STATISTICAL PHYSICS 06/07

Radial distribution function; Debye-Hueckel Theory **Tutorial Sheet 5**

The questions that follow on this and succeeding sheets are an integral part of this course. The code beside each question has the following significance:

- **K**: key question – explores core material
- **R**: review question – an invitation to consolidate
- **C**: challenge question – going beyond the basic framework of the course
- **S**: standard question – general fitness training!

5.1 **Debye screening length [r]**

Show that the continuum approximation which underlies the Debye-Hückel theory is valid provided that

$$q^3 N^{\frac{1}{2}} \left(\frac{\beta}{\epsilon}\right)^{\frac{3}{2}} \ll 1$$

where $\beta = 1/kT$.

5.2 **Multi species plasma [s]** Consider a plasma containing more than one species of mobile ion, with no fixed charges but overall charge neutrality (for example, an ionized gas). By considering the distribution of ions around a point charge (of arbitrary sign) show that the Debye screening length λ_D obeys

$$\lambda_D^{-2} = \frac{\sum_i q_i^2 n_i}{\epsilon kT}$$

where q_i is the charge of species i and n_i is the particle density (at infinity) of species i.

5.3 **Radial distribution Function [s]** For a plasma containing two ionic species with opposite charges but the same density ρ, calculate the radial distribution functions $g_{++}(r)$, $g_{-+}(r)$, where $\rho_i g_{ij}(r)$ is the conditional probability density for finding a particle of type i in a small volume at distance r from one of type j. You may assume Debye-Hueckel theory is valid and should use the results of the previous question.

5.4 **Semi-infinite plasma [s]** A semiinfinite sample of a one-component plasma ends at a flat wall which carries a positive surface charge density σ. Assuming the Debye-Hueckel equation applies, calculate and sketch the potential ϕ as a function of z, the distance from the wall. Sketch also, on the same horizontal axis, the density of free charges in the plasma.

5.5 **Virial equation of state [c]** Complete the proof, outlined in the notes, of the virial equation of state

$$P = \rho kT - \frac{\rho^2}{6}\int_0^\infty \left(r\frac{d\phi}{dr}\right) g(r)\, 4\pi r^2 dr$$

where $g(r)$ is the radial distribution function and $\phi(r)$ is the interatomic potential and here ρ is the particle density.