Methods of Mathematical Physics 2007/8

Synopsis

A course on advanced methods of mathematical physics. The course aims to demonstrate the utility and limitations of a variety of powerful calculational techniques and to provide a deeper understanding of the mathematics underpinning theoretical physics. The course will review and develop the theory of: Complex analysis and applications to Special Functions; Asymptotic Expansions including Method of Steepest Descent; Dirac delta and generalised functions; Ordinary and Partial Differential Equations, in particular Integral Transform techniques and Sturm-Liouville Theory; Green Function techniques, in particular calculation by Fourier inversion. The generality of approaches will be emphasised and illustrative examples from electrodynamics, quantum and statistical mechanics will be given.

Syllabus

Sections marked * will only be covered in lectures if time permits

1. Introduction; Revision of Infinite Series; Convergence of series: convergent, divergent and asymptotic series.
2. Review of complex analysis; residue theorem; analytic continuation.
3. Gamma function: definition; analytic continuation; integral representation; Stirling's Formula; Hankel's contour; beta functions.
4. Approximating Integrals; Laplace's method; Method of Stationary Phase;
5. The Saddle-point method a.k.a. 'method of steepest descents'; asymptotic expansions.
6. Delta functions: limit sequence of functions; integral representations; fourier transform; theory of distributions
7. Ordinary differential equations: generalities; theory of linear homogeneous differential equations; Wronskians
8. Second order ODE: finding a second solution; Green function method; series solutions; *WKB method
9. Some properties of 'Special Functions'; integral representations; generating functions; asymptotic expansions
10. Fourier transforms application to ODEs; inversion by contour integration and Green functions
11. * Sturm–Liouville Theory: eigenvector expansions; Hilbert spaces; self-adjoint operators; eigenfunction expansions; existence of eigenvalues and completeness of eigenfunctions; spectral theory.

12. Laplace transforms: application to ODEs; inversion contour; asymptotic expansions and applications to dynamical problems

13. Partial differential equations: examples; integral transform techniques; *method of characteristics. Classification of second order PDE s hyperbolic, parabolic and elliptic equations.

14. Green function methods for PDEs: Wave equation (hyperbolic case);

15. Diffusion equation (parabolic case); initial value Green functions;

16. Laplace's equation (elliptic case); * method of images

Textbooks

- Jon Mathews and Robert Walker, *Mathematical Methods of Physics*, Benjamin/Cummings Publishing Co. ISBN 0805370021. £60.

- G. Arfken and H. Weber, *Mathematical Methods for Physicists (4th or 5th edition)*, Academic press.

- K.F. Riley, M. P. Hobson and S. J. Bencce *Mathematical Methods for Physics and Engineering (3rd edition)*,

  Cambridge Univeristy Press.

  £35

- D. Richards, *Advanced Mathematical Methods with Maple*, Cambridge University Press. ISBN 0521779812 . £34.95.

Remarks: Quite a few parts of the course go beyond the scope of previous editions of Riley, Hobson and Bence (which many students may have a copy of), but the new, third edition seems to cover just about everything. Arfken (and Weber) covers nearly all the material in a comprehensive way but is a little uninspiring. Mathews and Walker is great but expensive and difficult to get hold of (but definitely worth getting if you are thinking of PhD). Richards covers all the topics and has an emphasis on Maple use to give a better feel for the techniques. Other references will be given for particular topics in the course.

MRE, September 19, 2007