School of Physics and Astronomy

Junior Honours **Thermodynamics** GJA 2019-2020

Tutorial 4: Cycles and the Second Law

1 Questions: Cycles and the Second Law

1. **Statements of the Second Law of Thermodynamics**

   Show that if the Clausius statement of the Second Law of Thermodynamics is false, the Kelvin-Planck statement of the Second Law of Thermodynamics must be false also.

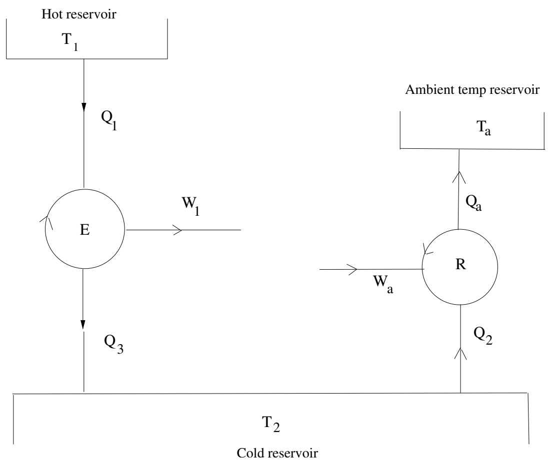
   Hint: Make a composite heat engine consisting of a Clausius-statement-breaking refrigerator which transfers an amount of heat Q_2 per cycle from a cold to a hot body together with an engine which delivers the same amount of heat, Q_2, per cycle to the cold body.

2. **Efficiency of engines part 1**

   Which gives the greater increase in the efficiency of a Carnot engine: increasing the temperature of the hot reservoir or lowering the temperature of the cold reservoir by the same amount?

3. **Efficiency of engines part 2**

   The maximum efficiency of a heat engine can be increased by reducing the temperature of the lower temperature reservoir. Consider the composite engine shown in the diagram opposite. Engine E operates between a high temperature T_1 and a body at T_2, lower than the ambient temperature. This lower temperature body is maintained at lower than ambient temperature by a refrigerator which extracts heat Q_2 from the lower temperature body and emits heat at the ambient temperature T_a. Show that the composite engine ER has a maximum efficiency equal to that of a single engine operating between the temperatures T_1 and T_a, in other words that nothing is gained by artificially cooling the lower temperature reservoir.

   For the composite engine use the usual formula for efficiency, in terms of the **net** *work done on the surroundings and the (heat) energy supplied at the* **highest** *temperature. Then exploit the relationships between heat flows and temperatures.*

4. **The best possible fridge** Using reasonable values for the temperatures inside and outside a domestic refrigerator, calculate its maximum possible efficiency.

5. **Domestic heating**

   A building is heated to 27°C. How much heat per second could be supplied by

   (a) An electric heater, with power input 20kW

   (b) A heat pump connected to an adjacent river at 7°C, with power input 20kW.

6. **Multipurpose device.**

   A company markets a device which, it claims, can extract 400W heat from a fridge compartment *and* deliver 1kW heating to the living room using just 100W of electricity. Consider both First and Second Laws to determine whether the claim is thermodynamically plausible?

7. **Yet another cycle**

   An engine cycle using an ideal gas consists of the following steps
   (i) an isobaric compression from Volume V_a to V_b at pressure P_a

(ii) an increase in pressure from P_a to P_b at a constant volume V_b
(iii) an adiabatic expansion from (P_b, V_b) to the original state at (P_a, V_a)

Sketch this cycle on a PV plot. Describe the steps where heat enters and leaves the system.

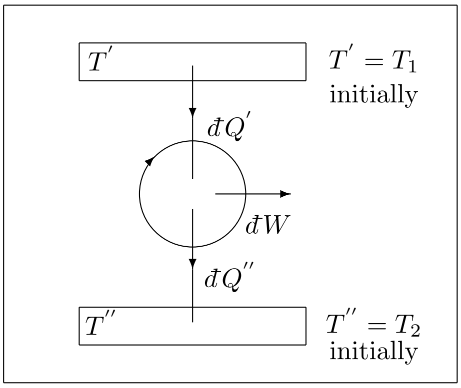
What is the efficiency of a heat engine expressed in terms of the magnitudes of the heat inputs and outputs? Show that the efficiency of the cycle described above is

$$\eta = 1 - \frac{\gamma P_a}{V_b}\frac{V_a - V_b}{P_b - P_a}$$

why is it peculiar to see γ in this expression?

8. **Challenge question: Work extracted while approaching thermal equilibrium**

A small Carnot engine operates between two identical bodies each having a finite heat capacity C_P, initially at temperatures T_1 and T_2 respectively, as indicated in the diagram.
Heat flows from the higher temperature body to the lower temperature body until the two bodies eventually reach the same temperature, T_f. Calculate the total amount of work done by the Carnot engine before the temperatures of both bodies reach T_f.
How would the result differ if the cold reservoir was large enough that its temperature remains constant?



Hint: It is necessary to specify an intermediate stage between start (here the two temperatures T_1 and T_2) and finish (here the common temperature T_f). To avoid an ambiguous notation, use T' as the intermediate temperature for the body whose initial temperature was T_1; and T'' as the intermediate temperature for the body whose initial temperature was T_2. Then small temperature changes for an intermediate cycle can be represented – loosely – by dT' and dT'', and heat flows by $C_P dT'$ and $C_P dT''$. However, great care has to be taken when allocating/associating signs with these four quantities! An application of the general relationship between heat flows and temperatures, in this context, provides an equation which can be integrated between limits denoted by T_1, T_2 and T_f, as appropriate. The relationship between T_f, T_1 and T_2 turns out to be $T_f^2 = T_1 T_2$, which you should derive.