Leptons, Loops and Dipoles in Randall-Sundrum Models

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Outline:

1. Brief intro to RS
2. General strategy
3. Calculations in 5D
4. Lepton phenomenology
Minimal RS model: Setup

Slice of AdS in interval \([1/k, 1/T]\) in conformal coordinates

\[
    ds^2 = \left(\frac{1}{kz}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - d^2 z)
\]

\[
    M_{Pl}^{4d} \approx \frac{M_{Pl}^{5d}}{k}
\]

\[
    \varepsilon \equiv \frac{T}{k} \approx 10^{-16} \approx \frac{1}{M_{Pl}^{4d}}
\]

proper distance between branes (\(=\) boundaries):

\[
    1/k \times \ln(k/T)
\]

Minimal RS model: All SM fields except the Higgs in the bulk; no additional fields

Custodially protected RS: Add \(SU(2)_R\) gauge fields and increase fermion content

lowest KK resonance mass \(\sim 2.5T\)
Minimal RS model: Setup

- in the KK mode language

Note that:

\[ Y^{AD}_{ij} \sim f^{(0)}_{Li} \left( \frac{1}{T} \right) y^{(5D)}_{ij} g^{(0)}_{E_i} \left( \frac{1}{T} \right) \]
Minimal RS model: Setup

- the equation in a picture

Note that:

\[ Y_{ij}^{AD} \sim f_{L_i}^{(0)} \frac{1}{T} y_{ij}^{(5D)} g_{E_i}^{(0)} \frac{1}{T} \]
Boundary Conditions and Spectrum

Boundary conditions (≡ parities under $S^1/Z_2$ orbifold $Z_2$ symmetries)

$$
\partial_z A_\mu \big|_{z=1/T, 1/k} = 0 \\
A_5 \big|_{z=1/T, 1/k} = 0 \\
L_R \big|_{z=1/T, 1/k} = 0 \\
E_L \big|_{z=1/T, 1/k} = 0
$$

Only $L_L$, $E_R$ have zero modes ($m_0 = 0$). KK excitations describe massive gauge bosons and vector-like fermions.

- 5D gauge fixed such that $A_\mu$ and $A_5$ decouple $\rightarrow \xi$
- wrong-chirality Higgs couplings (WCHC)

$$
\int d^4x \left[ (\bar{L}\Phi)E + h.c. \right] \big|_{z=1/T} = \int d^4x \left[ (\bar{L}\Phi)E_R + (\bar{L}_R\Phi)E_L + h.c. \right] \big|_{z=1/T}
$$

The WCHC $(\bar{L}_R\Phi)E_L$ vanishes for a brane-localized Higgs due to the boundary condition. Too naive!
RS & Lepton-flavour observables

- Exhaustive phenomenology of tree-level processes (electroweak, flavour)
  from gluon FCNCs KK gluon mass $\sim 20$ TeV w/o extra flavour structure [e.g. Csaki, Falkowski, Weiler, 2008]

- Higgs production in gluon-gluon fusion [Casagrande et al., 2010; Azatov et al., 2010; Carena et al., 2012; Malm et al., 2013, Archer et al. 2014]

- quark FCNCs: $b \to s\gamma$, $b \to sg$ [Gedelia, Isidori, Perez 2009; Blanke et al., 2012], and $c \to ug$ [Delaunay et al., 2012]

- lepton observables
  - $a_\mu - a^{SM} = 239(63)(48) \times 10^{-11}$
  - $\text{Br}(\mu \to e\gamma) < 5.7 \times 10^{-13} (10^{-14})$ MEG (upgraded)
  - $\text{Br}(\mu \to 3e) < 1.0 \times 10^{-12}$ SINDRUM
  - $\text{Br}^{Au(Al)}(\mu N \to eN) < 7 \times 10^{-13} (10^{-16})$ SINDRUM II (DeeMe, Mu2E, COMET)
Lepton Flavour Observables

- $g - 2$
  - pure loop-induced effect in RS; flavour diagonal; [Davoudiasl et al. 2000]

- $d_e$
  - pure loop-induced effect in RS; flavour diagonal, $CP$ violating;

- $\mu \rightarrow e\gamma$
  - pure loop-induced effect in RS; LFV [Agashe et al., 2006, Csaki et al., 2010]

- $\mu \rightarrow 3e$
  - tree-dominated effect (?) [Agashe et al., 2006, Grojean et al., 2003; Csaki et al., 2010]

- $\mu \rightarrow e$ in Au
  - tree-dominated effect (?) ; provides “orthogonal” information [Agashe et al., 2006; Chang & Ng, 2005; Csaki et al., 2010]

This talk:
→ a general strategy
→ How do you calculate loops in 5D?
Lagrangian (minimal model)

\[ S_{5D} = \int d^4x \int_{1/k}^{1/T} dz \sqrt{G} \left\{ - \frac{1}{4} F_{MN}^M F_{MN} - \frac{1}{4} W_{a, MN}^a W_{MN}^a \right\} + \sum_{\psi=E,L} \left( e^M_m \left[ \frac{i}{2} \bar{\psi}_i \Gamma^m (D_M - \bar{D}_M) \psi_i \right] - M_{\psi_i} \bar{\psi}_i \psi_i \right) \right\} + S_{\text{GF+ghost}} \\
+ \int d^4x \left\{ (D^\mu \Phi)^\dagger D_\mu \Phi - V(\Phi) - \left( \frac{T}{k} \right)^3 \left[ y_{ij}^{(5D)} (\bar{L}_i \Phi) E_j + \text{h.c.} \right] \right\} \\
\Gamma^m = (\gamma^\mu, i\gamma_5) \quad e^M_m = (kz) \delta^M_m \quad M_{\psi_i} = c_L/E_i \cdot k \\

Two scales:

* \( k \) determines the size of all parameters
* \( T \) put in by hand to address the quantum corrections to the Higgs mass
Strategy

- distinct scale hierarchy $k \gg T \gg v \gg m_\ell$

  matching scale $\mu$

- strategy:

  1. Step (in symmetric phase $\to$ no vev):
     integrate out the “bulk” $\to$ match onto an $SU(2)_L \times U(1)_Y$ symmetric effective theory

     $$\mathcal{L}_{RS} \to \mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{T^2} \sum_i C_i O_i$$

     [Buchmüller & Wyler]

  2. Step: change into the “broken” phase

  3. Step: compute observables in using 4d effective theory
1. Step: EFT before EWSB

- distinct scale hierarchy $k \gg T \gg v \gg m_\ell$
  
- strategy: integrate out the “bulk” by matching onto an $SU(3) \times SU(2) \times U(1)_Y$

invariant Lagrangian at a scale $T \gg \mu \gg v$ in the unbroken theory:

$$\mathcal{L}_{RS} \rightarrow \mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{T^2} \sum_i C_i O_i$$

relevant operators include

$$\sum_i C_i O_i \supset a_{B,ij} \bar{L}_i \Phi \sigma_{\mu\nu} E_j B^{\mu\nu} + a_{W,ij} \bar{L}_i \tau^a \Phi \sigma_{\mu\nu} E_j W^{a,\mu\nu} + \text{h.c.}$$

+ $c_{1,i} (\bar{E}_i \gamma_\mu E_i)(\Phi^\dagger iD^\mu \Phi)$ + $c_{2,i} (\bar{L}_i \gamma_\mu L_i)(\Phi^\dagger iD^\mu \Phi)$

+ $c_{3,i} (\bar{L}_i \gamma^\mu \tau^a L_i)(\Phi^\dagger i \tau^a D_\mu \Phi)$

+ $b_{1,ijkl} (\bar{L}_i \gamma_\mu L_i)(\bar{L}_k \gamma^\mu L_i) + b_{2,ij} (\bar{L}_i \gamma_\mu L_i)(\bar{E}_j \gamma^\mu E_j)$

+ $b_{3,ij} (\bar{E}_i \gamma_\mu E_i)(\bar{E}_j \gamma^\mu E_j)$ + ...

+ $b_{L\tau Q,ij} (\bar{L}_i \gamma_\mu \tau^a L_i)(\bar{Q}_j \gamma^\mu \tau^a Q_j) + b_{LQ,ij} (\bar{L}_i \gamma_\mu L_i)(\bar{Q}_j \gamma^\mu Q_j)$

+ $b_{EQ,ij} (\bar{E}_i \gamma_\mu E_i)(\bar{Q}_j \gamma^\mu Q_j) + b_{LQ,ij} (\bar{L}_i \gamma_\mu L_i)(\bar{u}_j \gamma^\mu u_j)$

+ $b_{Ed,ij} (\bar{E}_i \gamma_\mu E_i)(\bar{d}_j \gamma^\mu d_j) + b_{Eu,ij} (\bar{E}_i \gamma_\mu E_i)(\bar{u}_j \gamma^\mu u_j)$

+ $b_{Ed,ij} (\bar{E}_i \gamma_\mu E_i)(\bar{d}_j \gamma^\mu d_j) + \ldots$
1. Step: EFT before EWSB

relevant operators include

\[ \sum_i C_i O_i \supset a_{B,ij} \bar{L}_i \Phi \sigma_{\mu\nu} E_j B^{\mu\nu} + a_{W,ij} \bar{L}_i \tau^a \Phi \sigma_{\mu\nu} E_j W^{a,\mu\nu} + \text{h.c.} \]

+ \( c_{1,i} (\bar{E}_i \gamma_{\mu} E_i) (\Phi^{\dagger} iD^{\mu} \Phi) \)
+ \( c_{2,i} (\bar{L}_i \gamma_{\mu} L_i) (\Phi^{\dagger} iD^{\mu} \Phi) \)
+ \( c_{3,i} (\bar{L}_i \gamma^\mu \tau^a L_i) (\Phi^{\dagger} i\tau^a D_{\mu} \Phi) \)
+ \( b_{1,ijkl} (\bar{L}_i \gamma_{\mu} L_i) (\bar{L}_k \gamma^\mu L_l) + b_{2,ij} (\bar{L}_i \gamma_{\mu} L_i) (\bar{E}_j \gamma^\mu E_j) \)
+ \( b_{3,ij} (\bar{E}_i \gamma_{\mu} E_i) (\bar{E}_j \gamma^\mu E_j) + \ldots \)
+ \( b_{L\tau Q,ij} (\bar{L}_i \gamma_{\mu} \tau^a L_i) (\bar{Q}_j \gamma^\mu \tau^a Q_j) + b_{LQ,ij} (\bar{L}_i \gamma_{\mu} L_i) (\bar{Q}_j \gamma^\mu Q_j) \)
+ \( b_{EQ,ij} (\bar{E}_i \gamma_{\mu} E_i) (\bar{Q}_j \gamma^\mu Q_j) + b_{Lu,ij} (\bar{L}_i \gamma_{\mu} L_i) (\bar{u}_j \gamma^\mu u_j) \)
+ \( b_{Ld,ij} (\bar{L}_i \gamma_{\mu} L_i) (\bar{d}_j \gamma^\mu d_j) + b_{Eu,ij} (\bar{E}_i \gamma_{\mu} E_i) (\bar{u}_j \gamma^\mu u_j) \)
+ \( b_{Ed,ij} (\bar{E}_i \gamma_{\mu} E_i) (\bar{d}_j \gamma^\mu d_j) + \ldots \)
2. Step: After EWSB

Changing to the 'broken' phase

\[ \Phi \rightarrow \left( \frac{1}{\sqrt{2}}(v + H + iG) \right) \]

\[ E_i \rightarrow V_{ij} P_R \psi_j, \quad L_i \rightarrow U_{ij} P_L \left( \frac{\nu_j}{\psi_j} \right) \]

gives

\[ \sum_i C_i O_i \rightarrow \frac{\alpha_{ij} + \alpha_{ij}^*}{2} \frac{v}{\sqrt{2}} \bar{\psi}_i \sigma_{\mu\nu} \psi_j F_{\mu\nu} + \frac{\alpha_{ij} - \alpha_{ij}^*}{2i} \frac{v}{\sqrt{2}} \bar{\psi}_i \sigma_{\mu\nu} i\gamma_5 \psi_j F_{\mu\nu} \]

\[ + \beta_{ijkl} (\bar{\psi}_i \gamma^\mu P_L \psi_j)(\bar{\psi}_k \gamma_\mu P_R \psi_l) \]

\[ + \gamma_{1,ij} \frac{v}{2} (\bar{\psi}_i P_L \gamma_\mu \psi_j)(i\partial^\mu H) + \left[ \gamma_{2,ij} + \gamma_{3,ij} \right] \frac{v}{2} (\bar{\psi}_i P_R \gamma_\mu \psi_j)(i\partial^\mu H) \]

\[ + \gamma_{3,ij} \frac{v}{\sqrt{2}} (\bar{\psi}_i P_R \gamma^\mu \nu_i)(-i\partial_\mu \phi^-) + \gamma_{3,ij} \frac{v}{\sqrt{2}} (\bar{\psi}_i P_R \gamma^\mu \nu_i)(eA_\mu \phi^-) \]

\[ + \text{h.c. of previous line} + \ldots \]

the Greek Wilson coefficients are Latin ones dressed with flavour rotation matrices
3. Step: Compute Observables

Example: $g - 2 \, \& \, \mu \rightarrow e \gamma$ correspond to the flavour conserving and violating part of

UV and IR divergences require regularisation

BUT finite due to $\frac{1}{\epsilon} \times \epsilon$

Scheme dependent $\rightarrow$ dependence must cancel with the dependence of the 5D loop in $\alpha$
3. Step: Compute Observables

Example: $g - 2 \, \& \, \mu \rightarrow e\gamma$ also receive enhanced contributions at two loops

- Barr-Zee type contributions
- arise from FCNC Higgs couplings (effect on $g - 2$ negligible)
- effect on e.g. $\mu \rightarrow e\gamma$ studied

Chang, Hou, Keung '93
3. Step: Matching from effective theory onto effective theory

Example: $\mu \rightarrow e$ & $\mu \rightarrow 3e$

- computation straightforward
- must include insertions of the flavour-changing dipole that also mediates $\mu \rightarrow e\gamma$ (same order in the $1/T$ counting)
So all we need is . . .

Wilson Coefficients

- treat RS as (non-renormalisable) QFT in 5D
- derive Feynman rules for 5D theory [Randall & Schwartz 2001]

for the moment we do not consider metric fluctuations

- vertices are simple, e.g.

\[ -i \frac{1}{kz} \epsilon^{abc} \eta_{\mu \nu} \left( \partial_z \big|_{\text{on } a'} - \partial_z \big|_{\text{on } b'} \right) \]

- propagators are only treated in Fourier space in the flat directions
  \( \rightarrow \) mixed representation
  \( \rightarrow \) each vertex is accompanied by an integral over the fifth dimension
5D Formalism

- work in a 5D QFT
  no KK sums; vertices and propagators are five dimensional \cite{Randall, Schwartz, 2001}
- zero-mode (\sim SM fields) must be separated explicitly

\[
\begin{align*}
\mathcal{f}_L^{(0)}(z) &= \sqrt{\frac{1 - 2c_L}{1 - e^{1 - 2c_L}}} \sqrt{T(kz)^2(Tz)^{-c_L}} \\
\mathcal{g}_E^{(0)}(z) &= \sqrt{\frac{1 + 2c_E}{1 - e^{1 + 2c_E}}} \sqrt{T(kz)^2(Tz)^{c_E}}
\end{align*}
\]

- use mixed coordinate-momentum representation for propagators in the unbroken theory \cite{Randall, Schwartz, 2001}
  \rightarrow propagators depend on 4D momentum and start/end coordinate in the fifth dimension, e.g.

\[
\left[\frac{1}{kz}\right]^4 \mathcal{D} \Delta_L(p, z, z') = i\delta(z - z') 1 \quad \mathcal{D} = p + i\Gamma^5(\partial_z - \frac{2}{z}) - \frac{c_L}{z}
\]

\[
\Delta_L(p, z, z') = -P_L F^+_L(p, z, z')_p P_R - P_R F^-_L(p, z, z')_p P_L
\]

\[
\begin{align*}
&+ P_L d^+_L(p, z, z') P_L + P_R d^-_L(p, z, z') P_L \\
\sim & \text{mass term}
\end{align*}
\]
5D Formalism

- exact solution in the unbroken phase

\[ F^+_L (p, x, y) = \Theta(x - y) \frac{ik^4 x^{5/2} y^{5/2} \tilde{S}^+_+(p, x, 1/T, c_L) \tilde{S}^+_+(p, y, 1/k, c_L)}{S^- (p, 1/T, 1/k, c_L)} \]
\[ + \Theta(y - x) \frac{ik^4 x^{5/2} y^{5/2} \tilde{S}^+_+(p, y, 1/T, c_L) \tilde{S}^+_+(p, x, 1/k, c_L)}{S^- (p, 1/T, 1/k, c_L)} \]

\[ S^\pm (p, x, y, c) = I_{c \pm 1/2} (px) K_{c \pm 1/2} (py) - K_{c \pm 1/2} (px) I_{c \pm 1/2} (py) \]
\[ \tilde{S}^\pm (p, x, y, c) = I_{c \pm 1/2} (px) K_{c \mp 1/2} (py) + K_{c \pm 1/2} (px) I_{c \mp 1/2} (py) \]

- similar expressions for the different boson propagators
Tree-level coefficients are ’for free’

\[ b_{ij} = -i(-g_5')^2 \frac{Y_L Y_E}{4} T^2 \int_{1/k}^{1/T} dx \, dy \, \frac{f_{L_i}^{(0)}(y)}{(ky)^4} \frac{g_{E_j}^{(0)}(x)}{(kx)^4} \Delta_\perp(q = 0, x, y) \]

the hypercharge boson zero-momentum propagator is

\[ \Delta_\perp(q, x, y) \xrightarrow{q \to 0} \Theta(x - y) \frac{ik}{\ln k T} \left( - \frac{1}{q^2} + \frac{1}{4} \left\{ \frac{1/T^2 - 1/k^2}{\ln k T} - x^2 - y^2 + 2x^2 \ln(xT) \right. \right. \]
\[ \left. + 2y^2 \ln(yT) + 2y^2 \ln \frac{k}{T} \right\} + \mathcal{O}(q^2) \right) + (x \leftrightarrow y), \]

all integrals are elementary

very similar to computation of \( \Delta F = 2 \) tree-level processes

\( \leftrightarrow \) agrees with KK sum calculation [Casagrande et al. 2008]
So all we need is ...

the dipole operator(s) \( \cos \Theta_w a^B - \sin \Theta_w a^W \)

\[ \mathcal{L} \supset a^B \bar{L} \Phi \sigma_{\mu \nu} B^{\mu \nu} E + a^W \bar{L} \Phi \tau^A \sigma_{\mu \nu} W_A^{\mu \nu} E \]

- actual 5D loop with different particle species in the loop
- two (3?) different diagram classes

one Yukawa interaction to contribute to \( \bar{L} \Phi \sigma \cdot F E \)

three Yukawa interactions to contribute to \( \bar{L} \Phi \sigma \cdot F E \)
Dipole operator matching–gauge part

- In external line put $\Phi$ in vev, and take the superposition of $B$ and $W^3$ corresponding to the photon

$\leftarrow$ Diagrams in minimal model

- In the custodial model there are 24 non-abelian diagrams and 23 abelian diagrams in 15 distinct topologies
gauge part – extraction the short-distance contribution

There are three different contributions from each diagram

1. the SM corrections; all 5D propagators propagate the zero mode. The loop integral does not contain the short-distance scales $T, k$ explicitly, and is purely long-distance needs to be eliminated by subtraction of the zero-mode from one gauge boson propagator

2. contributions from momentum scales $l \ll T$, at least one of the 5D propagators propagates a KK mode. One-loop long-distance matrix element of tree insertion of four-fermion and fermion-Higgs operator.

3. The short-distance contribution: loop momentum of order $l \sim T$ (or $k$)

expand in external fermion momenta $p, p'$ to linear/quadratic order eliminates region (2), since the denominator no longer contains long-distance scales
off-shell terms

- non-1PR diagrams are necessary!
- obvious if external leg propagates a massive particle → short distance effect
- but even if the particle is on-shell (mass-less mode) the diagram is relevant
- known from $B$ physics → matching of power-suppressed heavy-to-light SCET currents

\[ \frac{p}{p^2} \]

\[ \Lambda^\mu = \Lambda^\mu + p \Lambda^{\text{off}, \mu}_1 + \Lambda^{\text{off}, \mu}_2 \]

- after cancelling the propagator the off-shell vertex-function is part of the short-distance coefficient
gauge part – remarks

gauge invariance

- calculation done for general 5d gauge parameter $\xi$
- one can show analytically that the set of 21 1-loop diagrams is gauge invariant (three gauge invariant subsets)

other checks: scheme independence (naive $\gamma_5$ vs general scheme), all integrals can be solved analytically in certain limits (e.g. large loop-momentum)
**anapole moment**

Most general $U(1)_{em}$ invariant vertex function for on-shell fermions:

$$\Gamma^\mu(p, p') = ie Q_\mu \bar{u}(p', s') \left[ \gamma^\mu F_1(q^2) + \frac{i \sigma^{\mu\nu}}{2m_\ell} q_\nu F_2(q^2) + \frac{i \sigma^{\mu\nu}}{2m_\ell} q_\nu \gamma_5 F_3(q^2) + \left( q^2 \gamma^\mu - \not{q} q^\mu \right) \gamma_5 F_4(q^2) \right] u(p, s)$$

- $F_1$ — charge form factor;
- $a_\mu = (g - 2)_\mu / 2 = F_2(0)$
- $F_3$ — EDM from factor
- $F_4$ — anapole moment, $SU(2) \times U(1)_Y$ gauge dependent [Musolf, Holstein, 1991]

$$\not{q} q^\mu \gamma_5 \rightarrow 2m_\ell q^\mu \gamma_5$$

The $EF^{\mu\nu}$ vertex contains both $(p + p')^\mu P_R$ and $(p - p')^\mu P_R$. The latter is associated with the anapole moment.

consistency check (analytical even in RS) coefficient of $(p - p')^\mu P_R$ must be proportional to $m_\ell$. 

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gauge part – result

- general structure of the gauge diagram contribution with full flavour dependence

\[ a_{ij}^{gauge} = i e Q_\mu \frac{\alpha_{em}}{4\pi} \frac{1}{T^2} \times \text{Loop}(c_{Li}, c_{Ej}) \cdot \log \frac{k}{T} \cdot f_{L_i}^{(0)}(1/T) Y_{ij} g_{E_j}^{(0)}(1/T) \left( \frac{T^3}{k^4} \right) = M_{ij} \]

- Loop\((c_{Li}, c_{Ej})\) (here for csRS, RS min is even less sensitive)

\[ \rightarrow \text{expect contribution to } g - 2 \text{ to be independent of the Yukawa structure & FCNCs are suppressed} \]

Moch, JR 2014
Phenomenology and Results (gauge part)

- \( a_{ij}^{\text{gauge}} \) relative to the mass matrix

\[
a_{ij}^{\text{gauge}} = \text{Const} \times \text{Loop}(c_{L_i}, c_{E_j}) \cdot M_{ij}
\]

\( \hookrightarrow \) supressed FCNCs

- \( g_\mu - 2 \)

\[
\Delta a_\mu^{\text{gauge}} = 27.2(8.8) \cdot 10^{-11} \frac{1 \text{ TeV}^2}{T^2}
\]

compared to

\[
a_\mu^{\text{exp}} - a_\mu^{\text{the}} = 287(63)(49) \times 10^{-11}
\]
Dipole operator matching– Higgs part

Subtlety: a brane localised Higgs should be described as having a delta-function-like profile

\[ \delta(z - 1/T) = \lim_{\delta \to 0} \frac{T}{\delta} \Theta(z - \frac{1 - \delta}{T}). \]

→ limit of a distribution of width \( \delta/T \) for \( \delta \to 0 \)

→ introduces an additional large scale \( \frac{T}{\delta} \) for any finite value

BUT we also have a dimensional regulator → \( \epsilon \to 0 \) before or after \( \delta \to 0 \)?
Dipole operator matching– Higgs part

- answer both options are valid part of the RS model
- the order of limits determines if the Higgs can be resolved or not

simple answer (minimal RS):

$$\Delta a_{ij} = (iQ_\mu e) \times \frac{1}{6} \times \frac{1}{16\pi^2} \frac{1}{T^2} \times f_{L_i}^{(0)}(1/T)[YY^\dagger Y]_{ij} g_{E_j}^{(0)}(1/T) \frac{T^3}{k^4}$$

$$+ (iQ_\mu e) \frac{1}{192\pi^2} \times f_{L_i}^{(0)}(1/T)Y_{ik}F_{E_k}^{(0)}(0, 1/T, 1/T)Y_{kh}^{\dagger}f_{L_h}^{(0)}(1/T)Y_{hj}g_{E_j}^{(0)}(1/T)$$

$$+ (iQ_\mu e) \frac{1}{192\pi^2} \times f_{L_i}^{(0)}(1/T)Y_{ik}g_{E_h}^{(0)}(0, 1/T, 1/T)Y_{kh}^{\dagger}F_{L_h}^{(0)}(0, 1/T, 1/T)Y_{hj}g_{E_j}^{(0)}(1/T)$$
gauge diagram contribution is independent of all model parameters (only overall scales $T$, $k$ matter)

$$\Delta a_\mu \approx 27 \times 10^{-11} \frac{1 \text{ TeV}^2}{T^2}$$

Higgs contribution depends on the Yukawa size
→ smaller than gauge contribution for Yukawas < 1
→ but can reach $1 \cdot 10^{-9}$ for $Y > 3$. 

![Graph showing three distributions with $x$-axis from $10^{-12}$ to $10^{-8}$ and $y$-axis from 0 to 800.](image-url)
Phenomenology—minimal model

- $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ ($T = 8$ TeV)

- dipole boundary ($\mu \rightarrow 3e$ dominated by dipole operate)

- in general both tree-level operators (Higgs-fermion and four-fermion) and 5D dipoles are important (esp. in the more natural bulk Higgs case tree-approximation [used in all RS analyses up to now] does not work)

- gauge-contributions prevent reducing $\mu \rightarrow e\gamma$ by just changing the Yukawa magnitude.
Phenomenology—minimal model

- $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ ($T = 8$ TeV)

- essentially no correlations $\rightarrow$ genuinely “orthogonal” constraints
Phenomenology—custodial model

- $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion ($T = 8$ TeV)

Yukawa size cannot help to escape bounds

next generation experiments can rule out the parameters space $m_{gluon}^{(1)} < 20$ TeV

one can invent flavour symmetries to avoid bounds, but it is quite hard to avoid all bounds (different operators contribute differently)
Comparisons

- Davoudiasl, Hewett, Rizzo [hep-ph/9911262]
  + KK sums
  + dropped external insertion and half the internal ones
  + Higgs diagrams appear when they should not
  + subset of abelian diagrams remains
  + similar order of magnitude but negative sign

- Csaki, Grossman, Tanedo, Tsai [hep-ph/1004.2037]
  + 5D calculation but no matching onto the effective lagrangian
  + depending on version without external Higgs insertions
  + fixed gauge
  + extracts dominant contributions
Comparisons


+ studies all LFV muon observables
+ only Higgs loop-diagrams
+ consider radiative transition for bulk Higgs
+ do not include loop contributions to $\mu \rightarrow e$, $\mu \rightarrow 3e$
+ general conclusions ✓
+ first to find that Yukawa size cannot be used to evade bounds
  ▶ first to mention wrong chirality couplings
Summary

- complete computation of the leptonic dimension-six Wilson coefficients including the dipole operators without approximations

- without imposing very specific flavour structures the new experiments will rule out KK modes in excess of 20 TeV

- 'gauge' corrections to $g - 2$ is model independent (like $S$ and $T$ parameter) but too small to help

- Higgs contribution is doubly model-dependent

- unmentioned: dimension-eight contributions, gravity, electric dipole moments, . . .