School of Physics and Astronomy

Junior Honours **Thermodynamics** GJA 2019-2020

Problems Set 3: Work and heat, the First Law, expansion processes

A related Exam-style question is online: it will be tackled in the Tutorials.

NOTE: the second part of question 7 uses the symbol for integral at constant entropy $)_S$ to mean "reversible adiabatic".

1 Questions: Work and heat, the First Law

1. **Work done in the expansion of an ideal gas**

   The volume of a given system containing n moles of a monatomic ideal gas is given by $V = V(P, T)$, where P is the pressure and T the temperature, and $PV = nRT$.

   (a) Obtain an expression in terms of the change in pressure, dP, for the incremental work done on the system, $đW$ in an isothermal expansion.

   (b) The gas expands isothermally to twice its original volume. Using the formula for the incremental work on the system in terms of dP (previous part), obtain an expression for the total work done on the system.

   (c) Write down an expression in terms of the change of temperature, ΔT, for the work done on the system in an *isobaric* expansion.

2. **Work, heat, PV diagrams**

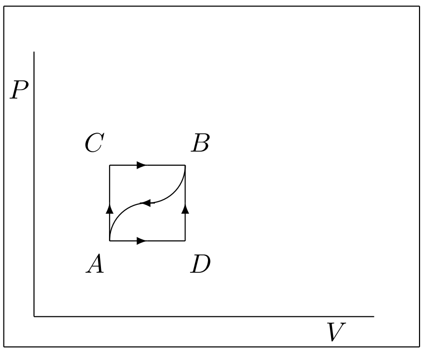
   A gas, contained in a cylinder fitted with a frictionless piston, is taken from the state A to the state B along the path ACB shown in the diagram. On this path, 80 J of heat flows into the system and the system **does** 30 J of work.

   (a) Using the First Law, write down the difference in internal energy between the states A and B.

   (b) How much heat flows into the system for the process represented by the path ADB if the work done by the system on this path is 10 J?

   (c) When the system returns from state B to state A along the **curved** path AB, the work done **on** the system is 20 J. What is the heat transfer?

   (d) If the internal energy of the system in state A is U_A while in the state D, $U_D = U_A + 40$ J, find the heat absorbed in the processes AD and DB.



3. **Free expansion of a van der Waals gas**

   Assuming that helium obeys the van der Waals equation of state, determine the change in temperature when one kilomole of helium gas, initially at 20°C and with a volume of 0.12 m^3, undergoes a free expansion to a final pressure of one atmosphere. You should use the following expression:

$$\left(\frac{\partial T}{\partial V}\right)_U = -\frac{a}{C_V}\left(\frac{n}{V}\right)^2$$

For this gas the relevant parameters in the van der Waals equation of state are given by $a = 3.44 \times 10^3$ J m^3 kmol^{-2}; $b = 0.0234$ m^3 kmol^{-1}; and in this case $C_V/nR = 1.506$.
(Hint: You may approximate. First show that the initial pressure is much higher than the final one. Then you may assume that the final volume is much larger than the initial volume.)
Numerical answer: -2.3 K.

4. **Free expansion experiments and internal energy of ideal gas**

In experiments on the free expansion of low-pressure (i.e. 'ideal') gases it was found that there was no measurable temperature change of the gas, i.e.

$$\left(\frac{\partial T}{\partial V}\right)_{U,ideal} = 0$$

Show that this observation means that the internal energy of the gas, U, must be a function of temperature T *only. Hint: you will need to use the* reciprocity *relation between partial derivatives:*

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

In this case the relevant variables (x,y,z) are T, U, and V.

5. **Adiabatic processes**

Use the First Law and the Ideal Gas equation to show that the pressure and volume of an ideal gas in a reversible adiabatic expansion are related by $PV^\gamma = c$ where c and $\gamma = C_P/C_V$ are constants. Show that the work done by the gas in a reversible adiabatic expansion from (P_1, V_1) to (P_2, V_2) is

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

Hint: Remember that $c_P - c_V = R$ for 1 mole of an ideal gas, and use the Ideal Gas equation in differential form RdT=PdV+VdP.

6. **Isobaric processes**

Use the First Law to find the change in internal energy of a monatomic ideal gas in an isobaric expansion at 1 atm from a volume of 5 m^3 to a volume of 10 m^3. Show that this is independent of the number of moles of gas. γ for a monatomic ideal gas is 5/3.

7. **Cooling in expansions**

By considering adiabats and isenthalps on an indicator diagram, *explain* why the adiabatic expansion produces more cooling than either a free expansion (Joule process) or a throttling (Joule-Kelvin) process for an ideal gas or similar material.

Given that:

$$\left(\frac{\partial H}{\partial P}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_P$$

use the triple product rule for partial differentials to derive the expression

$$\mu_{JK} = \left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left(T\left(\frac{\partial V}{\partial T}\right)_P - V\right)$$

Show that over a range of T where μ_{JK} is independent of temperature, the cooling in a throttling (Joule-Kelvin) process with a pressure change from P_1 to P_2 is

$$\Delta T = \int_{P_1}^{P_2} \left(\frac{\partial T}{\partial P}\right)_H dP$$

In a similar way, show that the cooling in an adiabatic reversible expansion from a pressure P_1 to P_2 is

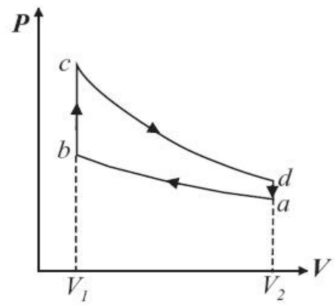
$$\Delta T = \int_{P_1}^{P_2} \left(\frac{\partial T}{\partial P}\right)_S dP$$

Hence *prove* that, for a given pressure change, the adiabatic expansion produces more cooling than a throttling process, i.e. that the difference in the integrands $(\partial T/\partial P)_S - (\partial T/\partial P)_H$ is positive.)

8. **EXAM STYLE TUTORIAL PROBLEM: Efficiency of engines**

An engineer applies to a venture capital company for money to market a new heat engine, which is claimed to extract 5000 J of heat from a reservoir at 400 K, reject 3500 J to a reservoir at 300 K, and do 1500 J of work per cycle on the surroundings. How should the venture capitalists go about studying details of the device.

An ideal gas is taken through the reversible cycle (The Otto cycle) shown in the diagram where **ab** and **cd** are adiabatics. The temperature at a is T_a and so on.



Briefly describe the working cycle identifing the processes during which heat flows in or out of the system. Calclate expressions for these heat flows in terms of an appropriate heat capacity. Give an expression for the net work output per cycle and identify this quantity on a sketch of the $P-V$ diagram.

The 'Otto cycle' is an idealisation of the internal combustion engine. Which segments represent fuel compression, burning, and exhaust?

Show that the efficiency of the Otto cycle is

$$1 - \frac{T_d - T_a}{T_c - T_b}$$