School of Physics and Astronomy

Junior Honours **Thermodynamics** GJA 2019-2020

Tutorial 5: Entropy

1 Questions: Thermodynamic Potentials

1. **Calculation of entropy change**

   5 kg of hot water at 25°C cools to the 5°C temperature of its surroundings. How much heat flows? What is the entropy change of the surroundings? What is the entropy change of the water? (c_P for water = 4.19×10^3 J kg^{-1} K^{-1}.)

2. **Variation on the same theme**

   An electric current of 10 A flows through a resistor of 20 ohms which is kept at 27°C by being immersed in running water. What is the entropy change per second of the resistor, the water and the universe?

3. **Entropy and ideal gases**

   The internal energy of an ideal gas is all kinetic energy, and so it depends only on the temperature.

   Use this fact to derive an expression for the difference, $S_2 - S_1$, in the entropies S_1 and S_2 of one mole of an ideal gas at volumes V_1 and V_2 respectively, at the same temperature.

   Then show that for an ideal gas, C_P is independent of pressure P.

4. **General entropy changes in a ideal gas** The heat capacity of an ideal gas is not fully defined by the equation of state, it depends on details of the molecule. Consider the case of a near ideal gas with equation of state $PV = nRT$ and $c_v = A + BT$ where A and B are constants, and show that the change in entropy between state (V_1, T_1) and state (V_2, T_2) is

   $$\Delta S = A\ln(T_2/T_1) + B(T_2 - T_1) + R\ln(V_2/V_1)$$

   Hint: From the Central Equation, dS can be written as a sum of two terms, one involving dU and the other dV. A glance at the result sought shows that the term involving dU has to be re-cast as one involving dT before doing the integration to get ΔS.

5. **Another look at the Carnot cycle** Sketch a Carnot cycle, not on a PV diagram but on a *TS* diagram (i.e. temperature versus entropy). Show that the area within the closed path is equal to the heat absorbed per cycle provided that the path is traced out clockwise.

   Hint: consider what is happening on each 'leg' of the Carnot cycle, in terms of entropy and temperature.

6. **Entropy change in a (simplified) gin and tonic**

   A cube of ice of mass 30 g melts in a glass of water at 0°C in a room at 20°C. The water is stirred slowly to keep its temperature at 0°C, but gently enough that the work done can be neglected. Calculate the changes in entropy of the ice, the water and the air in the room. (The latent heat of fusion of ice is 334 kJ kg^{-1}.)

7. **Entropy change, reversible and irreversible processes**

   A block of lead with heat capacity C_P=1000 J K^{-1} is cooled from T_1=200K to T_2=100K by:

   (a) plunging the block into a large bath of liquid at 100 K,

   (b) first plunging the block into a large bath at 150 K, equilibrating, then plunging into a second bath at 100 K.

   (c) a reversible process

Calculate the entropy change of the universe in each case, and give an explanation for how the reversible process could be implemented in practice.

Exam-style version of 6 & 7

A cube of ice of mass 30 g melts in a glass of water at 0°C in a room at 20°C. The water is stirred slowly to keep its temperature at 0°C, but gently enough that the work done can be neglected.

Define each system carefully, explaining how you treat the molecules which start as ice and melt, or which start as water and evaporate, and calculate the changes in entropy of the following systems:

(a) the ice, [4 marks]

(b) the water [4 marks]

(c) the air in the room. [4 marks]
(The latent heat of fusion of ice is 334 kJ kg^{-1}.)
A block of lead with heat capacity C_P=1000 J K^{-1} is cooled from T_1=200K to T_2=100K. Calculate the entropy change of the universe in each of the following cases,

(d) plunging the block into a large bath of liquid at 100 K, [3 marks]

(e) first plunging the block into a large bath at 150 K, equilibrating, then plunging into a second bath at 100 K. [4 marks]

(f) a reversible process [3 marks]

(g) Give an explanation for how the reversible process could be implemented in practice. [3 marks]