Measuring g with Compound and Kater Pendula

Abstract: A compound pendulum and a Kater pendulum have been used to measure the gravitational acceleration, g, to a precision of 0.13% and 0.03%, respectively. The measured values of g, $9.821(13)\text{ms}^{-2}$ and $9.810(3)\text{ms}^{-2}$, respectively, are both in excellent agreement with the value of $9.8159870(2)\text{ms}^{-2}$ measured on top of Blackford Hill in 1976.

The Compound Pendulum: For small angular displacements, a compound pendulum executes simple harmonic motion with a period T_O given by the equation:

$$T_O = 2\pi\sqrt{\frac{(k^2+h_O^2)}{gh_O}}$$

where h_o is the distance from the suspension point to the centre of mass, and k is the radius of gyration. This equation can be linearised to read:

$$T_O^2 h_O = 4\pi^2\left[\frac{k^2+h_O{}^2}{g}\right]$$

And value of g can thus be determined from the variation of T_O with h_O.

The period of oscillation was determined for 10 different values of h_O using a photogate timer. The period of the pendulum for each value of h_O, and its uncertainty, was determined from the times of 20 individual periods. A histogram of 20 such measurements is shown in Figure 1, from which the mean period was estimated as 1.99274(3)s. A plot of the raw data is shown in Figure 2 and a plot of the linearised data is shown in Figure 3. In both cases the error bars are smaller than the symbols used to plot the data

From a non-weighted least-squares fit to the data using LINEST, the value of g was determined to be $9.821(13)\text{ms}^{-2}$, in good agreement with the value of $9.8159870(2)\text{ms}^{-2}$ measured on top of Blackford Hill in 1976. A plot of the residuals from the least-squares fit – shown in Figure 4 – suggests that the uncertainty of 1mm on the measured values of h_O was overestimated, particularly at small values of h_O. This is reflected in the reduced value of χ^2 for the least-squares fit which is 0.69.

The Kater Pendulum: For a Kater pendulum with two pivot points a distance l apart (Figure 5), when the period, T, of the pendulum is identical about each pivot point, then the period is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

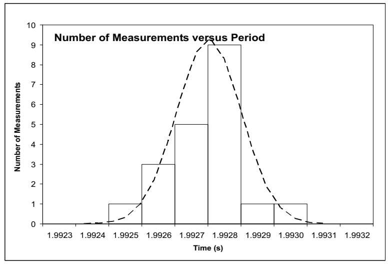

Figure 1: Histogram of the measured period of the compound pendulum. The dashed Gaussian curve was calculated from the mean (1.99274s) and standard deviation (0.00011s) estimated from these measurements.

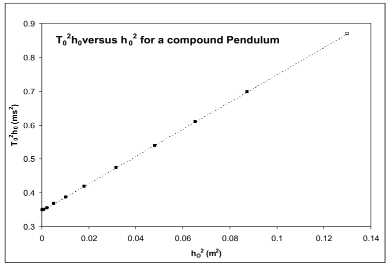

Figure 3: The linearised data. The dashed line is the best-fitting straight line to the data. The uncertainties on each data point are smaller than the symbols used to plot the data.

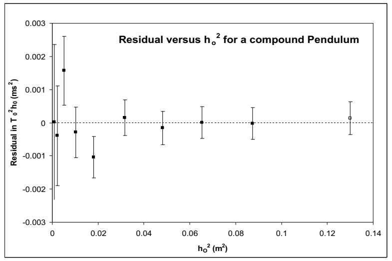

Figure 4: The residuals from the least-squares fit. The distribution of data points suggests that the uncertainties have been overestimated.

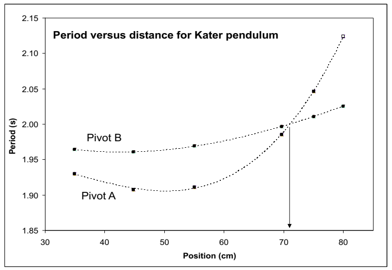

Figure 6: Period of the Kater pendulum about pivots A and B as a function of the position of the moveable mass. The periods are equal at a position of 71cm.

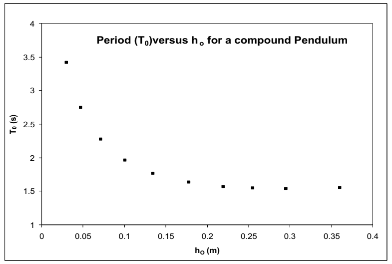

Figure 2: The period of the compound pendulum as a function of the distance between pivot point and centre of mass. The uncertainties on each data point are smaller than the symbols used to plot the data

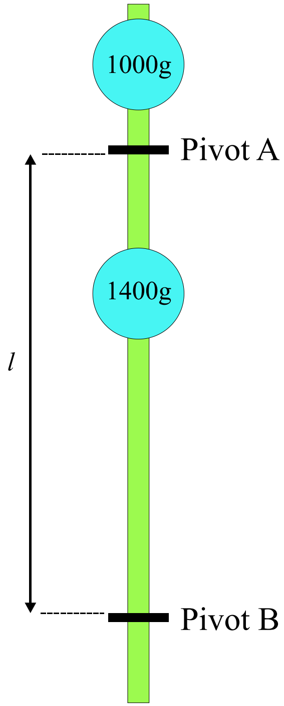

Figure 5: The Kater pendulum, showing the positions of the two masses, and the locations of pivots A and B.

For a Kater pendulum with two pivot points a measured distance l=0.9939(1)m apart, the period of the pendulum about pivots A and B (Fig. 5) as the adjustable 1.4kg mass is moved along the length of the pendulum is shown in Figure 6. The mean period at each point, and its uncertainty, was determined from the periods of 25 individual oscillations.

The position of the adjustable mass required to make the periods about each pivot equal is clearly near 71cm. The period of the pendulum was therefore carefully remeasured at several points close to this position. The position of the moveable mass was determined relative to that of the fixed mass, and the position were measured to a precision of ±0.05mm by using vernier calipers. The variation of the period with position is shown in Figure 7. The period about each pivot was determined from the times of 25 individual oscillations.

From linear least-squares fits to the period data for each pivot, the position of the mass required to make the periods equal is 11.9974cm. At this position, the period of the pendulum about each pivot is 1.9999(3)s, where the uncertainty is the typical standard deviation in the measurements of the period.

The values of g and its uncertainty Δg are given by

$$g = \frac{4\pi^2 l}{T^2} \quad\text{and}\quad \Delta g = g\sqrt{(\Delta l/l)^2 + (2\Delta T/T)^2}$$

and from the measured values of l and T and their uncertainties, the measured value of g is $9.810(3)\text{ms}^{-2}$. The measured value is determined to a precision of 0.03%, a 4-fold improvement over the precision obtained with the compound pendulum. The value of g is also is excellent agreement with the literature value of $9.8159870(2)\text{ms}^{-2}$.

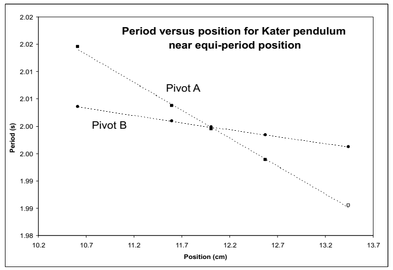

Figure 7: Period versus position in the vicinity of the equi-period position. The uncertainties on each data point are smaller than the symbols used to plot the data.