

Junior Honours

Electromagnetism

Problem Sheet 2

Electrostatics: Electric field, Electric potential, Gauss' Law

The questions that follow on this and succeeding sheets are an integral part of this course. Cross references to the questions are given in the lecture notes. The code beside each question has the following significance:

- **K**: key question – explores core material
- **R**: review question – an invitation to consolidate
- **C**: challenge question – going beyond the basic framework of the course
- **S**: standard question – general fitness training!

2.1 Curvature and field [K]

Consider two conducting spheres of radius R_A and R_B ($R_A < R_B$) connected by a long conducting wire.

Using the potential for a charged sphere $V = \frac{q}{4\pi\epsilon_0 R}$ show that the charge on the two spheres is in the ratio

$$\frac{q_A}{q_B} = \frac{R_A}{R_B}$$

but that the electric field is of greater magnitude on the smaller sphere A . [Hint: recall the relation between surface charge *density* and normal surface field for a conductor.]

2.2 Sparks might fly [S]

(i) Find the \underline{E} field at a distance u from the axis of a long conducting cylinder of radius a carrying charge per unit length Λ .

(ii) Dry air cannot support a field above about 10^6 V/m without breaking down (causing a spark). Estimate the maximum potential difference to earth sustainable by a long cylinder of radius 1 cm in dry air, within an earthed concentric cylinder of radius 1 m.

2.3 Potential example [S]

The potential in a region of space is given by $V(\underline{r}) = -kx^3$, where k is a constant and $\underline{r} = (x, y, z)$.

(i) Calculate the electric field $\underline{E}(\underline{r})$.

(ii) Use Gauss' law (integral form) to find the total charge Q within a cube of side a with one corner at the origin and its three edges along the positive x, y, z directions.

[Hint: think first about which faces can make a contribution to the surface integral in Gauss's law.]

(iii) Using Poisson's equation, determine the charge density $\rho(\underline{r})$.

(iv) Show by doing a volume integral over the cube that this is consistent with the previous result for Q .

2.4 Gauss's Law in spherical symmetry [K]

(a) Use Gauss' Law (integral form) to obtain the field, everywhere, of charge of uniform density ρ occupying the region $a < r < b$, where r is the distance from the origin. Find the potential. [Hint: note the spherical symmetry of the problem]

(b) Show that in the limit $b \rightarrow a$, $\rho \rightarrow \infty$, with $(b - a)\rho = \sigma$ remaining finite, the electric field suffers a discontinuity of amount σ/ϵ_0 in crossing the layer of charge. What happens to the potential?

2.5 Gauss's Law in planar symmetry [K]

(i) Find $V(z)$ and hence $\underline{E}(z)$ (on the axis) for a disc of radius a carrying uniform surface charge density σ .

(ii) Check that when $a \rightarrow \infty$ you recover the field found by applying Gauss's law to an infinite charged sheet. Explain why this would be doubled if the same surface charge were to reside on the surface of a conductor.

(iii) A large horizontal sheet of surface charge density σ has a hole of radius b cut out. Find the field as a function of height z directly above the centre of the hole.

2.6 Hiding in its shell [S]

A charge Q resides at the centre of a spherical shell of copper, of inner radius a and outer radius b . The copper shell carries no net charge.

(i) Use Gauss's law to find the field $\underline{E}(r)$ everywhere, and sketch the field lines.

(ii) Work out the surface charge density on both the inner and the outer surface of the copper shell.

(iii) How would the field for $|r| > b$ be altered if the charge was moved off-centre within the shell?

2.7 Charge on hemisphere [S/C]

A charge Q is distributed uniformly over the surface of a hemisphere of radius a . Calculate the potential and thus the electric field at any point on the axis inside the hemisphere. Show that at the centre of the hemisphere the field has magnitude $\sigma/4\epsilon_0$, where σ is the surface charge density. [Hint: you will need to recall the cosine rule]