School of Physics and Astronomy

Junior Honours **Thermodynamics** GJA 2019-2020

Tutorial 10: Chemical Potential

1 Questions: Chemical Potential

1. **Irn Bru This question is very similar to the 2018 Exam question B2.**

   A 1.05l bottle of Irn Bru contains 1l of drink and is pressurised to 5atm with CO_2 gas.

   At this pressure, the CO_2 concentration in the drink is given by Henry's law C = 0.031 P_g where C is the concentration in moles/L, and P_g is the partial pressure of the gas (5.0 atm).

   What is the chemical potential of the CO_2 in this system relative to the gas at atmospheric pressure?

   The cap is briefly loosened, so that the gas comes fully into equilibrium with the atmosphere, but the dissolved CO_2 remains in solution. The bottle is then sealed.

   What are the chemical potentials of the CO_2 in the drink and the gas above?

   Now the bottle is shaken, such that the contents come into equilibrium. Describe what happens, and calculate the pressure inside the bottle now?

   Approximately how many times can this process be repeated before the drink goes flat?

2. **Chemical Potential: Nature's boundary condition**

   Show that the molar chemical potential for an ideal gas at temperature T is given by

   $$\mu = RT\ln(P/P_0) + \mu_0$$

   where $\mu_0 = \mu(T, P_0)$.

   Under standard temperature and pressure, dilute carbon dioxide has μ_0 = 394kJ/mol in air and μ_0 = 386kJ/mol. in water.

   If the atmosphere contains 0.04% CO_2, estimate the concentration of CO_2 dissolved in the ocean.

3. **Chemical Potential Change in Mixing**

   A rigid, thermally isolated container holding 1mol of argon at 1atm, 300K is connected to an identical container holding 1mol of krypton at 1atm, 300K. No heat is added and no work is done on the system. Without calculation, explain what you expect to happen and the final equilibrium state. How do the pressure, temperature and entropy change?

   How would the result change if the initial amounts were different, in volumes V_A and V_K, still at the same initial T and P and still totalling 2 moles of atoms?

4. **Regular solution and solubility limits**

   A total of one mole of fluids, comprising two atomic types, A and B, are mixed at constant temperature. The fluids are ideal except that they repel one another, which adds a term to the internal energy $\frac{Z}{v_A v_B}$.

   Explain why this is a reasonable form for the interaction.

   Calculate the change in Gibbs Free Energy when the two are mixed, with mole fractions x_A and $x_B = 1 - x_A$, and plot this as a function of x_A for various values of $\frac{RT}{Z}$

   For $Z = 3RT$ what values of x_A minimise Δg? Calculate the chemical potential for species A at these values.

   Describe the mixing process as x_A increases from 0 to 1.

5. **Supercool**

   Salt is added to a mixture of ice and water at 0^oC. Assuming that salt cannot dissolve in ice, what is the change in chemical potential of the water with salt concentration $X = 0.1$?

$$\left(\frac{\partial \mu_L}{\partial X}\right)_T$$

   What is the final temperature of the mixture? Assume that the partial pressure of each component in the brine is proportional to its concentration. Take the latent heat of fusion to be 18kJ/mol. For convenience, you can assume that for small changes in absolute temperature l/T^2 is temperature independent.

6. **Simplified Osmosis**

   A surface-dwelling single-celled, spherical marine creature contains protein molecules and water, and is separated from the sea by a semipermeable membrane[1] which water can pass, but not protein. Treating the protein as a monotonic ideal gas, and assuming there are 2% as many protein molecules as water molecules, calculate the total pressure inside the cell.

   If the membrane has diameter 10μm and thickness 10nm, then what is the stress in the membrane?

[1] i.e. the cell wall. Real cells have all sorts of funny shaped or charged holes to let through some molecules and not others