Junior Honours

Electromagnetism **Problem Sheet 9**

Magnetic field $\underline{H}$; Boundary conditions at interfaces

The questions that follow on this and succeeding sheets are an integral part of this course. The code beside each question has the following significance:

- **K**: key question – explores core material
- **R**: review question – an invitation to consolidate
- **C**: challenge question – going beyond the basic framework of the course
- **S**: standard question – general fitness training!

9.1 Completing the circuit [S]

Consider a coil with $N = 200$ turns and circular cross section (radius $a = 1$ cm) that is bent into a toroid of radius $r = 10$ cm.

(i) Find the $\underline{B}$ field inside the toroid if it is filled with a material of relative permeability $\mu_r = 1000$ and carries a current of 5A.

(ii) If a small gap of length $l = 1$ mm is cut through the material, use the continuity conditions to find the $\underline{B}$ field in the gap. Show that this is smaller than in (i) but much larger than if there was no magnetic material present at all.

(iii) Sketch the field lines for $\underline{H}$ and for $\underline{B}$.

[Data: $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}.]

9.2 On current reflection [S]

A long wire runs horizontally at height a above a large horizontal slab of material of relative permeability $\mu_r = 3$; the wire carries a current I_1.

(i) Write down the continuity conditions for $\underline{B}$ and $\underline{H}$ at the surface of the slab.

(ii) Show that these can be satisfied if

(a) Exterior to the slab, the $\underline{B}$ field is that of the current I_1 in the wire plus that of a parallel current I_2 carried by an 'image wire' (both calculated as though in a vacuum);

(b) Within the slab, the $\underline{B}$ field is that of a current I_3 along the original wire (calculated as though the material were present throughout space).

[Hint: you are not asked to write out explicit expressions for the resulting $\underline{B}$ field everywhere, so can restrict attention to $\underline{B}$ on the matching plane itself.]

(iii) Show that $I_2 = \frac{\mu_r - 1}{\mu_r + 1} I_1$ and find I_3.

(iv) What is the force per unit length on the wire?

9.3 Currently polarized [K]

A wire of radius a carries a current I; it is surrounded by a magnetic material.

(i) For static fields and with media present, Ampère's law reads $\underline{\nabla} \times \underline{H} = \underline{J}$. Use this equation in integral form to find $\underline{H}(\underline{r})$ at points outside the wire.

(ii) Assume that the medium is linear so that its magnetisation obeys $\underline{M}(\underline{r}) = \chi_M \underline{H}$. Find $\underline{M}$, and the field $\underline{B}(\underline{r}) = \mu_0(\underline{H} + \underline{M})$ in the material.

(iii) Show that the bulk magnetisation current $\underline{J}_M(\underline{r})$ in the material is zero.

(iv) Find the surface magnetisation current $\underline{j}_M$ where the magnetic material meets the conductor.

(v) Show that when this is included in the total current, the fundamental version of Ampère's law (that for the $\underline{B}$ field) is obeyed.

9.4 Feeling the force [S]

A solenoidal inductor is maintained at constant current I. It contains a ferrite core of large μ_r. If this is displaced so that only part of the solenoid has the core within it, does the ferrite feel a force expelling it, drawing back in, or neither?

[N.B. A detailed calculation is not required.]

9.5 An induced dipole [S]

A sphere of relative permittivity ϵ_2 is surrounded by a large region of relative permittivity ϵ_1. The $\underline{E}$ field far from the sphere is uniform and of magnitude E_0.

(i) Show that the continuity conditions for fields $\underline{D}$ and $\underline{E}$ can be satisfied by an E-field that is uniform within the sphere, and that outside the sphere is a superposition of a uniform field with one from an electric dipole at the sphere centre.

(ii) Hence find $\underline{E}$ inside the sphere. Sketch the field lines for $\underline{E}$, in the case where $\epsilon_2 \gg \epsilon_1$.

9.6 At the interface [S]

At the interface between one linear dielectric and another the electric field lines bend. Consider a plane interface: for $z > 0$ the permittivity is ϵ_1, while for $z < 0$ it is ϵ_2. If the electric field for $z > 0$ is towards the interface, and makes an angle θ_1 to the normal (i.e. the z-axis), show that the electric field in $z < 0$ is away from the interface, and makes an angle θ_2 to the normal, where $\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_2}{\epsilon_1}$. Find also the relative magnitudes of the fields.

Derive similar results for the bending of magnetic fields at the interface between materials of permeability μ_1 and μ_2.

9.7 Integration of curl [C]

In lecture 16 we used the identity

$$\int_V \underline{\nabla}' \times \left(\frac{\underline{M}(\underline{r}')}{|\underline{r} - \underline{r}'|} \right) \mathrm{d}V' = - \oint_S \frac{\underline{M}(\underline{r}') \times \mathrm{d}\underline{S}'}{|\underline{r} - \underline{r}'|}$$

Prove this by using the divergence theorem for a vector field $\underline{F} = \underline{v} \times \underline{c}$ where $\underline{c}$ is a constant vector.