Quantum Mechanics

Assessed problem sheet 2 - to be handed to TO by Thurs 10th November 5pm.

1. In one dimension, consider a particle coming in from the left with energy $E > 0$ and scattering on the potential barrier:

$$V(x) = g\delta(x)\,,$$

where g is some positive real number.

   (a) Compute the transmission and reflection coefficients. **[4]**

   (b) Compare with the result for the finite potential step, Eq. (5.20) of the lecture notes, in the limit where $V \to \infty$, $a \to 0$, and $Va = g$ stays constant. **[4]**

   (c) What happens if $g < 0$. Compare with the classical result. **[4]**

2. The angular momentum operator in two dimensions can be defined as:

$$\hat{L} = \hat{X}\hat{P}_y - \hat{Y}\hat{P}_x\,.$$

Rewrite the operator $\hat{L}$ as a differential operator in polar coordinates. Show that the eigenvalues of L are $\hbar m$, where m is an integer, and find the eigenfunctions. **[3]**

3. Write down the time-independent Schrödinger equation for a particle in a two-dimensional circular infinite well of radius R. Separate the variables by writing the wave function as:

$$\psi(r,\theta) = R(r)\Phi(\theta)\,.$$

   (a) Find the differential equation for the radial part of the wave function $R_m(r)$ for each value of the angular momentum m. **[4]**

   (b) The solution of the radial equation is given by the Bessel function $J_{|m|}(kr)$, where k is related to the energy E by $E = \hbar^2 k^2/(2m)$. Write down the boundary condition for this problem. Let us denote by $a_{n,m}$ the n-th zero of $J_{|m|}$. Find the energy levels as functions of the zeroes of the Bessel functions. **[2]**

   (c) For $m = 0$ the first zero of the Bessel function $J_0(z)$ occurs for $z = 2.405$. Deduce the value of the energy for the ground state of the system. Compare to the ground state energy of the two-dimensional infinite square potential well of size $L = \sqrt{\pi}R$. **[4]**

L Del Debbio, November 2011.