MUSICAL ACOUSTICS
Tutorial 4: Solution Guide

1. Comparing clavichord, harpsichord and piano

Compare the abilities of the clavichord, harpsichord and piano to
(a) play very loudly:

The clavichord has a very low maximum loudness; the main reason is that the tangent stays in contact with the string after striking, and the striking point is therefore (approximately) a node for all the modes of vibration of the string.
The harpsichord's plucking action is in principle capable of generating a loud sound, but historically based instruments have relatively light strings and small soundboards, so the level is lower than a modern piano.
The relatively heavy hammers, thick strings at high tension, and large soundboard of the modern piano make it capable of very loud playing.

(b) play with a big dynamic range:

The clavichord and piano actions both allow the player to vary the loudness of the sound by varying the key velocity. On the harpsichord the loudness is almost independent of key velocity.

(c) choose different timbres for the sound:

On the clavichord and piano, the timbre can be varied by changing the key velocity, although this also changes the loudness. On the piano pedals can also be used to change the timbre. On the harpsichord stops can be used to introduce or remove different ranks of strings with different pitches or tonal qualities.

(d) repeat notes very rapidly:

The simple clavichord action makes fast repetitions very easy. On the harpsichord there is also direct control of the plectrum by the player's finger through the key, so rapid reiterations are possible. The double escapement action of the piano stops the hammer falling back to its rest position after a first strike, facilitating rapid reiterations.

(e) vary the sound after a key has been pressed:

On the clavichord the player can vary the pitch of the note by varying the force on the key. On the harpsichord and piano there is no contact with the string after the note is sounded until the key is allowed to rise, although pedalling can be used to vary the sound on the piano.

2. Multiple stringing on the piano

Over most of the piano's compass, pressing a key makes a hammer head strike three strings tuned in unison. Assuming that the tuning is perfect, and that the hammer strikes each string with equal force, what will be the decrease in vibrational energy transmitted to the soundboard through the bridge if two of the strings are prevented from vibrating by inserting a felt wedge between them? How realistic are these assumptions?

If the hammer strikes three identical strings in exactly the same way, they will vibrate with the same frequency, amplitude and phase. The total force on the bridge, and the amplitude of the resulting

bridge vibration, will therefore be three times that of a single string. The energy transmitted is proportional to the square of the displacement amplitude, so it will be nine times greater than the energy transmitted by a single string.
However, the assumptions are not very realistic - in practice the strings are not hit at exactly the same time, or with exactly the same force. There are also usually small frequency differences between the three strings. As a result phase differences develop, reducing the rate at which energy is transferred through the bridge. This is important in allowing the sound to sustain for several seconds.

3. Spectral brightness in harpsichord and piano

The sound of a harpsichord note is normally much brighter (stronger in high frequency components) than the sound of a piano note. Discuss possible explanations for this difference.

The dominant factors are the width of the piano hammers, the fact that they are covered with felt, and the fact that they spend a significant amount of time in contact with the strings before rebounding. These factors mean that the high frequencies are heavily damped by the hammer, whereas they are allowed to ring on undamped after the harpsichord plectrum has plucked the string.

4. Inharmonicity in piano sounds

The frequency of the n^{th} mode of a string is given to a good approximation by the formula

$$f_n = n f_0(1 + Bn^2).$$

The factor $(1 + Bn^2)$ is a correction which accounts for the stiffness of the string, and makes the mode frequencies slightly inharmonic. For a perfectly flexible string, $B = 0$. What does f_0 represent?

If $B = 0$, then $f_1 = f_0$. In words, f_0 is the fundamental frequency which the string would have if it had negligible stiffness.

Taking $B = 0.0002$ for a piano C4 string (fundamental frequency $f_1 = 261.6\,\text{Hz}$), calculate the frequency of the fifth mode of the string, and the pitch differences between this mode, the fifth harmonic of C4, and the nearest note on the equally tempered piano keyboard. Comment on the musical significance of these differences.

For $n = 1$, $f_1 = (1 + B)f_0$, so $f_0 = f_1/(1 + B) = 261.6/1.0002 = 261.55\,\text{Hz}$. The fifth mode frequency is $f_5 = 5f_0(1 + 25B) = 1314\,\text{Hz}$.

The fifth harmonic of C4 is $5 \times 261.6 = 1308\,\text{Hz}$.

The nearest note on the equally tempered keyboard is E6 at 1319 Hz.

The pitch difference between the fifth mode and a true fifth harmonic is $3986\log(1314/1308) = 8$ cents. This is barely noticeable musically. The difference between the fifth mode and the fundamental of E6 is $3986\log(1314/1319) = -7$ cents. Again, a barely noticeable pitch difference. However, if the notes C4 and E6 are played simultaneously, a beat at $1319 - 1314 = 5\,\text{Hz}$ may be audible between these two frequencies.