

Interlude A

Postulates

The Postulates of Quantum Mechanics

We can now try to summarize the minimal set of assumptions that we have discussed so far to set up quantum mechanics. There is no unique choice and you will find a variety of formulations in the various recommended texts. This is a quick review of the concepts discussed in lectures 1-4; it will not be presented in a lecture, but should be used as a reference for the basic concepts. The rest of the course will present further developments of quantum mechanics that rely on these postulates.

Postulate 1:

Every possible physical state of a given system corresponds to some wavefunction $\Psi(x, t)$ that is a single-valued function of the parameters of the system and of time, and from which all possible predictions of the physical properties of the system can be obtained.

Note: the parameters could be, for example, coordinates but may also refer to internal variables such as ‘spin’.

Postulate 2:

Every observable is represented by a Hermitean operator. To each such observable, $\mathcal{A}$, there corresponds an operator, $\hat{A}$, with a complete orthonormal set of eigenfunctions, $\{u_i(x)\}$, and a corresponding set of real eigenvalues, $\{A_i\}$:

$$\hat{A} u_i(x) = A_i u_i(x)$$

The only possible values which any measurement of $\mathcal{A}$ can yield are the eigenvalues $A_1, A_2, A_3, \dots$

Notes:

- Orthonormality means as usual that

$$\int_{-\infty}^{\infty} u_i^*(x) u_j(x) dx = \delta_{ij}$$

- completeness means that an arbitrary wavefunction $\Psi(x, t)$ can be expanded as:

$$\Psi(x, t) = \sum_i c_i(t) u_i(x)$$

with coefficients c_i given by orthogonal projection:

$$c_i(t) = \int_{-\infty}^{\infty} u_i^*(x) \Psi(x, t) dx$$

The set of functions $\{u_i(x)\}$ is referred to as *the eigenbasis of $\hat{A}$* .

- the eigenvalues of $\hat{A}$ may be discrete or continuous.

Postulate 3:

If the observable $\mathcal{A}$ is measured on a system which, immediately prior to the measurement, is in the state $\Psi(x, t)$ then the strongest predictive statement that can be made about the result is

$$P(A_j), \text{ the probability of getting } A_j = \left| \int_{-\infty}^{\infty} u_j^*(x) \Psi(x, t) dx \right|^2 = |c_j(t)|^2$$

Notes:

- measurements are assumed to be *ideal*, i.e. to yield a single, errorless real number;
- the integral $\int_{-\infty}^{\infty} u_j^*(x) \Psi(x, t) dx$ is sometimes called an *overlap integral*;
- in general, *we cannot predict with certainty the outcome of a measurement*; only in the special case where $\Psi(x, t)$ coincides with an eigenfunction of $\hat{A}$, for example, $u_k(x)$ at the instant t , in which case

$$c_j(t) = \int_{-\infty}^{\infty} u_j^*(x) u_k(x) dx = \delta_{jk}$$

so that A_k will be obtained with probability 1;

- a measurement of $\mathcal{A}$ on each of two identically prepared systems, both in the same quantum state Ψ , will not necessarily yield the same result.

Successive Measurements

What can we say about the state of a system *after* making a measurement of $\mathcal{A}$ on it ? Suppose that the result of our measurement was A_k . Then it is plausible that were we to immediately remeasure $\mathcal{A}$, we should get the same result A_k . Postulate 3 asserts that we can only be *certain* to get the result A_k if the system is described by the eigenfunction u_k corresponding to the eigenvalue A_k .

Postulate 4:

A measurement of an observable $\mathcal{A}$ generally causes a drastic, uncontrollable change in the state of the system. Regardless of the form of $\Psi(x, t)$ just before the measurement, immediately after the measurement the wavefunction will coincide with the eigenfunction of $\hat{A}$ corresponding to the eigenvalue obtained in the measurement of $\mathcal{A}$.

Notes:

- this is sometimes referred to as the *collapse of the wavefunction*; we also speak of *forcing the system into an eigenstate*;
- we have assumed that the eigenvalues and eigenfunctions are in 1-1 correspondence i.e. that there is no degeneracy;
- Postulate 3 guarantees that if, after measurement of $\mathcal{A}$, the wavefunction coincides with $u_k(x)$, then the probability of getting A_k is unity if we immediately remeasure $\mathcal{A}$;
- if the wavefunction, $\Psi(x, t)$ *before* the measurement does not coincide with an eigenfunction of $\hat{A}$, then the observable $\mathcal{A}$ cannot be said to have a value in the state $\Psi(x, t)$;
- more generally, we speak of a series of *successive measurements* being made, if the state of the system immediately prior to the $(n + 1)$ th measurement (of the same, or some other, observable) is that which resulted from the n th measurement, in contrast to the case of *repeated measurements* which are always made with the system in the same state immediately prior to each measurement.

Postulate 5:

The time development of a quantum system is determined by the Time-Dependent Schrödinger Equation:

$$\hat{H} \Psi(x, t) = i\hbar \frac{\partial}{\partial t} \Psi(x, t)$$

where the Hamiltonian operator, $\hat{H}$, is formed from the corresponding classical Hamiltonian function by operator substitution, and represents the total energy of the system.

Notes:

- $\hat{H}$ possesses a complete orthonormal set of eigenfunctions $\{u_n(x)\}$ and a corresponding set of real eigenvalues $\{E_n\}$;
- if $\Psi(x, 0)$ is normalised to 1 then $\Psi(x, t)$ is also normalised to 1 for all t .