Quantum Mechanics

Assessed problem sheet 1 - to be handed to TO on Fri 22/10.

Consider a one-dimensional harmonic oscillator defined by the potential:

$$V(x) = \frac{1}{2} m\omega^2 x^2 \,.$$

At $t = 0$ the state of the oscillator is:

$$\Psi(x, 0) = \sum_{n=0}^{\infty} c_n u_n(x) \,,$$

where $u_n(x)$ are the eigenfunctions of the Hamiltonian.

1. Write down the equations satisfied by the wave functions $u_n(x)$. Recall the possible values of the energy E_n, and the corresponding eigenfunctions.

2. Write the wave function describing the system at time t.
   Deduce the mean value of any observable A at time t:

   $$\langle \Psi(t) | \hat{A} | \Psi(t) \rangle$$

   as a function of the matrix elements:

   $$A_{mn} = \langle u_m | \hat{A} | u_n \rangle \,.$$

   Compare with the time-dependence of the mean value of A in a stationary state, i.e. in the case where only one of the coefficients c_n is different from zero.

Let us now consider the case where $c_0 = \cos\eta$, $c_1 = \sin\eta$, and $c_n = 0$ for $n > 1$. You should use this initial state in all subsequent questions.

3. What are the possible outcomes of a measurement of the energy? What are the probabilities of each outcome?

4. Consider the observables X and P. Using the explicit expressions for the eigenfunctions $u_0(x)$ and $u_1(x)$ given in the lecture notes, determine which are the matrix elements X_{mn} and P_{mn} that do not vanish for $m, n = 0, 1$. (You don't need to compute the integrals, just identify those that do not vanish!)
   Deduce that the mean values $\langle \Psi(t) | \hat{X} | \Psi(t) \rangle$ and $\langle \Psi(t) | \hat{P} | \Psi(t) \rangle$ are sinusoidal functions of time with angular frequency ω.

5. Verify that this result is consistent with Ehrenfest's theorem.

L Del Debbio, October 2010.