

# Thermal Physics Assignment 3: Feedback

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## Question a

This question was done OK, with most people scooping both available marks. By far the easiest method is to form an expression for the differential of the Gibbs free energy  $dG$ , using an appropriate form for  $dU$  which includes the contribution of magnetisation to the internal energy, and then read off the coefficients of  $dP$ ,  $dT$  and  $dH$  in order to find the appropriate partial derivatives of  $G$ . It is possible to brute force the partial derivatives if one is careful, but it takes longer and is messy. Almost all errors made in this section were due to the mishandling of the internal energy (e.g. assuming  $(\frac{\partial U}{\partial T})_{P,H} = 0$ , which is in general false, or forgetting that the internal energy contains information about magnetic behaviour) or jumping directly to the final result without stopping to prove anything (saying that a cyclic relation will make the remaining results follow by corollary isn't quite sufficient for a derivation question).

I did observe here that some people have developed some interesting notation. In general, it is standard to refer to an infinitesimal variation in, say,  $x$  by  $dx$ , **not**  $\partial x$ . Likewise, and this applies to all subsequent questions as well, it is **always** important to keep track of what is being held constant when taking a partial derivative. If for the sake of neatness one wishes to hold, say, the pressure constant throughout all operations, it suffices to say that this is being done at the start of a derivation, but it should nonetheless be said! This applies all the more when one finds oneself trying to use such expressions as

$$\left(\frac{\partial x}{\partial y}\right)_{x,z} \left(\frac{\partial w}{\partial x}\right)_{x,z} = \left(\frac{\partial w}{\partial y}\right)_{x,z} \dots$$

(Hint: This is not so valid!) I have removed a maximum of one mark per assignment when I have judged notation to be too confusing.

## Question b

This question went smoothest if one simply consulted the notes to find an expression for  $C_{P,H}$  in terms of a partial differential relation between entropy and temperature, although it can again be brute forced if one considers a magnetic version of enthalpy. This was mostly fine, besides a few unconvincing derivations.

## Question c

Almost everyone got this right, which is reassuring. Perhaps some proofs could be fleshed out a little more by mentioning that the order of partial derivation could be swapped only assuming the good nature of the Gibbs free energy, but I didn't actively penalize anyone for this.

## Question d

This was mostly fine, but a few slip-ups, mostly relating to the sign of the final result. One needs to use the previously-derived Maxwell relation as well as either the cyclic rule or something to the effect of  $dS = 0$  (as the process is adiabatic); failing to handle the latter correctly results in the sign error. Where a result has been fudged (either accidentally or deliberately) I have reintroduced the sign error and deducted one mark. There is a little confusion about which rule is called which; the cyclic rule is the one with the minus sign, the chain rule without (the chain rule being the simple generalization of the single-variable calculus result of the same name, the cyclic rule being something slightly more interesting).

## Question e

Most people got the right idea of how to approach this. However, there were some issues with implementation of the method, for which marks have been deducted:

- Some of the gradients taken from the graph are rather shallow, probably due to not taking a tangent line right at the transition.
- It may seem harsh, but I have deducted marks for not stating what the units of the gradient are. This is because, in my experience, such gradients are an excellent source of real error, as graphs can be in rather silly units in the literature and it always pays to check.
- Likewise, I have removed marks for final answers which purport to have more accuracy than is actually possible. I think it's going to be very hard to measure the gradient to within one percent; thus, why are some students writing answers to 4s.f? Likewise, I think we can certainly measure to within ten percent, so why quote the final result with less accuracy?

In the end, most students derived numerically appropriate answers. Those who did not tended to mess up taking account of the molar mass somehow.

## Question f

Few people got both marks for this question. The main issue was that the first derivative of  $\frac{M}{\rho}$  with respect to  $T$  being discontinuous doesn't tell us much if we already know that  $\frac{M}{\rho}$  is discontinuous (which it is; you won't find much experimental data of a transition jump which is so nicely defined!); thus, I

wouldn't give marks for both. The other main feature of the graph is the hysteresis (heating and cooling curves not matching); the presence of hysteresis makes it very likely that the transition (which we know is occurring due to the jump in  $\frac{M}{\rho}$ ) is first order, not second order.

Some students seem to have assumed that the discontinuity in  $\frac{M}{\rho}$  is due to discontinuity in the density, and so indicates a mechanical phase transition (such as solid/liquid, or liquid/gas). Whilst this is technically plausible, you do know from your lecture notes that magnetic transitions are a real thing, and the assignment does seem to be talking a lot about magnetic transitions, so I haven't always looked kindly upon this.

## Question g

In order to have a chance to do well at this question, the student has to point out that the Gibbs free energy (densities) must match on the transition line (otherwise one phase would be preferred over the other and so the transition would not be happening there). Without this fact, any derivation is potentially fatally flawed as otherwise the free energies and state variables in the different phases wouldn't have to be related in any way whatsoever. Once this is done, Taylor's theorem may be used to relate quantities either side of the transition (only as we move along the transition line, of course).

## Question h

This question was extremely conceptually hard, and as such few people managed to score highly here. It doesn't help that the result derived in the previous question isn't actually all that useful here, as we aren't moving along transition lines. The thing to notice is that the change is adiabatic, and that during the transition thermal entropy is converted into magnetic entropy, thus lowering the temperature. If one doesn't look at the problem in this way, it's almost impossible to get anywhere near the solution. The other thing to be considered is that it is the change from a magnetised state to a demagnetised state which does most of the cooling, and so the vast majority of the cooling happens when the field is moving between 1T and 0T. Like I said, though, this question is **hard**, so one shouldn't feel too bad if it didn't go well.

## Question i

I might've been slightly harsh here, but I really wanted to reward students who had thought hard about the thermodynamics involved in magnetic refrigeration, and maybe read up on ferromagnetic transitions; thus, I didn't look kindly upon answers involving chemical safety, etc, or were vague about why their suggestion would help the fridge.