College of Science and Engineering
School of Physics & Astronomy

Thermodynamics
SCQF Level 9, PHYS09021
Wednesday 9th May, 2012
14:30 - 16:30

Chairman of Examiners
Professor A Trew

External Examiner
Professor D McMorrow

Answer ALL of the questions in Section A
and TWO questions from Section B.

The bracketed numbers give an indication of the value assigned
to each portion of a question.

Only the supplied Electronic Calculators may be used during this examination.

ANONYMITY OF THE CANDIDATE WILL BE MAINTAINED DURING THE MARKING OF
THIS EXAMINATION.

Section A: Answer ALL of the questions in this Section

A.1 (i) Without referring to temperature, explain what conditions need to be satisfied for two bodies to be in thermal equilibrium. **[3]**

(ii) State the zeroth law of thermodynamics. **[2]**

A.2 Explain why the heat supplied to a simple fluid in contact with a constant pressure reservoir can be equated to the change of enthalpy of the fluid. You should be careful to explain why this result does not depend on whether the changes occur reversibly. **[5]**

A.3 Show that the entropy of one mole of an ideal gas at constant volume is related to the temperature by
$$S = C_v \ln[T] + \text{const},$$
where C_v is the constant volume heat capacity. **[5]**

A.4 State the third law of thermodynamics. What value does $\left(\frac{\partial U}{\partial V}\right)_T$ tend to as $T \to 0$ (explain your reasoning)? **[5]**

Section B: Answer TWO questions from this Section

B.1 This question examines phase transitions and a thermodynamic process for ocean thermal energy conversion. An example of ocean thermal energy conversion is power generation from the difference in temperature of surface sea water (typically 25 ^{o}C in the tropics) and deep ocean water (5 ^{o}C at 1 km depth).

(a) Explain carefully what is meant by thermodynamic efficiency. What is the numerical value for the maximum thermodynamic efficiency for a heat engine working between the above two temperatures? [4]

An idea for a non-cyclic 'engine' using the water itself as the 'working substance' is shown below (all air is removed from the system). The difference in vapour pressure between the water at the two temperatures is used to drive a turbine, generating electricity. In the following we consider this process for pure water.
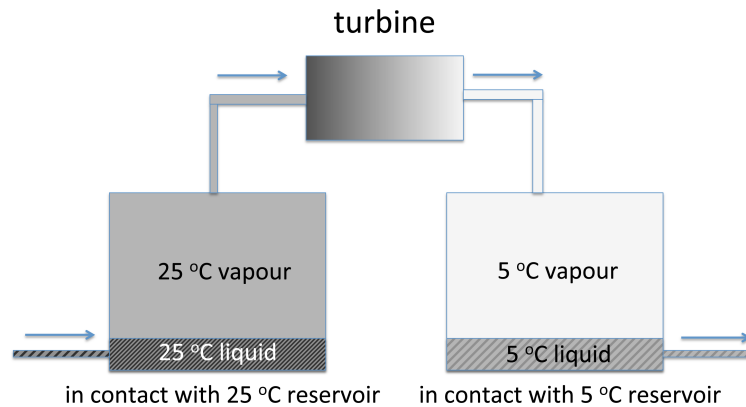
(b) Outline the derivation of the Clausius-Claperon equation that expresses the change of vapour pressure with temperature for a pure substance in terms of the changes in entropy and volume crossing the liquid-vapour phase boundary:
$$\frac{dP_{\text{vap}}}{dT} = \frac{\Delta S}{\Delta V}.$$ [7]

(c) Assuming the vapour can be approximated as an ideal gas, show that
$$P_{\text{vap}} \approx P_0 e^{-L/RT},$$
where P is the vapour pressure, P_0 is a constant, and L is the latent heat of vaporisation. State clearly any other approximations you make. [5]

(d) Assuming the vapour can be approximated as an ideal gas, what is the maximum work obtainable per mole of vapour passing through a thermally isolated turbine (i.e. in a continuous flow process) connecting vapour at these two temperatures and pressures ($U = 3RT$ for one mole of water vapour)? A numerical answer is expected (there is no need to check whether such a process is thermodynamically allowed). [4]

B.2 This question concerns the Eriksson cycle plotted on a temperature-entropy indicator diagram in the figure below.
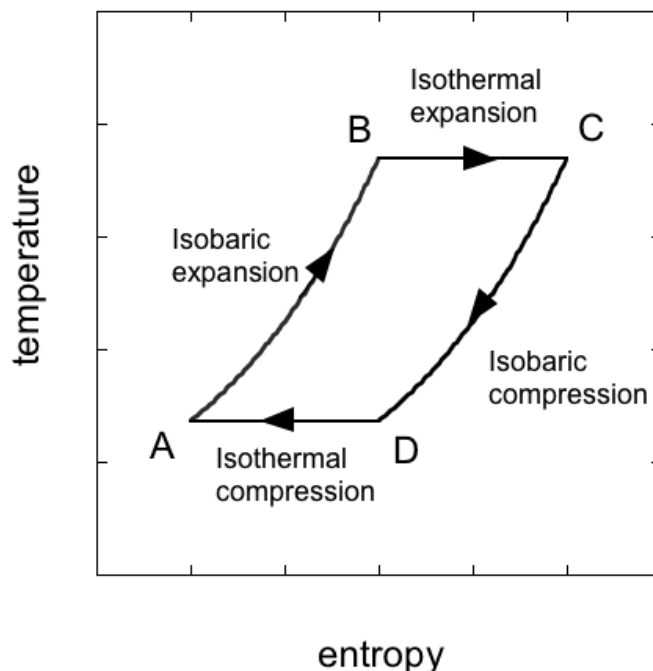
(a) For a monatomic ideal gas (starting from $dH = TdS + Vdp$, or otherwise) show that the equation relating molar entropy s to T for a reversible isobaric process is:

$$\ln\left[\frac{T}{T_1}\right] = \frac{(s - s_1)}{(5/2)R},$$

where s_1 and T_1 are the coordinates of one point on the line. [4]

(b) (i) Explain how you can read off from the above figure the direction of heat transfer to the working substance for each leg of the process.

(ii) What would a process with zero heat transfer look like on the above figure? [3]

(c) Assuming the working substance is one mole of a monatomic ideal gas and all the processes shown are reversible, show that the heat transfers to the working substance along AB and BC are [5]

$$Q_{AB} = \frac{5}{2}R(T_B - T_A),$$

$$Q_{BC} = T_B R \ln\left[\frac{P_B}{P_C}\right].$$

P_B and P_C denote the pressure at points B and C. T_A and T_B denote the temperature at points A and B.

(d) (i) What is the change of internal energy of the gas going once around the closed cycle? [2]

(ii) Hence, what is the relationship between the sum of the heat transfers and the work done? [2]

(e) Hence, show that the work done per cycle is,

$$W = (T_B - T_A)R \ln\left[\frac{P_B}{P_C}\right].$$

[4]

B.3 This question examines some analytic thermodynamic relations for a magnet. Changes of the internal energy of a uniformly magnetised magnetic needle can be written:

$$dU = TdS + B_a dM - pdV,$$

with B_a the applied magnetic induction (assumed uniform), M the total magnetic moment, p the pressure and V the volume. Consider the following equation:

$$\left(\frac{\partial C_{B_a,V}}{\partial B_a}\right)_{T,V} = T\left(\frac{\partial^2 M}{\partial T^2}\right)_{B_a,V}.$$

(a) $C_{B_a,V}$ is the heat capacity at constant volume and applied magnetic field. What is meant by this quantity and how does it relate to changes of entropy? **[2]**

(b) Explain why in the derivation of the equation we can drop the $-pdV$ term in the equation for dU and effectively work with $dU = TdS + B_a dM$. **[2]**

(c) Derive the following Maxwell relation, justifying the critical steps of your derivation:

$$\left(\frac{\partial S}{\partial B_a}\right)_{T,V} = \left(\frac{\partial M}{\partial T}\right)_{B_a,V}.$$

[6]

(d) Hence or otherwise derive the above expression for $\left(\frac{\partial C_{B_a,V}}{\partial B_a}\right)_{T,V}$. **[5]**

(e) The magnetic moment of a needle held at constant volume is observed to obey the following law in zero magnetic field:

$$M^2 = M_0^2\left(1 - \frac{T^2}{T_c^2}\right),$$

where T_c is the Curie temperature and M_0 is the zero field moment (both constants). Sketch how the heat capacity divided by temperature is initially expected to vary with field at both $T = 0$ and $T \rightarrow T_c$. **[5]**