Frixione, Mitov; to appear.

## Introduction: Why do we care about the top quark mass?

- ✓ Precision EW tests: the place in collider physics that is most sensitive to  $m_{\text{top}}$ .  
With the discovery of the (presumably SM) Higgs boson the SM is complete and the tests are over-determined. Everything looks good. The “bottleneck” is the uncertainty on the W mass. Top mass will be competitive once the ultimate W mass precision (at LHC) is achieved.
- ✓ All other places in collider physics are even less sensitive to  $m_{\text{top}}$ .
- ✓ However: there is very strong dependence on  $m_{\text{top}}$  in models that rely on bottom-up approaches  
These take some data at EW scale (measured) and then predict (through RG running)  
how the model looks at much larger scales, say  $\mathcal{O}(M_{\text{Plank}})$ .
- ✓ Two types of uncertainties appear:
  - ✓ Due to running itself Chetyrkin, Zoller '12-13  
Bednyakov, Pikelner, Velizhanin '13
  - ✓ Due to boundary condition at EW. It is here  $m_{\text{top}}$  is crucial.
- ✓ Examples:
  - Higgs inflation. Model very predictive; relates SM and  $\Lambda_{\text{CDM}}$  parameters. Agrees with Planck data.
  - Vacuum stability in SM. Change of 1 GeV in  $m_{\text{top}}$  shifts the stability bound for SM from  $10^{11}$  to the Plank scale. Bezrukov, Shaposhnikov '07-'08  
De Simone, Hertzberg, Wilczek '08  
Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia '12

This is the place where high precision in  $m_{\text{top}}$  is needed most.

# The fate of the Universe might depend on 1 GeV in $M_{top}$ !

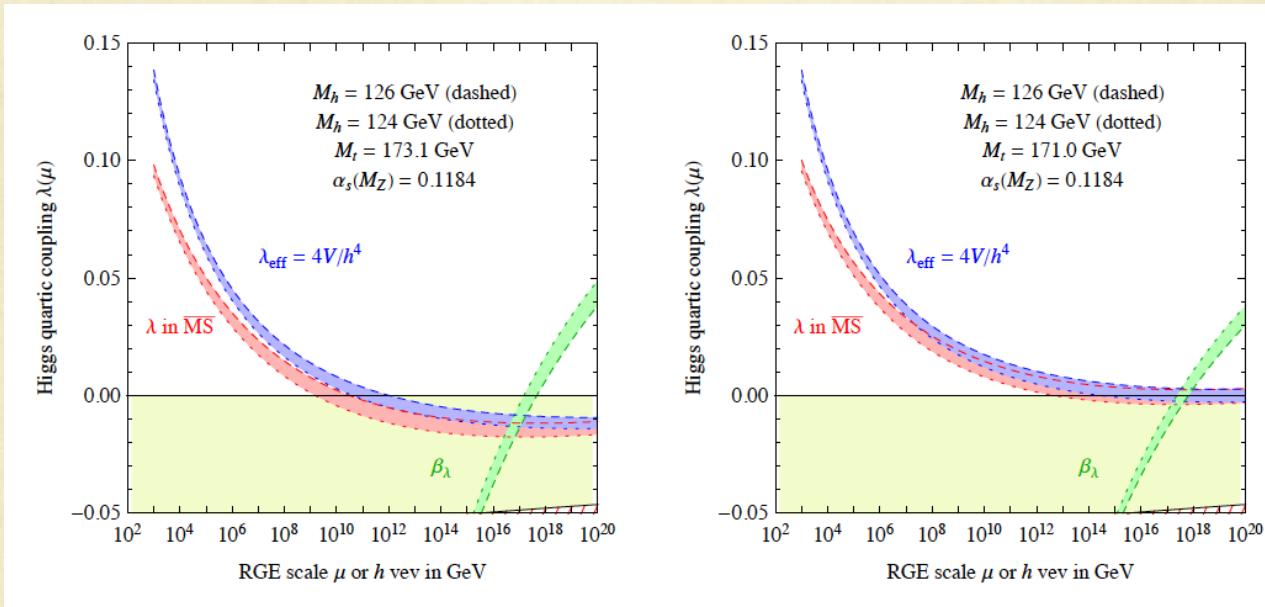
Higgs mass and vacuum stability in the Standard Model at NNLO.

Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia '12

Vacuum stability condition:

$$V_{\text{eff}} = -\frac{m^2}{2}h^2 + \frac{\lambda}{4}h^4 + \Delta V$$

Quantum corrections  
(included)



Possible implication:

For the right values of the SM parameters (and we are right there) SM might survive the Desert.

- ✓ Currently a big push for better understanding of the top mass. Precision is crucial here...

## Introduction: goals regarding top mass determination at hadron colliders

✓ The apparent sensitivity to  $m_{top}$  requires convincing  $m_{top}$  determination

✓ What do I mean by convincing?

✓  $m_{top}$  is not an observable; cannot be measured directly.

✓ It is extracted indirectly, through the sensitivity of observables to  $m_{top}$

$$\sigma^{\text{exp}}(\{Q\}) = \sigma^{\text{th}}(m_t, \{Q\})$$

✓ The implication: the “determined” value of  $m_{top}$  is as sensitive to theoretical modeling as it is to the measurement itself

✓ The measured mass is close to the pole mass (top decays ...)

✓ One needs to go beyond the usual MC's to achieve theoretical control

✓ Lots of activity (past and ongoing). A big up-to-date review:

➤ A worry: can there be an additional systematic  $O(1 \text{ GeV})$  shift in  $m_{\text{top}}$  ?

➤ Two types of possible hidden errors:

✓ QCD related. As follows from the equation:

$$\sigma^{\text{exp}}(\{Q\}) = \sigma^{\text{th}}(m_{\text{top}}, \{Q\})$$

the precision in  $m_{\text{top}}$  determination reflects the experimental uncertainty, as well as the error on the theory input. Unaccounted theory sources might have impact.

Typical situation: using a MC to construct a likelihood and find the likeliest value of  $m_{\text{top}}$ . Combine with other methods/measurements to improve errors, etc. etc.

At each step the error seemingly decreases. But this is not so, because we have irreducible error that the MC generator simply may not know about and no improvement in the measurement will take care of it. Such errors are the scariest since they are hidden (bias).

✓ bSM related. Unexplored territory. Conceptually the same as above, but the role of higher order terms is now played by bSM physics: it contributes to the measurement but is not accounted for on the theory side. Basically, a kind of bias again.

## Issues in top mass determination

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- ✓ MC modeling.

Most methods for extraction of  $m_{\text{top}}$  rely on modeling the measured final state with typically LO+LL MC generators. The extracted mass then reflects the mass parameter in the corresponding MC generator. Identifying the nature of this mass parameter and relating it to common mass schemes, like the pole mass, is a non-trivial and open problem.  
It may be associated with ambiguities of order 1 GeV.

Buckley, Butterworth, Gieseke et al Phys. Rep. '11

The effect of the top and bottom masses on parton-shower radiation patterns is generally included already in the LO+LL MC's and they screen collinear singularities.

- ✓ Non-perturbative corrections:

Mostly affect the MC modeling of the final state. Includes hadronization, color reconnection, Underlying Event, final state interactions (especially with jet vetoes).

Many such systematics are accounted for through the JES.  
Color reconnection small at e+e- but O(500 MeV) at hadron colliders.

Recommendation: try methods with alternative systematics (unrelated to MC).

- ✓ Reconstruction of the top pair.

Typically, the existing methods for extraction of the top quark mass implicitly or explicitly rely on the reconstruction of the top pair from final state leptons and jets.

This introduces uncertainties of both perturbative origin (through higher-order corrections) and non-perturbative origin (related to showering and non-factorizable corrections).

Methods that do not rely on such reconstruction are therefore complementary and highly desirable; two examples are  $\text{J}/\Psi$  methods and dilepton distributions.

- ✓ This is correlated with the attempt to define a pseudo top. How needed/useful is that?

- ✓ Alternative top mass definitions.

Alternative mass definitions that reflect the physics are beneficial (known from e+e-). Less clear at hadron colliders.

- ✓ Renormalon ambiguity in top mass definition.

Pole mass of the top quark suffers from the so-called renormalon ambiguity. This implies an additional irreducible uncertainty of several hundred MeV's on the top pole mass. Not an issue for short distance masses. Currently, at hadron colliders, this is a subdominant uncertainty.

- ✓ Higher-order corrections.

Important source of uncertainty. State of the art NLO QCD; not always included.

- ✓ Unstable top and finite top width effects.

Understood for e+e-.

Computed at NLO for hadron colliders. Could affect certain distributions.

G. Bevilacqua, M. Czakon, A. van Hameren, C. G. Papadopoulos and M. Worek, JHEP **1102**, 083 (2011) [arXiv:1012.4230]

A. Denner, S. Dittmaier, S. Kallweit and S. Pozzorini, JHEP **1210**, 110 (2012) [arXiv:1207.5018]

Melnikov, Schulze

Not really used so far in top mass studies.

- ✓ Bound-state effects in top pair production at hadron colliders.

When the ttbar pair is produced with small relative velocity (i.e. close to threshold) bound-state formation begins. These effects can affect the shape of differential distributions within few GeV away from the threshold. Special care must be taken if a measurement is sensitive to such effects.

In usual “inclusive” observables (like total x-section) this effect is diluted to about 1%.

## Methods for $m_{\text{top}}$ determination

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## Methods for $m_{\text{top}}$ determination: Matrix Element Methods

- ✓ The backbone of the Tevatron studies as well as the most precise LHC ones.  
Performed in all final states.
- ✓ Measured objects are compared with expectations from the LO  $t\bar{t}$  production and decay diagrams convoluted with the detector response.
- ✓ Method's power comes from the fact that the likelihood for each event to be consistent with both  $t\bar{t}$  and background production is calculated; greater weight is assigned to events that are more likely to be from  $t\bar{t}$  when measuring  $m_{\text{top}}$ .
- ✓ Issue: incorrect modeling due to missing theory corrections.

# Methods for $m_{\text{top}}$ determination: Matrix Element Methods

Projections based on CMS lepton-plus-jet analysis:

S. Chatrchyan *et al.* [CMS Collaboration], JHEP **1212**, 105 (2012) [arXiv:1209.2319]

	Ref.[2]	Projections				
CM Energy	7 TeV	14 TeV				
Cross Section	167 pb	951 pb				
Luminosity	$5\text{fb}^{-1}$	$100\text{fb}^{-1}$	$300\text{fb}^{-1}$	$3000\text{fb}^{-1}$		
Pileup	9.3	19	30	19	30	95
Syst. (GeV)	0.95	0.7	0.7	0.6	0.6	0.6
Stat. (GeV)	0.43	0.04	0.04	0.03	0.03	0.01
<b>Total</b>	<b>1.04</b>	<b>0.7</b>	<b>0.7</b>	<b>0.6</b>	<b>0.6</b>	<b>0.6</b>
Total (%)	0.6	0.4	0.4	0.3	0.3	0.3

Scenario	Dominant Uncertainties
Ref.[2]	Jet Energy Scale, Hadronization, Soft QCD, ISR/FSR
$100\text{ fb}^{-1}/19$ PU	Jet Energy Scale, Hadronization, Soft QCD, ISR/FSR
$100\text{ fb}^{-1}/30$ PU	Jet Energy Scale, Hadronization, Soft QCD, ISR/FSR, Pileup
$300\text{ fb}^{-1}/19$ PU	Jet Energy Scale, Hadronization, Soft QCD, ISR/FSR
$300\text{ fb}^{-1}/30$ PU	Jet Energy Scale, Hadronization, Soft QCD, ISR/FSR, Pileup
$3000\text{ fb}^{-1}/95$ PU	Jet Energy Scale, Hadronization, Soft QCD, ISR/FSR, Pileup

TABLE II: Dominant systemic uncertainties for each scenario

- ✓ Projections beyond 14 TeV require full detector simulation. Not done here.
- ✓ Pileup and UE become more important at higher energy/pileup.
- ✓ ISR/FSR become dominant uncertainties at high luminosity (unlike current measurements)
- ✓ Extra 300MeV uncertainty added by hand.

# Methods for $m_{top}$ determination: CMS endpoint method

S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1304.5783

A kinematical method: utilizes the strong correlation between the maximum of the  $M_{bl}$  distribution and  $m_{top}$ .

	Ref.[8]	Projections		
CM Energy	7 TeV	14 TeV		
Cross Section	167 pb	951 pb		
Luminosity	$5fb^{-1}$	$100fb^{-1}$	$300fb^{-1}$	$3000fb^{-1}$
Syst. (GeV)	1.8	1.0	0.7	0.5
Stat. (GeV)	0.90	0.10	0.05	0.02
<b>Total</b>	<b>2.0</b>	<b>1.0</b>	<b>0.7</b>	<b>0.5</b>
Total (%)	1.2	0.6	0.4	0.3

Scenario	Dominant Uncertainties
Ref.[8]	Jet Energy Scale, Hadronization, Soft QCD
$100 fb^{-1}$	Jet Energy Scale, Hadronization, Soft QCD
$300 fb^{-1}$	Jet Energy Scale, Hadronization, Soft QCD
$3000 fb^{-1}$	Jet Energy Scale, Hadronization

TABLE IV: Dominant systemic uncertainties for each scenario

- ✓ ISR/FSR and pileup do not play a role at high luminosity. (unlike conventional methods)
- ✓ Does not rely on MC for internal calibration (analytical with data-driven backgrounds).
- ✓ Less likely to be affected by bSM corrections.
- ✓ Nonetheless, higher order effects do affect the endpoint position (particularly top widths)  
NLO calculations do exist – not utilized.

G. Bevilacqua, M. Czakon, A. van Hameren, C. G. Papadopoulos and M. Worek, JHEP **1102**, 083 (2011) [arXiv:1012.4230]

A. Denner, S. Dittmaier, S. Kallweit and S. Pozzorini, JHEP **1210**, 110 (2012) [arXiv:1207.5018]

## Methods for $m_{\text{top}}$ determination: $J/\Psi$ method

A. Kharchilava, Phys. Lett. B **476**, 73 (2000) [hep-ph/9912320]

A different method: no reconstruction is involved. Known at NLO.

	Ref. analysis	Projections			
		8 TeV	14 TeV	33 TeV	100 TeV
CM Energy	8 TeV	14 TeV		33 TeV	100 TeV
Cross Section	240 pb	951 pb		5522 pb	25562 pb
Luminosity	$20 fb^{-1}$	$100 fb^{-1}$	$300 fb^{-1}$	$3000 fb^{-1}$	$3000 fb^{-1}$
Theory (GeV)	-	1.5	1.5	1.0	1.0
Stat. (GeV)	7.00	1.8	1.0	0.3	0.1
Total	-	2.3	1.8	1.1	1.0
Total (%)	-	1.3	1.0	0.6	0.4

TABLE VI: Extrapolations based on the  $J/\Psi$  method.

Estimates from NLO QCD.

S. Biswas, K. Melnikov and M. Schulze, JHEP **1008**, 048 (2010) [arXiv:1006.0910]  
(see also) A. Denner, S. Dittmaier, S. Kallweit and S. Pozzorini, JHEP **1210**, 110 (2012) [arXiv:1207.5018]

NNLO accuracy assumed in some extrapolations.

Main source: B-fragmentation. Likely will be irreducible unless new e+e- data.

## Methods for $m_{\text{top}}$ determination: $m_{\text{top}}$ from kinematic distributions

### ✓ Total cross-section:

Allows extraction with about 3% uncertainty due to limited sensitivity to  $m_{\text{top}}$ .

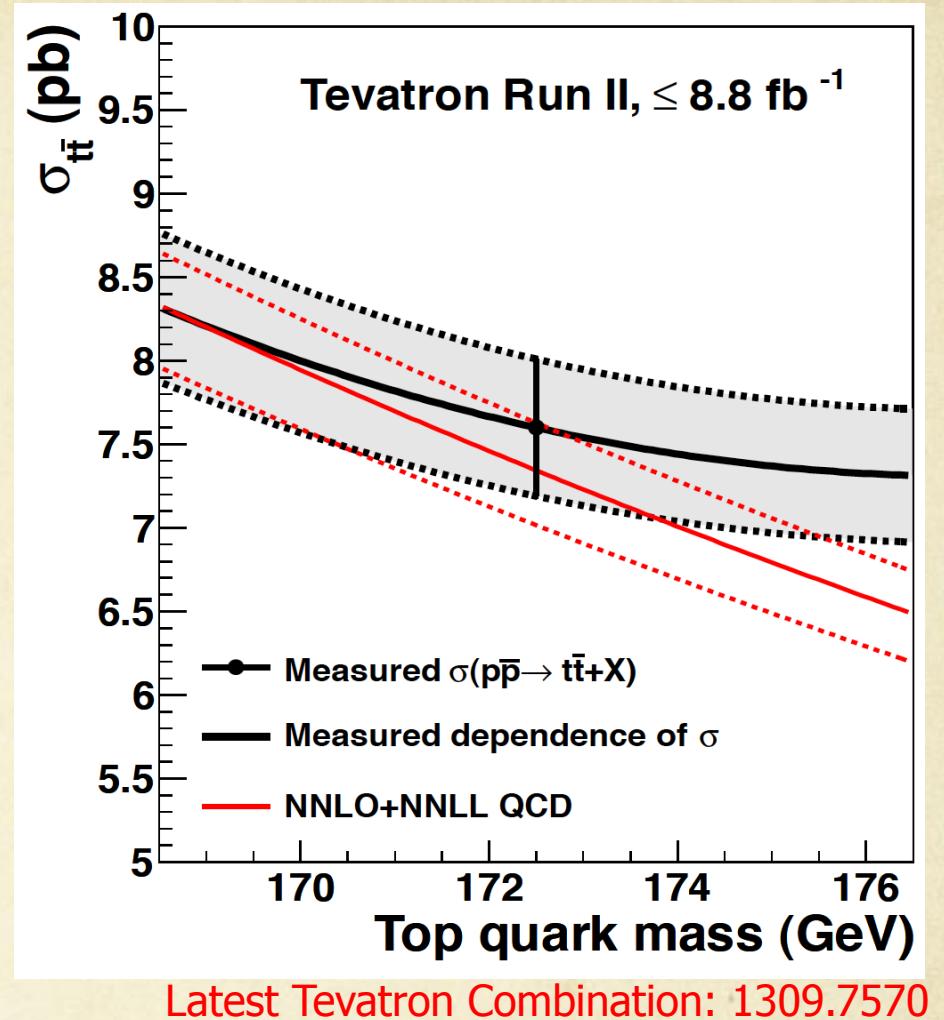
### ➤ Positive features:

Good theory control (NNLO)  
Small non-perturbative and width effects

### ➤ Negatives:

Small sensitivity (unlikely to improve)

- ✓ At present there are inconsistently applied acceptance corrections (i.e. LO or NLO not NNLO). Still, likely a small effect.



- ✓ Extraction suggested from  $t\bar{t}+\text{jet}$ .

S. Alioli, P. Fernandez, J. Fuster, A. Irles, S. -O. Moch, P. Uwer and M. Vos, arXiv:1303.6415

Estimates for contributions from unknown corrections – below 1 GeV.

Method is MC dependent and involves  $t$  ( $t\bar{t}$ ) reconstruction

- ✓ Dilepton distributions

- No reconstruction
- Minimal shower and NP sensitivity. Reliably computable at fixed order.
- Potential for 14 TeV at 1.5 GeV.

S. Biswas, K. Melnikov and M. Schulze, JHEP **1008**, 048 (2010) [arXiv:1006.0910]

- Further studies in progress

Frederix, Frixione, Mitov, in progress.      <<< Second part of the talk

- ✓ The machine where the ultimate precision of 100MeV or less can be achieved.
- ✓ Best approach is threshold scan.
- ✓ Continuum production also possible.
- ✓ Similar at ILC and CLIC.
- ✓ Interesting question: is it possible to measure  $m_{top}$  at c.m. energy of, say, 250GeV, i.e. below the threshold?
- ✓ Given the presumed ILC schedule this might imply few more years of waiting ...

- ✓ One hardly mentioned problem!
- ✓ There is the possibility that undetected corrections to top production might shift the top mass measurements (measure top+bSM but theory assumes pure SM).

Example: stop  $\rightarrow$  top+X

If the stop is light, the event looks top-like!

- ✓ The strongest constraint on bSM contributions to  $m_{\text{top}}$  comes from the CMS end-point method

S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1304.5783

- ✓ The method is kinematic: it measures the position of the end-point of the spectrum of top decay products. This is independent of the top production mechanism.
- ✓ The total error from the measurement is just above 2.0 GeV and agrees with the world average
- ✓ From here we can conclude that bSM contributions to  $m_{\text{top}}$  are not larger than  $\sim 2 \text{GeV}$ .
- ✓ Dedicated studies are welcome. Likely they will be model dependent; any model-independent arguments would be very valuable.

# **Top mass from leptonic distributions**

Frederix, Frixione, Mitov; to appear

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The message I'd like to convey: the questions I raised so far are not "academic".

Example: look at the spread across current measurements

➤ Current World Average:  $m_{\text{top}} = 173.34 \pm 0.76 \text{ GeV}$

arXiv:1403.4427

➤ New CMS (I+j):  $m_{\text{top}} = 172.04 \pm 0.19 \text{ (stat.+JSF)} \pm 0.75 \text{ (syst.) GeV.}$  TOP-14-001

Comparable uncertainties; rather different central values!

This is possible in the context of my discussion: different theory systematics.

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In order to properly understand and estimate the theory systematics  
we propose a particular observable

$$pp \rightarrow t\bar{t} + X$$

$$t \rightarrow W + b + X$$

$$W \rightarrow \ell + \nu_\ell$$

These are ttbar dilepton events,  
subject to standard cuts:

$$|\eta_\ell| \leq 2.4 , |\eta_b| \leq 2.4 ,$$

$$p_{T,\ell} \geq 20 \text{ GeV} , p_{T,b} \geq 30 \text{ GeV}$$

- Construct the distributions from leptons only
- Require b-jets within the detector (i.e. integrate over)

The definition of the observable possesses several important properties:

- It is inclusive of hadronic radiation, which makes it well-defined to all perturbative orders in the strong coupling,
- It does not require the reconstruction of the  $t$  and/or  $\bar{t}$  quarks (indeed we do not even speak of  $t$  quark),
- Due to its inclusiveness, the observable is as little sensitive as possible to modelling of hadronic radiation. This feature increases the reliability of the theoretical calculations.

The top mass is extracted from the shapes of the following distributions: (not normalizations)

kinematic distribution

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$$\begin{aligned} p_T(\ell^+) \\ p_T(\ell^+ \ell^-) \\ M(\ell^+ \ell^-) \\ E(\ell^+) + E(\ell^-) \\ p_T(\ell^+) + p_T(\ell^-) \end{aligned}$$

Working with distributions directly is cumbersome.  
Instead, utilize the first 4 moments of each distribution

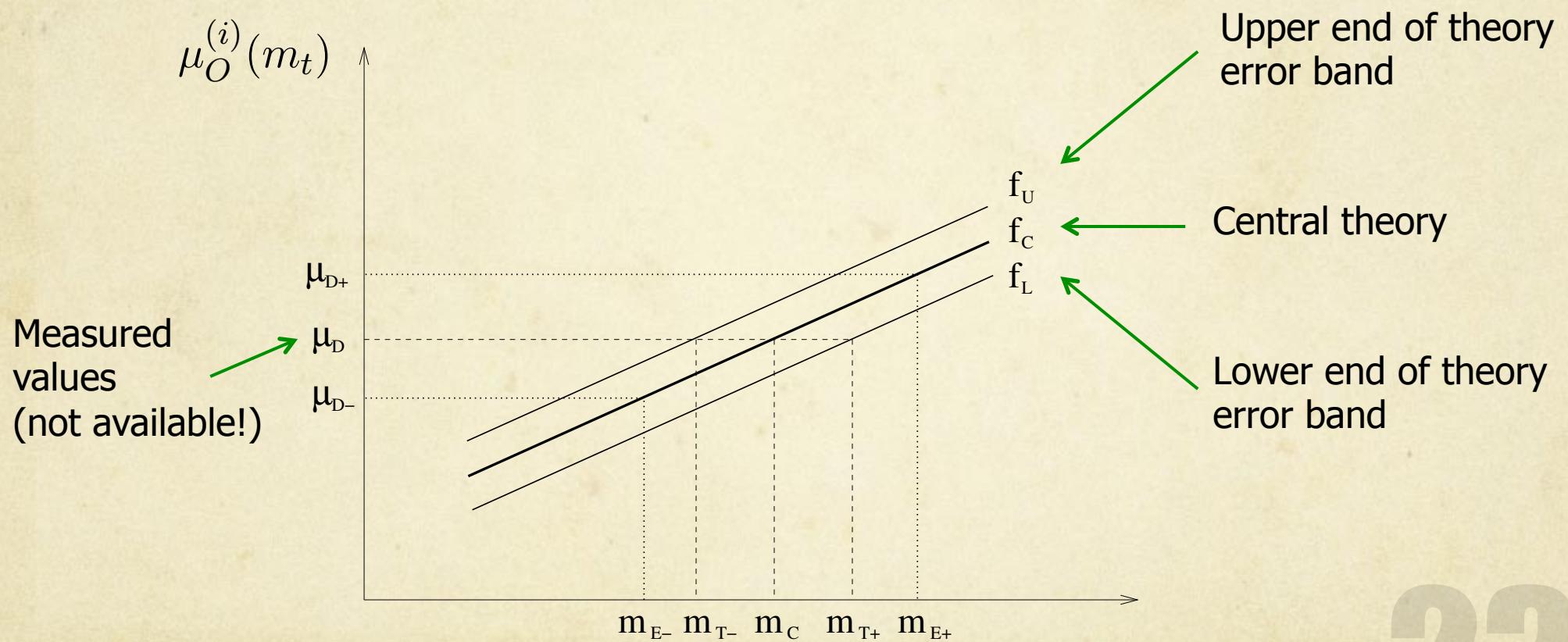
$$\sigma = \int d\sigma \quad \mu_O^{(i)} = \frac{1}{\sigma} \int d\sigma O^i \quad \mu_O^{(0)} = 1, \quad \mu_O^{(1)} = \langle O \rangle$$

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Note: both are subject to cuts (or no cuts); we tried both.

Here is how it all works:

- 1) Compute the dependence of the moments  $\mu_O^{(i)}(m_t)$  on the top mass
- 2) Measure the moment
- 3) Invert 1) and 2) to get the top mass (would be the pole mass, since this is what we use)



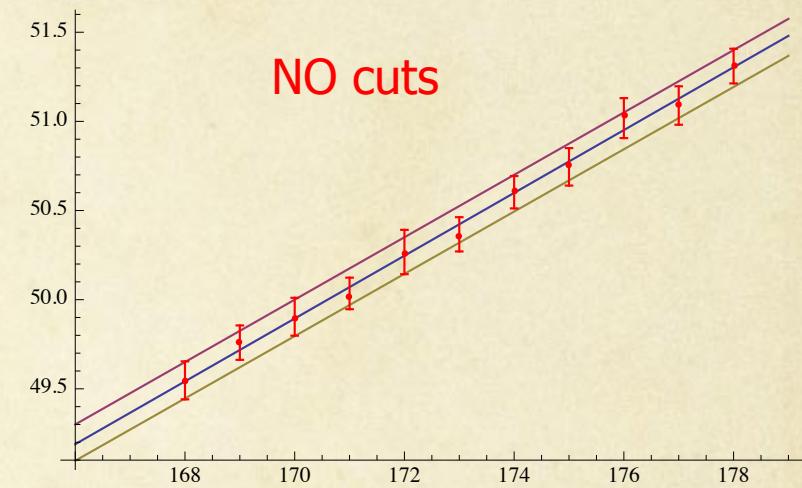
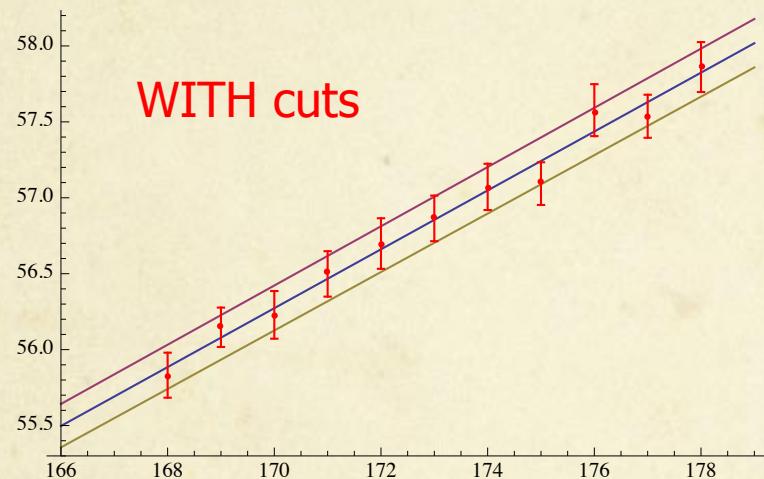
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How to compute the theory error band for  $\mu_O^{(i)}(m_t)$  ?

- Compute  $\mu_O^{(i)}(m_t)$  for a finite number of  $m_t$  values:  $m_t = (168, 169, \dots, 178)$  GeV  
Then get best straight line fit (works well in this range).

Example:

- Single lepton  $P_T$
- Subject to cuts



- ✓ Errors: pdf and scale variation; restricted independent variation

$$0.5 \leq \xi_F, \xi_R \leq 2 \quad \xi_{F,R} = \mu_{F,R}/\hat{\mu} \text{ and } \hat{\mu} \text{ is a reference scale}$$

- ✓ There are statistical fluctuation (from MC even generation) No issue for lower moments  
1M events; 30% pass the cuts.

## Theory systematics

- We access them by computing the observables in many different ways.
- For a fair (albeit biased) comparison across setups and moments we use pseudodata (PD) generated by us
- Compare the systematics by comparing the top mass “extracted” by each setup from PD.

6 Setups:

label	fixer order accuracy	parton shower/fixed order	spin correlations
1	LO	PS	-
2	LO	PS	MS
3	NLO	PS	-
4	NLO	PS	MS
5	NLO	FO	-
6	LO	FO	-

3 FR Scales:

$$\begin{aligned}\hat{\mu}^{(1)} &= \frac{1}{2} \sum_i m_{T,i}, \quad i \in (t, \bar{t}), \\ \hat{\mu}^{(2)} &= \frac{1}{2} \sum_i m_{T,i}, \quad i \in \text{final state}, \\ \hat{\mu}^{(3)} &= m_t,\end{aligned}$$

All is computed with aMC@NLO (with Herwig)

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# Theory systematics: impact of shower effects

obs.	$m_t^{(3)} - m_t^{(5)}$	$m_t^{(3)} - m_t^{\text{pd}}$	$m_t^{(1)} - m_t^{(6)}$	$m_t^{(1)} - m_t^{\text{p}}$
1	$-0.35^{+1.14}_{-1.16}$	+0.12	$-2.17^{+1.50}_{-1.80}$	-0.67
2	$-4.74^{+1.98}_{-3.10}$	+11.14	$-9.09^{+0.76}_{-0.71}$	+14.19
3	$+1.52^{+2.03}_{-1.80}$	-8.61	$+3.79^{+3.30}_{-4.02}$	-6.43
4	$+0.15^{+2.81}_{-2.91}$	-0.23	$-1.79^{+3.08}_{-3.75}$	-1.47
5	$-0.30^{+1.09}_{-1.21}$	+0.03	$-2.13^{+1.51}_{-1.81}$	-0.67

NLO

LO

label	fixer order accuracy	parton shower/fixed order	spin correlations
1	LO	PS	-
2	LO	PS	MS
3	NLO	PS	-
4	NLO	PS	MS
5	NLO	FO	-
6	LO	FO	-

- Setups 2,3 are anomalous (More later).
- Clearly big impact of NLO corrections (shower matters more at LO).

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NOTE: proper PS study would require Pythia etc. Not done here.

# Theory systematics: impact of NLO vs LO effects

obs.	$m_t^{(4)} - m_t^{(2)}$	$m_t^{(4)} - m_t^{\text{pd}}$	$m_t^{(3)} - m_t^{(1)}$	$m_t^{(3)} - m_t^{\text{pd}}$	$m_t^{(5)} - m_t^{(6)}$	$m_t^{(5)} - m_t^{\text{pd}}$
1	$+1.16^{+1.43}_{-1.60}$	$+0.41$	$+0.79^{+1.43}_{-1.60}$	$+0.12$	$-1.03^{+1.22}_{-1.43}$	$+0.47$
2	$-2.79^{+1.27}_{-1.65}$	$-1.18$	$-3.05^{+1.35}_{-1.64}$	$+11.14$	$-7.41^{+1.64}_{-2.72}$	$+15.87$
3	$-0.73^{+3.21}_{-3.45}$	$+0.84$	$-2.18^{+3.03}_{-3.30}$	$-8.61$	$+0.09^{+2.42}_{-2.91}$	$-10.13$
4	$+1.74^{+3.27}_{-3.78}$	$+0.16$	$+1.23^{+3.10}_{-3.61}$	$-0.23$	$-0.70^{+2.79}_{-3.09}$	$-0.38$
5	$+0.99^{+1.42}_{-1.72}$	$+0.25$	$+0.70^{+1.40}_{-1.72}$	$+0.03$	$-1.13^{+1.23}_{-1.33}$	$+0.33$

PS+MS

PS

-

label	fixer	order	accuracy	parton shower/fixed order	spin correlations
1		LO		PS	-
2		LO		PS	MS
3		NLO		PS	-
4		NLO		PS	MS
5		NLO		FO	-
6		LO		FO	-

- Setups 2,3 are anomalous (More later).
- Clearly big impact of NLO corrections.

# Theory systematics: impact of Spin-Correlations effects

obs.	$m_t^{(4)} - m_t^{(3)}$	$m_t^{(4)} - m_t^{\text{pd}}$	$m_t^{(2)} - m_t^{(1)}$	$m_t^{(2)} - m_t^{\text{pd}}$
1	$+0.29^{+1.17}_{-1.14}$	+0.41	$-0.08^{+1.66}_{-1.96}$	-0.75
2	$-12.32^{+1.62}_{-2.13}$	-1.18	$-12.58^{+0.90}_{-0.94}$	+1.60
3	$+9.45^{+2.36}_{-2.16}$	+0.84	$+8.00^{+3.74}_{-4.26}$	+1.57
4	$+0.39^{+2.93}_{-3.16}$	+0.16	$-0.11^{+3.42}_{-4.16}$	-1.58
5	$+0.22^{+1.12}_{-1.28}$	+0.25	$-0.06^{+1.65}_{-2.07}$	-0.73

NLO+PS

LO+PS

label	fixer	order	accuracy	parton shower	/fixed order	spin correlations
1		LO		PS		-
2		LO		PS		MS
3		NLO		PS		-
4		NLO		PS		MS
5		NLO		FO		-
6		LO		FO		-

- NOTE setups 2,3 Huge dependence on spin correlations  
(the place with strongest sensitivity to spin-correlations known!)
- NLO corrections make a difference.

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# "Best" Theory Predictions (NLO+PS+MS): choice of scale and Moment

$$m_t^{\text{pd}} = 174.32 \text{ GeV}$$

$$\xi^2 \text{ per d.o.f.}$$

scale	$i = 1$	$i = 1 \oplus 2$	$i = 1 \oplus 2 \oplus 3$	$i = 1 \oplus 2 \oplus 3 \oplus 4$
1	$174.48^{+0.73}_{-0.77}[5.0]$	$174.55^{+0.72}_{-0.76}[5.0]$	$174.56^{+0.71}_{-0.76}[5.1]$	$174.06^{+0.67}_{-0.71}[7.2]$
2	$174.73^{+0.77}_{-0.80}[4.3]$	$174.74^{+0.76}_{-0.79}[4.3]$	$174.91^{+0.75}_{-0.79}[4.1]$	$175.51^{+0.73}_{-0.79}[4.0]$
3	$172.54^{+1.03}_{-1.07}[1.6]$	$172.46^{+0.99}_{-1.05}[1.6]$	$172.22^{+0.95}_{-1.04}[1.38]$	$171.90^{+0.92}_{-0.98}[1.3]$
$1 \oplus 2 \oplus 3$	$174.16^{+0.81}_{-0.85}$	$174.17^{+0.80}_{-0.84}$	$174.17^{+0.78}_{-0.84}$	$174.08^{+0.75}_{-0.80}$

All 5 observables  
NLO+PS+MS

scale	$i = 1$	$i = 1 \oplus 2$	$i = 1 \oplus 2 \oplus 3$	$i = 1 \oplus 2 \oplus 3 \oplus 4$
1	$174.67^{+0.75}_{-0.77}[3.0]$	$174.67^{+0.75}_{-0.77}[3.0]$	$174.61^{+0.74}_{-0.77}[3.17]$	$174.14^{+0.71}_{-0.73}[5.2]$
2	$174.81^{+0.83}_{-0.80}[6.2]$	$174.80^{+0.82}_{-0.80}[6.2]$	$174.85^{+0.82}_{-0.80}[6.1]$	$175.31^{+0.80}_{-0.80}[5.5]$
3	$172.63^{+1.85}_{-1.16}[0.2]$	$172.64^{+1.82}_{-1.15}[0.2]$	$172.58^{+1.81}_{-1.15}[0.2]$	$172.30^{+1.80}_{-1.07}[0.2]$
$1 \oplus 2 \oplus 3$	$174.44^{+0.92}_{-0.87}$	$174.44^{+0.92}_{-0.87}$	$174.43^{+0.91}_{-0.86}$	$174.32^{+0.88}_{-0.83}$

Observables 1,4,5  
NLO+PS+MS

scale	$i = 1$	$i = 1 \oplus 2$	$i = 1 \oplus 2 \oplus 3$
1	$174.73^{+0.80}_{-0.79}[0.2]$	$174.73^{+0.80}_{-0.79}[0.2]$	$174.72^{+0.80}_{-0.79}[0.2]$
2	$174.78^{+0.90}_{-0.90}[0.6]$	$174.78^{+0.90}_{-0.90}[0.6]$	$174.78^{+0.90}_{-0.90}[0.6]$
3	$172.73^{+2.0}_{-1.2}[0.5]$	$172.73^{+1.96}_{-1.19}[0.5]$	$172.73^{+1.96}_{-1.19}[0.5]$
$1 \oplus 2 \oplus 3$	$174.46^{+0.99}_{-0.92}$	$174.46^{+0.99}_{-0.92}$	$174.45^{+0.99}_{-0.92}$

Observable 1  
NLO+PS+MS

$$\begin{aligned}\hat{\mu}^{(1)} &= \frac{1}{2} \sum_i m_{T,i}, \quad i \in (t, \bar{t}), \\ \hat{\mu}^{(2)} &= \frac{1}{2} \sum_i m_{T,i}, \quad i \in \text{final state}, \\ \hat{\mu}^{(3)} &= m_t,\end{aligned}$$

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## Theory systematics: Predictions

observable; setup	$i = 1$	$i = 1 \oplus 2$	$i = 1 \oplus 2 \oplus 3$
all; LO+PS	$187.90^{+0.6}_{-0.6}[428.3]$	$187.71^{+0.60}_{-0.60}[424.2]$	$187.83^{+0.58}_{-0.60}[442.8]$
all; LO+PS+MS	$175.98^{+0.63}_{-0.69}[16.9]$	$176.05^{+0.63}_{-0.68}[17.8]$	$176.12^{+0.61}_{-0.68}[18.9]$
all; NLO+PS	$175.43^{+0.74}_{-0.80}[29.2]$	$176.20^{+0.73}_{-0.79}[30.1]$	$175.67^{+0.73}_{-0.76}[31.2]$
all; NLO <sub>FO</sub>	$174.41^{+0.72}_{-0.73}[96.6]$	$174.82^{+0.71}_{-0.73}[93.1]$	$175.44^{+0.70}_{-0.68}[94.8]$
all; LO <sub>FO</sub>	$197.31^{+0.42}_{-0.35}[2496.1]$	$197.19^{+0.42}_{-0.35}[2505.6]$	$197.48^{+0.36}_{-0.35}[3005.6]$
1,4,5; LO+PS	$173.68^{+1.08}_{-1.31}[0.8]$	$173.68^{+1.08}_{-1.31}[0.9]$	$173.75^{+1.08}_{-1.31}[0.9]$
1,4,5; LO+PS+MS	$173.61^{+1.10}_{-1.34}[1.0]$	$173.63^{+1.10}_{-1.34}[1.0]$	$173.62^{+1.10}_{-1.34}[1.0]$
1,4,5; NLO+PS	$174.40^{+0.75}_{-0.81}[3.5]$	$174.43^{+0.75}_{-0.81}[3.5]$	$174.60^{+0.75}_{-0.79}[3.2]$
1,4,5; NLO <sub>FO</sub>	$174.73^{+0.72}_{-0.74}[5.5]$	$174.72^{+0.71}_{-0.74}[5.6]$	$175.18^{+0.64}_{-0.71}[4.6]$
1,4,5; LO <sub>FO</sub>	$175.84^{+0.90}_{-1.05}[1.2]$	$175.75^{+0.89}_{-1.05}[1.2]$	$175.82^{+0.89}_{-1.04}[1.2]$

$$m_t^{\text{pd}} = 174.32 \text{ GeV}$$

$$\xi^2 \text{ per d.o.f.}$$

## Conclusions

- ✓ New developments have resurrected the interest in knowing  $m_{top}$  precisely
  - ✓ Vacuum Stability in SM
  - ✓ Higgs Inflation
- ✓ There are many dedicated hadron collider measurements.  
They return consistent values around  $m_{top} = 173$  GeV  
and uncertainty (mostly on the measurement!) of below 1 GeV.
- ✓ Questions remain: can there be a significant additional theoretical systematics  $O(1$  GeV) ?
- ✓ This is not an abstract problem:  $m_{top}$  is not an observable and so is a theoretically defined concept.
- ✓ The issue of various mass definitions is a non-issue at present for hadron colliders.
- ✓ e+e- colliders offer the real possibility of measuring  $m_{top}$  with  $\times 10$  precision, i.e.  $O(100$  MeV)  
But how long would we have to wait for a ttbar threshold scan?
- ✓ New physics contributions to  $m_{top}$  are a totally open question. Upper limit of  $O(2$  GeV)  
likely can be placed at present.
- ✓ Proposed new method, with emphasis on control over theory systematics.  
NLO vs LO: 1 GeV; spin correlations crucial. Awaiting the measurement (100k events exist!).