Junior Honours

Electromagnetism **Problem Sheet 8**

Electric Displacement Field; Radiation Pressure

HAND-IN DEADLINE: you must bring your solution to Q8.4 to the teaching office on Friday 15th March before 4pm

You are strongly advised to work through the preceding questions, obtaining help from tutors where necessary, **before** attempting the hand-in question.

The code beside each question has the following significance:

- **K**: key question – explores core material
- **R**: review question – an invitation to consolidate
- **C**: challenge question – going beyond the basic framework of the course
- **S**: standard question – general fitness training!

8.1 **Cleaning the plates [S]**

A 100pF parallel plate capacitor in a vacuum is charged to a potential of 50 V.

(i) Find the charge on the capacitor and its energy U_o.

(ii) Maintaining the connection to the battery, the capacitor is immersed in an insulating oil of relative permittivity $\epsilon_r = 3$. Find the new charge on the capacitor and the new energy U_1.

(iii) The plates are then disconnected from the battery, and withdrawn from the oil. Find the new potential difference, and the new energy U_3.

(iv) Does repeating this procedure represent a free source of unlimited energy?

8.2 **Obeying Gauss's law – on the surface of it [K]**

A conducting sphere of radius a, carrying positive net charge Q, is placed within a linear dielectric medium of electrical susceptibility χ_E.

(i) Use an appropriate Maxwell equation to find $\underline{D}(\underline{r})$ outside the sphere.

(ii) Deduce expressions for $\underline{E}(\underline{r})$ and $\underline{P}(\underline{r})$ in the dielectric.

(iii) Show that the bulk polarization charge density ρ_b in the region occupied by dielectric is zero.

(iv) Show that there is a nonzero polarization surface charge density σ_P at the surface of the dielectric touching the sphere, and give its magnitude and sign. [Take care with the sign: what direction is the surface normal?]

(v) Calculate the total polarization charge Q_{pol} at this surface of the dielectric, and show that, when this is included as part of the full charge density, the microscopic version of Gauss's law (that for the $\underline{E}$ field) is obeyed.

8.3 **Concentric capacitor[S]**

A capacitor consists of two concentric spherical conducting shells of radius $r = a$ and $4a$. There is a charge Q on the inner shell and $-Q$ on the outer. The region with $a < r < 2a$ contains a medium of relative permittivity $\epsilon_r = 7$. The region with $2a < r < 4a$ is empty.

(i) Find $\underline{D}$ and $\underline{E}$ everywhere within the capacitor.

(ii) Find the potential difference between the outer and inner shells, and hence find the capacitance.

8.4♣ **Coaxial conundrum**

A long cable consists of two coaxial conducting cylindrical shells of radius a and $3a$. The region with radius ρ between a and $2a$ is filled with a material of relative permittivity $\epsilon_r \neq 1$ and relative permeability $\mu_r = 1$; the remaining space between the cylinders is empty.

(i) Suppose the cable carries charge per unit length $\pm\lambda$ on the inner and outer cylinders. Find $\underline{D}$ and $\underline{E}$ everywhere within it.

(ii) Hence calculate the magnitude of the potential difference between the inner and outer shell, and deduce an expression for the capacitance per unit length of cable.

(iii) Suppose the cable forms part of a circuit in which a current I flows down the inner cylinder and back along the outer cylinder. Find $\underline{B}$ everywhere within it. Compute the Poynting vector and comment on its direction.

(iv) Hence calculate the flux Φ_B that cuts the current loop formed by the cable, and deduce an expression for the inductance per unit length.

8.5 **Withdrawal symptoms [S]**

A slab of dielectric (relative permittivity ϵ_r) fills the space between two rectangular plates (l_1 by l_2) of a parallel plate capacitor at small separation d. It is then withdrawn by a distance x parallel to the l_1 direction, with a battery maintaining a constant potential V across the capacitor.

(i) Find $\underline{E}$ and $\underline{D}$ in both parts of the capacitor, and calculate the electrostatic energy $u_E = \frac{1}{2}\underline{D} \cdot \underline{E}$ as a function of x.

(ii) Hence work out the force on the slab $\underline{f}(x)$.

8.6 **More withdrawal symptoms [S]**

Return to the previous problem.

(i) Suppose the surface charge density on the conducting plates was everywhere held fixed at its initial value during the process of withdrawal; calculate the new $\underline{F}(x)$ and give a reasoned argument for its sign.

(ii) Give the sign of the force in the case where the capacitor is charged up to potential V but then disconnected from the battery before the slab is withdrawn. State with reasons whether either the above calculation, or the one in Q8.5, can give the correct force $\underline{F}(x)$ for this case.

8.7 **Withstanding the pressure [S]**

On the equator, the power per unit area of sunlight at normal incidence to the earth's surface is approximately $1.4 \times 10^3 \mathrm{Jm^{-2}s^{-1}}$.

(i) By considering the mean Poynting vector $\langle \underline{S} \rangle$, estimate the magnitudes of the fields E and B due to the incident radiation. You may assume that $\mu_r = 1$ for air.

(ii) Estimate the radiation pressure exerted on the ground.

(iii) For an asteroid of the same density as the earth, moving in roughly the same orbit, how small would the asteroid have to be for this pressure to significantly perturb its orbital motion?

[Data: the mass of the earth is 6×10^{24}kg, its radius is 6.4×10^6m, and that of its orbit 1.5×10^{11}m.]