Thermodynamics Assessment Problem 1 The Ram Jet

Hand in solutions to teaching office by noon Tuesday Week 5 (17th Oct) 2017.

1/ You may work on the hand-ins in self-organised groups of up to six. There is no obligation to do this, nor is there any constraint by degree program.

2/ If you do, you must hand in a single submission, with ALL barcodes attached.

3/ Group submission will be expected to have a higher standard of presentation and detail of explanation. There is no other reduction in marks.

Indicative marking scheme, parts a-i (2,2,3,3,5,3,3,2,2)

Background : *This question examines the thermodynamics of jet engines assuming that air behaves as an ideal gas. It illustrates how enthalpy (or more precisely the specific enthalpy, i.e. enthalpy per unit mass) can be used to analyze a constant flow process. Two quantities which are not covered in lectures are used: thrust and useful power of a jet engine. They are fully defined and are essential to demonstrate when a ramjet is useful.*

The question considers a ramjet (Fig 1) operating at 8000m altitude, moving at speed v relative to the surrounding air. A ramjet only works when the aircraft is already moving at speed, but has the advantage of being light and simple. The objective is to estimate the thrust and the efficiency of this engine.

Definition of symbols and some data

(the actual values should be used only for part h)

$\gamma = \frac{C_p}{C_v} \approx \frac{(5/2)R+R}{(5/2)R} = 1.4$

air pressure at 8000 m$P_0 = 35$ kPa

temperature at 8000 m$T_0 = 236K = -37$ C

average molar mass of air$M = 0.0297$ kg/mole

density of air at 8000m$\rho_0 = \frac{PM}{RT} = 0.53$ kg m^{-3}

You may assume that air is an ideal diatomic gas with internal energy, $U = (5/2)RT$ per mole.

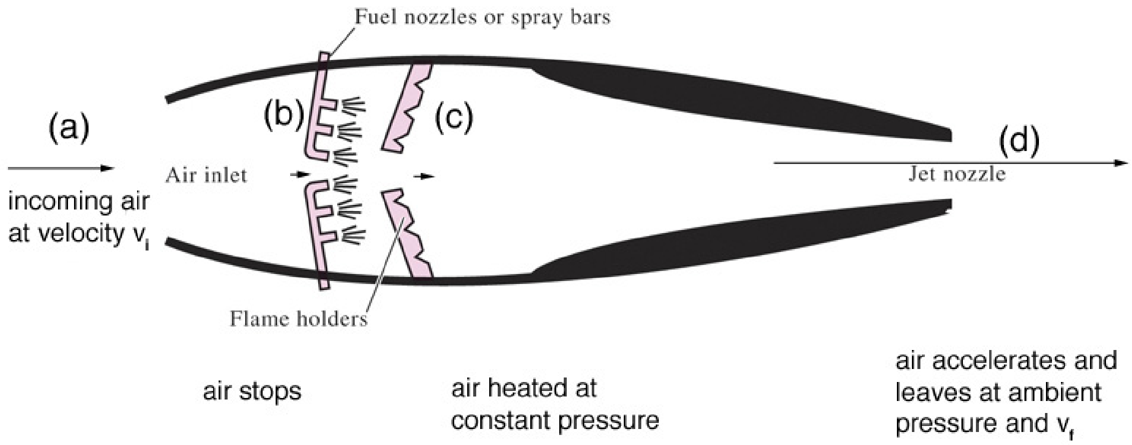

FIG. 1: A ramjet engine

It is most convenient to work in the inertial frame of reference of the engine. The incoming air speed is the same as the flying speed $v_i = v$ and the outgoing speed is denoted v_f.

The question leads you through the various steps, however you will have to put it all together, so it is useful to tabulate expressions for the state variables at each of the four stages as and when you work them out.

(a) Write down an expression for the enthalpy **per unit mass** of air at temperature T.

(b) Combustion of the fuel provides heat q per unit mass of air passing through the engine. From the expression for a continuous flow process derive the following expression for the velocity of the air exiting the engine (in the engine's rest frame),

$$v_f = \sqrt{v^2 + \frac{7R(T_0 - T_f)}{M} + 2q},$$

where T_f is the temperature of the ejected air. State any assumptions you made.

To determine v_f and the thrust we need to calculate T_f. This depends on the processes taking place in the engine. We will get the most thrust from the engine if all the processes are reversible (quasi-static) or equivalent to reversible processes. We therefore model the passage of the air through the engine by the following sequence of processes:

(i) The incoming air is compressed adiabatically and reversibly as its velocity is reduced to be almost stationary in the reference frame of the engine (see figure).
(ii) Heat is then supplied at constant pressure to the almost stationary gas.
(iii) The air expands adiabatically and reversibly, accelerating and exiting the engine at atmospheric pressure, P_0.

(c) Explain why the change of temperature in process (ii), brought about by the irreversible process of burning the fuel, is the same as if heat q was supplied reversibly and is given by $2Mq/7R$.

(d) By considering the continuous flow process in (i) find an expression for the temperature of the air, T_1, immediately after this process. Use this result and the result from part (c) to obtain an expression for the air temperature, T_2, immediately after process (ii). You are advised to check the units of the expression, paying attention to intensive and extensive quantities.

(e) For the reversible adiabatic expansion/compression of a fixed mass of an ideal gas prove the following relationship between pressure and temperature,

$$TP^{\xi} = const$$

with $\xi = -R/C_P$. Use this expression to deduce that $T_f/T_0 = T_2/T_1$ and combining this with the results from (b) and (d) or otherwise show that:

$$v_f = v\sqrt{1 + \frac{2Mq}{7RT_0 + Mv^2}}$$

(f) The thrust (work done) provided by an engine is equal to the rate of momentum transfer to the air, and the useful power of an engine is the product of the aircraft's velocity and thrust. Given that the engine draws in air at density ρ_0 over a cross-sectional area A, calculate expressions for the thrust (W) and the useful power delivered by the engine, $\dot{W}$.

(g) Noting that the rate of energy supplied burning the fuel is $\dot{Q} = Av\rho_0 q$, sketch a graph of the efficiency η of the engine, defined as $\eta = \dot{W}/\dot{Q}$, versus v for constant $\dot{Q}$ showing clearly the limiting behaviour for small and large v.

(h) If the rate of energy supplied by burning the fuel is $\dot{Q} = 20$ MW and the engine draws in air over a surface area 0.1m^2, calculate the thrust achieved when $v = 200\ \text{ms}^{-1}$

(i) Why does a ramjet-powered missile require additional rocket propulsion?

This is the end of the question. More information about the ramjet is online.

Further background information not needed to answer the question

In a jet engine FIG 2 the incoming gas is additionally compressed by a compressor before combustion. The power needed to run the compressor is recovered from a turbine located after the combustion stage. The engine works more efficiently than the ramjet at lower air speed since the compressor draws in air. Additionally the combustion occurs at a higher temperature due to the additional compression and the combustion process is more complete.

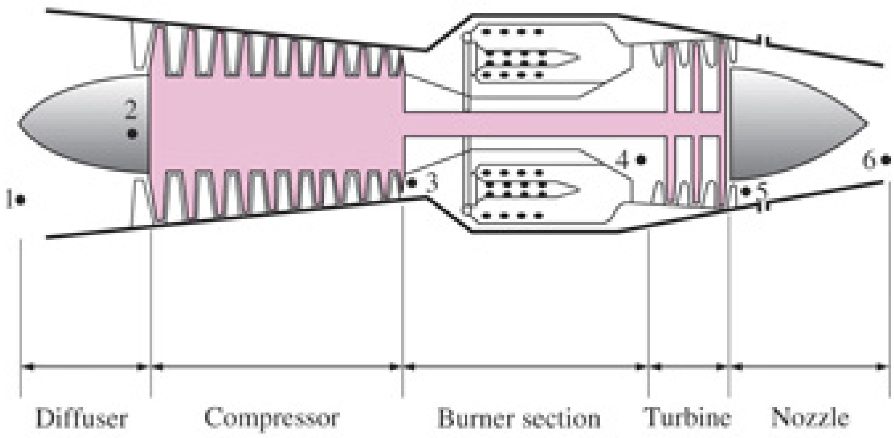

FIG. 2: A jet engine

Current commercial airline engines are Turbo fan engines (FIG 3). The Fan serves to accelerate a much bigger volume of air. Accelerating a bigger mass of air to lower speeds is a more fuel efficient way of generating the same thrust. A turbo-prop is more efficient still for the same reasons, but not yet a viable alternative for high altitude, high speed flight.

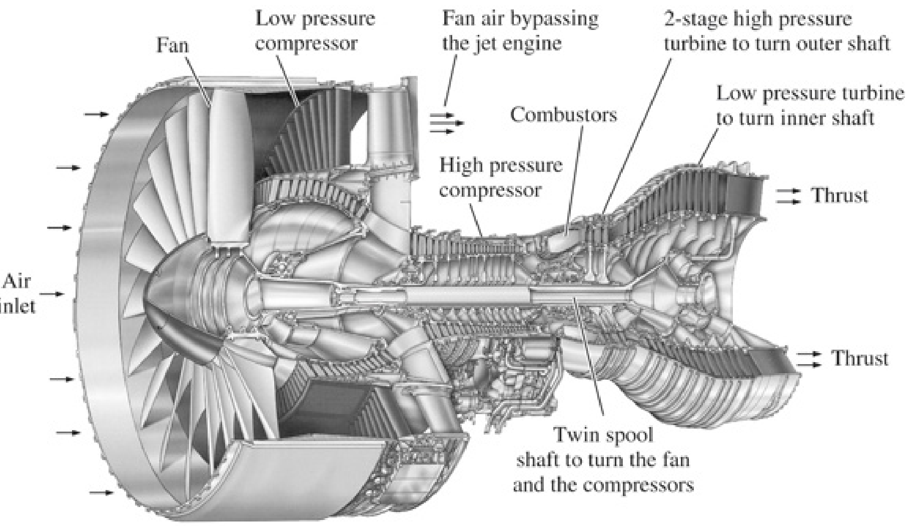

FIG. 3: A modern turbofan engine used in a Boeing 777

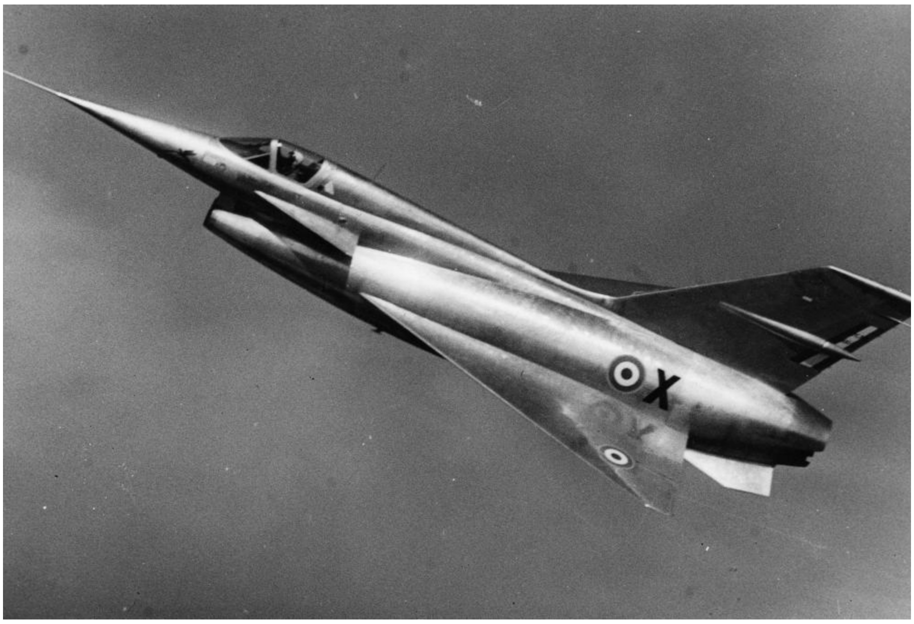

FIG. 4: The Nord 1500 Griffon, a 1950s ramjet-powered fighter aircraft