Quantum Physics 2011/12

Tutorial Sheet 9: Scattering

An asterisk denotes a harder problem, which you are nevertheless encouraged to try!

1. The UK has two major scattering facilities based at Rutherford Appleton Laboratory in Oxford, the Diamond synchrotron (X-rays) and the Isis spallation source (neutrons).

   With reference to the time dependent scattering matrix element

   $$\langle \mathbf{k}'|\hat{V}|\mathbf{k}\rangle = \frac{1}{L^3}\int\int\int V(\mathbf{r},t)\,\exp\left(-i\chi.\mathbf{r} - (\omega-\omega')t\right)\mathrm{d}\tau$$

   explain.

   a) What causes V in each case:

   b) Which technique would you use to study (i) the crystal structure of polonium (ii) the crystal structure of deuterated ice (iii) the phonon (vibrational) spectrum of sapphire (Al_2O_3)

2. Particles of mass m and momentum $\underline{p} \equiv \hbar\underline{k}$ are scattered by the potential

   $$V(r) = V_0\,\exp(-ar)$$

   Show that, in the first Born approximation, the differential and total cross-sections are given by

   $$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = \left(\frac{4V_0 ma}{\hbar^2}\right)^2 \frac{1}{(a^2+\chi^2)^4}$$

   where $\chi = 2k\sin\theta/2$ is the magnitude of the wave-vector transfer, and

   $$\sigma_T = \frac{64\pi m^2 V_0^2}{3a^4\hbar^4}\left\{\frac{16k^4+12a^2k^2+3a^4}{(a^2+4k^2)^3}\right\}$$

   Hint: the required integral may be obtained by maple or parametric differentiation of the integral

   $$\int_0^\infty \sin(\chi r)\exp(-ar)\,\mathrm{d}r$$

3. Particles are scattered from a box of size L^3 containing some gas molecules, represented by a time varying potential:

   $$V(r) = V_0 L^3 \delta(r)\sin(\alpha t)$$

   where δ is a Dirac delta function and the particle take time $t_0 >> 1/\alpha$ to pass through the potential. Show that the differential cross-section in the Born approximation is

   $$\frac{m^2V_0^2L^6t_0^2}{16\pi^2\hbar^4}[\delta(\omega'-\omega+\alpha)+\delta(\omega'-\omega-\alpha)]$$

   Given that this is a model for the interaction of light with a fluctuating dipole in a gas molecule, comment on the presence of the factor of L^3 in the potential, the angular dependence, the values of k' and the conservation of energy.

   Considering the Born series (lecture 13), can you guess what the second order term would mean here

4. Evaluate the differential cross-section in the Born approximation for the potential

$$V(r) = V_0/r^2$$

What happens to the total cross-section for this potential ?

You may assume that:

$$\int_0^\infty \frac{\sin x}{x}\, \mathrm{d}x = \pi/2.$$

5. Two beams of unpolarised electrons are scattered from one another. Taking the scattered wavefunction to be $f(\theta)$, find the scattered intensity for combinations of singlet and triplet spins with appropriate spatial wavefunctions. Show that this is the same scattering cross section given by the combination of 50% distinguishable and 50% indistinguishable fermion collisions, as discussed in the notes.

6. * Show that in a classical elastic two-body collision between particles of mass m_1 and m_2, the LAB frame scattering angle, θ and the CM frame scattering angle, θ^*, are related by

$$\tan\theta = \frac{\sin\theta^*}{\rho + \cos\theta^*} \quad \text{where} \quad \rho = m_1/m_2$$

and hence that the LAB and CM frame differential cross-sections are related by

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_L = \frac{\left(1+\rho^2+2\rho\cos\theta^*\right)^{3/2}}{|1+\rho\cos\theta^*|} \cdot \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{CM}$$

7. Show that the function $G(\boldsymbol{r}) = -\exp(ikr)/4\pi r$ is a solution of the Schroedinger equation with a delta function potential:

$$\frac{-\hbar^2}{2m}\nabla^2 G(\boldsymbol{r}) + \delta(\boldsymbol{r})G(\boldsymbol{r}) = EG(\boldsymbol{r})$$

and evaluate E. You may resort to a verbal argument to explain the delta function without evaluating its strength.

Show that for a potential $V(r) = \hbar^2 U(r)/2m$ the time independent Schroedinger equation gives:

$$\Phi(\boldsymbol{r}) = Ae^{i\boldsymbol{k}.\boldsymbol{r}} + \int G(r-r')U(r')Ae^{i\boldsymbol{k}.\boldsymbol{r}'}d^3\boldsymbol{r}' + \int G(r-r')U(r')G(r'-r'')U(r'')\Phi(\boldsymbol{r})d^3\boldsymbol{r}'d^3\boldsymbol{r}''$$

Under what circumstances is this useful?