School of Physics and Astronomy

Junior Honours **Thermodynamics** GJA 2019-2020

Tutorial 1: Some properties of materials

Use the following values where necessary in the questions:

$$\begin{aligned}
\text{latent heat of fusion (melting) of ice} &= 334\ \text{kJ kg}^{-1}\\
\text{latent heat of vaporization of water} &= 2256\ \text{kJ kg}^{-1}\\
\text{specific heat capacity of ice} &= 1.94\ \text{kJ kg}^{-1}\ \text{K}^{-1}\\
\text{specific heat capacity of water} &= 4.2\ \text{kJ kg}^{-1}\ \text{K}^{-1}\\
\text{specific heat capacity of steam} &= 2.04\ \text{kJ kg}^{-1}\ \text{K}^{-1}
\end{aligned}$$

1. ***In-class activity**

   Work in groups to place the following quantities of energy in order (use the internet)

   1/ The daily energy consumption of the UK

   2/ The binding energy of all the electrons in one kg of hydrogen molecules.

   3/ The rest mass energy of one kg of hydrogen molecules.

   4/ The zero point energy of one kg of hydrogen molecules (vibrational frequency 4161cm^{-1}).

   5/ The energy released when one kg of deuterium molecules fuse to create one kg of helium.

   6/ The thermal energy of one kg of hydrogen molecules at 300K.

   7/ The calorific value of a cold chicken sandwich at 280K.

   8/ The additional thermal energy of a cold chicken sandwich at 320K.

   9/ The energy required to remove the chicken sandwich from the earth's gravitational field.

   10/ The kinetic energy of the chicken sandwich on a train at 50m/s.

   11/ 1kg of coal, when burnt.

   12/ 1kg of uranium-235, when fissioned

   13/ 1000 cubic metres of air moving at 10m/s

   14/ 1000 cubic metres of water, raised by 5m

2. **Heating and metabolism**

   A class of 180 students sits in a lecture theatre for one hour, each student metabolising at 100 W. The lecture theatre is a cubic room twenty metres long on each side and 2.5 metres high. The specific heat capacity of air at constant volume is 1 kJ kg^{-1} K^{-1}. The density of air is about 1.2 kg m^{-3}. The initial temperature was 20°C. The ventilation is poor, the air conditioning is broken and the walls are well insulated. What is the room temperature at the end of the lecture?

   Compare this to the rate of heat production of the sun, which produces 3.86×10^{26}W.

3. **Thermal properties in food**

   A 500-gram box of strawberries is cooled in a refrigerator, from an initial temperature of 25 °C down to the fridge temperature of 4 °C.

   (a) *Estimate* how much heat is removed from the strawberries during the cooling, explaining your reasoning. *Hint: Strawberries have a water content of about 88%. For this and subsequent problems, some of the data given at the start of this section may be useful.*

(b) Fruits and vegetables actually respire continuously whilst in storage, taking in oxygen and converting it to carbon dioxide. In the case of strawberries take the heat produced by this reaction to be about 210 mW kg^{-1}. How does your estimate of heat removed change if respiration is taken into account? *Hint: You need to make a sensible assumption about timescales.*

(c) The strawberries are now removed from the fridge and put into a polystyrene (i.e. thermally insulating) container in the kitchen. How long will it take for the strawberries to reach room temperature?

(d) The nutritional value of 100g of strawberries is 33kcal (140kJ). Use this and the respiration rate to estimate the lifetime of strawberries.

4. **Phase changes: latent heat.**
An ice cube of mass 0.03 kg at 0°C is added to 0.2 kg of water at 20°C in an insulated container. (a) Does all the ice melt? (b) What is the final temperature of the drink?
Comment: The ice does all melt, but specify carefully the criterion which must be satisfied.

5. **Conservation of energy: gravity and heat**
In 1845 James Prescott Joule suggested that the water at the bottom of a waterfall should be warmer than at the top. In particular, for Niagara Falls (a height of about 50 m) the temperature difference would be approximately 0.12 °C.

How would you go about calculating this number?

How do you know this must be an overestimate?

Unlike in 1845 most of the water nowadays is diverted through hydroelectric power schemes. How does this affect Joule's prediction?

6. **The ideal gas law**
The ideal gas is defined by its *equation of state*, which relates the variables pressure P, temperature T and volume V:

$$PV = nRT$$

Here n is the number of moles of the gas sample, and R is the gas constant. Calculate the volume occupied by a sample of ideal gas at atmospheric pressure and a temperature of 25°C, given that its volume under *standard temperature and pressure* is $V_m = 2.2414 \times 10^{-2} m^3$.

n.b. Standard temperature and pressure means $T = 0$°C, $P = 1atm = 101325Pa$.

7. **Another 'ideal' gas** You are told that at a pressure of 1.2 atm and temperature of $T = 300K$, n moles of oxygen occupy a volume of $82cm^3$. Calculate n and the *mass* of the oxygen sample, assuming that under these conditions the gas behaves ideally. How would the answer change if the sample was *ozone* (O_3)?

8. **Melting, heating and boiling.** A 1 kg sample of ice at an initial temperature of -4°C is heated at constant pressure in an insulated container, heat being supplied at a constant rate of 1 kJ s^{-1}, until the sample is steam at 110°C. Using the data given below, sketch the temperature as a function of time. Discuss the differences in the times taken for melting, heating, and boiling.

A Bonus Question is available in the online version

9. **BONUS QUESTION: Conduction of heat (an idealised hot water bottle).**

A certain heat source (a *heat reservoir* or *thermal reservoir*), is always at temperature T_0. Heat is transferred from it through a slab of thickness L to an object which is initially at a temperature $T_1 < T_0$. The object, which is otherwise thermally insulated from its surroundings, has a mass m = 0.5 kg and a specific heat capacity $c = 4 \times 10^3$ J kg^{-1} K^{-1}. Heat is conducted through the slab at a rate (in J s^{-1}) specified by the formula $KA((T_0 - T)/L)$, where K is the thermal conductivity of the slab, A is the area of the slab through which heat is transferred and T is the instantaneous temperature of the object.

(a) Show that provided certain assumptions are made,

$$KA((T_0 - T)/L)\Delta t = mc\Delta T$$

where the temperature of the object changes from T to $T + \Delta T$ during the time interval Δt. (For simplicity, treat the arrangement as a 1-dimensional system with the x-axis perpendicular to the slab.)

(b) Show that for small Δt, and hence small ΔT, the above equation can be rearranged and integrated to give

$$T_2 - T_1 = (T_0 - T_1)\left(1 - \exp\left(-\frac{KA(t_2 - t_1)}{Lmc}\right)\right)$$

T_2 and T_1 are the temperatures of the object at the end, $t = t_2$, and the beginning, $t = t_1$, respectively, of the process described.

(c) Put $A = 100$ cm^2 and $L = 1$ cm. Calculate the value of $T_2 - T_1$ for $t_2 - t_1$ equal to (a) 2 s and (b) 2000 s (about half-an-hour) for $T_0 = 60$°C and $T_1 = 35$°C for the following slab materials: whose thermal conductivities are given in the table.

aluminium ($K = 200$ W m^{-1} K^{-1}) porcelain ($K = 1.5$ W m^{-1} K^{-1}) rubber ($K = 0.15$ W m^{-1} K^{-1}) wool ($K = 0.05$ W m^{-1} K^{-1}) air ($K = 0.025$ W m^{-1} K^{-1})

Comment: The mathematical analysis incorporates a step which is quite common in problems in thermodynamics. If the temperature of part of a composite system changes from T_i to T_f, work done and/or heat flow during the process can often be calculated most quickly by considering intermediate stages in which the temperature changes from T to $T + dT$, setting up the appropriate equations and integrating them.