STATISTICAL PHYSICS

Ising Model, Landau Theory, Universality **Tutorial Sheet 7**

The questions that follow on this and succeeding sheets are an integral part of this course. The code beside each question has the following significance:

- **K**: key question – explores core material
- **R**: review question – an invitation to consolidate
- **C**: challenge question – going beyond the basic framework of the course
- **S**: standard question – general fitness training!

7.1 Order parameter of an antiferromagnet [s]

The simplest representation of an antiferromagnet is afforded by the Ising model with a nearest neighbour interaction parameter J which is *negative*.

(i) By considering a one-dimensional system identify the two possible ground states and hence the two phases which one might expect to coexist at low enough temperatures. Find a suitable order parameter which differentiates between the two low temperature phases and is 0 in the high temperature disordered phase

(ii) Generalise to a three-dimensional system

7.2 Ising Model Representation of Binary Alloy [s]

The simplest model of a binary allow envisages an assembly consisting of $N/2$ atoms of species A and $N/2$ atoms of species B, distributed over sites forming a body centred cubic lattice. The energy of interaction of neighbouring pairs of atoms is ϵ_{AA}, ϵ_{AB} or ϵ_{BB} according to the pair involved. Show that the canonical partition function is that of the Ising model in zero field and with interaction parameter

$$J = -\frac{1}{4}\left(\epsilon_{AA} + \epsilon_{BB} - 2\epsilon_{AB}\right)$$

7.3 Mean-Field Theory of Ising Model Revisited [s]

(i) Review the derivation given in lectures of a mean field free energy function per spin

$$f(m) = -\frac{zJ}{2}m^2 - hm + kT\left[\left(\frac{1+m}{2}\right)\ln\left(\frac{1+m}{2}\right) + \left(\frac{1-m}{2}\right)\ln\left(\frac{1-m}{2}\right)\right]$$

(ii) Show that minimising $f(m)$ leads to the Weiss mean-field equation for the magnetisation

(iii) Expand $f(m)$ in powers of m keeping terms up to order m^4. You will need the expansion

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$$

7.4 Landau Theory [r/s]

Check and complete the Landau theory calculations, given in lectures, for the critical exponents β, γ, δ and α of the Ising model. For the latter you will need to first show the result $c_h = -T\left(\partial^2 f/\partial T^2\right)_h$.