Junior Honours

Electromagnetism **Problem Sheet 6**

Inductance

The questions that follow on this and succeeding sheets are an integral part of this course. The code beside each question has the following significance:

- **K**: key question – explores core material
- **R**: review question – an invitation to consolidate
- **C**: challenge question – going beyond the basic framework of the course
- **S**: standard question – general fitness training!

6.1 My generation [K]

A horizontal conducting rod is supported by parallel static conducting rails at separation L connected to a lightbulb of resistance R; the rod moves through a vertical $\underline{B}$ field, as shown, with horizontal velocity $\underline{v}$:

(i) Why does the lightbulb illuminate?

(ii) Find the current flowing (you should specify its direction round the circuit) and also the power P required to maintain it.

(iii) Find the force $\underline{F}$ on the rod caused by the $\underline{B}$ field acting on the current there, and hence show that $P = -\underline{F} \cdot \underline{v}$; interpret this (including the sign).

6.2 Stars of track and field [S]

Two horizontal parallel conducting rails are separated by a distance a, and are connected by a wire of resistance R. Between the rails is a uniform magnetic field $\underline{B}$ acting vertically. A straight rod of mass m rests across the rails at right angles at a distance l from the wire, so as to complete a conducting circuit. The rod is free to roll along the rails.

If the rod is set in motion with initial velocity $\underline{v_0}$ parallel to the rails, find its velocity at a subsequent time t. Is your answer consistent with Lenz's Law?

[Hint: consider changing the sign of both $\underline{B}$ and $\underline{v_0}$.]

6.3 Inductive reasoning [S]

(i) A long cable consists of two thin concentric conducting cylinders of radii a and $2a$. This forms part of a circuit in which a current I flows down the inner cylinder and back along the outer one. Use Ampères law and symmetry to find the $\underline{B}$ field between the conductors.

(ii) Hence find the self inductance per unit length of cable.

[Hint: if in doubt about how to define Φ_B for this circuit, find it for a particular current path and observe that all others will have the same value.]

(iii) Show that the magnetic energy stored in a given length of the cable, found from $U_{mag} = \int \frac{|\underline{B}|^2}{2\mu_0} d\tau$, is equal to the electromotive work $\mathcal{L}I^2/2$ required to establish the current I.

6.4 Displacement activity [K]

A circular parallel plate capacitor of radius 10cm and separation 1mm is discharged via a long wire with a steady current $I = 0.2$A. Sketch the $\underline{B}$ field lines between the plates, and find the $\underline{B}$ field 5cm from the axis of the capacitor. How does this compare with the $\underline{B}$ field at 5cm radius from a point on the long wire (far from the plates)? Interpret your result.

6.5 An honorable discharge [S]

A similar capacitor to that in the previous question carries charge Q. The external wire is left on open circuit ($I = 0$), but instead a material of nonzero conductivity σ is introduced between the plates. (You can assume the material has negligible dielectric or magnetic response.)

Show that, as the capacitor discharges, the current density $\underline{J}$ between the plates is exactly cancelled by the Maxwell displacement current density, $\epsilon_0 \dfrac{\partial \underline{E}}{\partial t}$. Deduce that no $\underline{B}$ field arises, and explain how this is consistent with the conditions in the external circuit.

6.6 The feeling is mutual [K]

(i) What is $\underline{B}$ at the centre of a circular current loop carrying current I?

(ii) A flat circular coil of N_a turns and radius a lies in a horizontal plane. At its centre lies a very small circular coil of radius b and N_b turns; its plane is at angle θ to the horizontal. Find the induced EMF in the larger coil when the current in the smaller one is changing at a rate $\dot{I}_b$.

[Hint: remember the result for the mutual inductance: $M = \Phi_{a|b}/I_b = \Phi_{b|a}/I_a$.]

(iii) Suppose instead that I_b is held constant but the small coil rotates at rate $d\theta/dt$. Find the resulting EMF in the large coil.

6.7 Totally wired [S]

A current I_2 flows in a rectangular wire (length a, width a) which is placed a distance d from a long wire carrying current I_1 as shown in the figure. Find the mutual inductance $\mathcal{M}$ and thus (interaction) energy of the two circuits. Hence show that the force on the rectangular wire from the long wire is

$$\underline{F} = \frac{\mu_0 I_1 I_2 ab}{2\pi d(d+b)} \underline{e}y$$