School of Physics and Astronomy

Junior Honours **Thermodynamics** GJA 2019-2020

The material in this tutorial will be covered in the Week 3 problems class. The TA will help you with any difficulties. Try to look over the problems before the class.

Tutorial 2 Temperature scales, work, equations of state

1. **Work done in other processes**

   (a) **In melting**: Ice at 0°C and at a pressure 1 atm, has a density of 916 kg m^{-3}, while water under these conditions has a density 1000 kg m^{-3}. How much work is done when 10 kg of ice melts into water? Explain why this work is done "by the atmosphere" rather than "against the atmosphere". Is there a difference between latent heat of melting at constant pressure, and latent heat of melting at constant volume?

   (b) **On a wire**: Calculate the work done when a copper wire of length 10cm holding a 2kg weight extends by 0.1% due to reversible heating. Given the linear thermal expansion coefficient is $16.6 \times 10^{-6} K^{-1}$, estimate the required temperature change.

   (c) **In a wire**: Calculate the electrical work done when the wire is connected to a 6V battery for 10sec. (assume resistance=4.2mΩ)

2. **Calculating properties from the equation of state, and vice versa**

   (a) The isothermal compressibility κ and the isobaric volume expansivity β are given by:

   $$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \qquad \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

   Using the equation of state, calculate the isobaric expansivity, β and isothermal compressibility, κ, of an ideal gas. Show that the bulk modulus for an ideal gas, $K = P$

   (b) A substance is found to have an isothermal bulk modulus $K = v/a$ and an isobaric expansivity $\beta = 2bT/v$ where a and b are constants and v is the molar volume. Show that the equation of state is $v - bT^2 + aP =$ a constant.

   [Hint: Integrate the expressions involving partial derivatives for κ and β, and merge/reconcile the outcomes.]

   (c) Show that for the ideal gas, the difference between the heat capacities at constant volume and constant pressure is actually given by:

   $$C_P - C_V = \frac{VT\beta^2}{\kappa}$$

   Remember $C_P - C_V = R$ and $PV = RT$ for one mole of an ideal gas.

3. **Reversible adiabatic expansion of ideal gas**

   (This question involves working through the final section of lecture 3)

   Explain why the first Law for an reversible adiabatic process gives $\Delta U = -PdV$, and why this equation doesn't hold for the Joule expansion.

   Assuming that for an ideal gas $U = C_V T$, prove that the First Law leads to the statement that PV^γ is constant in a reversible adiabatic process.

   A container of Helium and a container of Oxygen are initially the same pressure and temperature. Each undergoes a reversible adiabatic compression to half its original volume. Assuming they are ideal gases, how do their temperatures compare?

4. **Temperature scales: based on water**

   At atmospheric pressure, water has a density of 960kg/m^3 at 100^{o}C, the same as supercooled water at at -40^{o}C. The maximum density, 1000kg/m^3, is at 4^{o}C.

   If the equation approximating an isobar is $\rho = A + BT + CT^2 + DT^3$, evaluate the constants A-D. (*Hint - you are free to choose the zero of the temperature scale as you please*)

   A water thermometer comprises water in a glass tube of constant radius. At 4^{o}C, the column of water is 100mm high. At what temperature will it be 101mm high?

   Give reasons why mercury is used in preference to water in thermometers.

5. **Joule's experiment**

   In an experiment similar to Joule's paddle wheel experiment, a mass of 20 kg drops slowly through a distance of 2 m, driving the paddles immersed in 2 kg of water. Viscous dissipation generates heat in the water.

   (a) Ignoring heat losses, bearing friction, etc, calculate the rise in temperature of the water.

   (b) What would be the error in determining the mechanical equivalent of heat if the mass was still moving at 10 cm s^{-1} when it hits the ground?

   (The heat capacity of water is 4.2 kJ K^{-1}kg^{-1})

6. **BONUS QUESTION: Temperature scales based on electrical resistance**

   Idiosyncratic Roger invents a temperature scale using as his thermometric property the resistance $R(T)$ of a special wire. He decides to calibrate his scale using the ice temperature (273.15K) and the triple point of water (273.16K). His wire has resistance R_0 at the ice point temperature, which he defines as $T_R = 273.15$. It has resistance $R_0 + \Delta R$ at the triple point, which he defines as $T_R = 273.16$. He then defines other temperatures by $T_R = 273.15 + 0.01(R - R_0)/\Delta R$, which he determines by measuring the resistance R.

   Unbeknownst to Roger, the resistance of his wire is given by

   $$R = R_0(1 + \alpha T + \beta T^2)$$

   where T is the temperature in degrees Celsius measured on the ideal gas scale. The constants α and β are 3.8×10^{-3} K^{-1} and -3.0×10^{-6} K^{-2} respectively. What temperature on Roger's resistance scale corresponds to a temperature of 70°C on the ideal gas scale?

You may use the following result. Preferably without the proof.

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x} = \frac{\cancel{\mathrm{d}}}{\cancel{\mathrm{d}}x}\frac{1}{x} = \frac{}{x}\frac{1}{x} = -\frac{1}{x^2}.$$