Radioisotopes, Poisson Statistics and Half Lives

Abstract: Using a Geiger-Muller detector with a measured dead-time of 284(26)μs, the half lives of the radioisotopes V^{52} and Rh^{104} have been determined. From linear least-squares fits to the dead-time and background corrected decay data, the half life of V^{52} was determined to be 3.62(12)mins, and the two half-lives of Rh^{105} were determined to be 4.3(3)mins and 45.7(1.8)s. The literature values are 3.743mins, 4.43mins and 42.3s, respectively. Decay data were also collected from the unknown sample "element X" and its two half-lives were determined to be 186(3)s and 30.7(5)s.

Radioactive decay is a random process governed by the laws of quantum physics. If a sample contains a large number N_0 of unstable nuclei at time t=0, then the number remaining at some later time t is given by:

$$N = N_0 e^{-\lambda t}$$

where λ is the "decay constant".

Before measuring decay-data, the dead time of the Geiger-Muller detector was determined. The background radiation was first determined by counting for 20 consecutive 60s intervals and was found to be 24.4(1.1)counts/min. Using a split calibration source of Tl^{204}, the count rates for the two individual half sources were 221.7(1.4)counts/s and 215.1(1.3) counts/s, respectively, while the count rate for the combined sources was 409.7(1.8) counts/s. The resulting deadtime was determined to be 284(26)μs.

Half-life of V52: The decay data for the vanadium sample were collected for 14 minutes, and the raw counts corrected for the background and the dead-time of the detector. The resulting corrected and linearised decay data are shown in Figure 1. A non-weighted least-squares fit to the data using LINEST gave a decay constant of 0.00322(11)s^{-1}, and hence a half-life of 3.62(12)mins.

Half-life of Rh105: The decay data from the Rh105 sample were collected for 25minutes, and the raw counts corrected for the background and detector deadtime. The time dependence of the corrected count rate is shown in Fig. 2(a), and the log of the count rate in Fig. 2(b).

From a non-weighted least-squares fit to the high-t (>300s) data using LINEST (see Fig. 2(c)), the decay constant of the longer lived Rh105(5+) state was determined to be 4.3(3)mins. The contribution of this longer-lived state to the low-t data was determined and subtracted from the measured countrate. The subsequent least-squares fit to the low-t (<150s) data is shown in Fig. 2(c), from which the half-life of the Rh105(1+) state was determined to be 45.7(1.8)s.

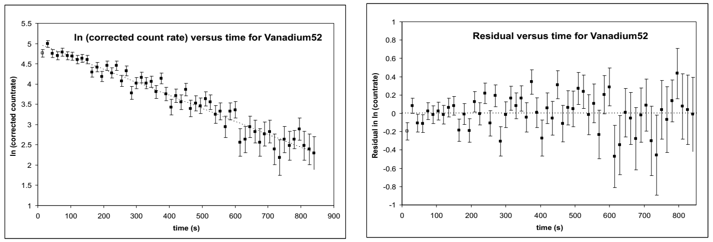

Figure 1: (Left) Natural log of the corrected decay rate data for V^{52}. The dashed line through the data point is the best-fitting straight line from the least-squares fit. (Right) The residuals from the least-squares fit.

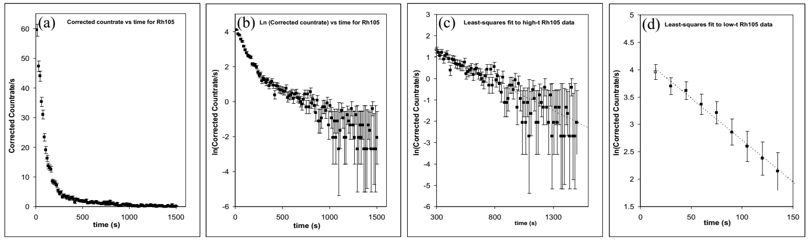

Figure 2: (a) The corrected count rate from Rh105, and (b) the natural log of the countrate. Graph (c) shows the high-time data, and the dashed line through the data points is the best-fitting straight line from least-squares fit. Graph (d) shows the least-squares fit to the low-time data after the contribution from the longer-lived isotope has been subtracted.

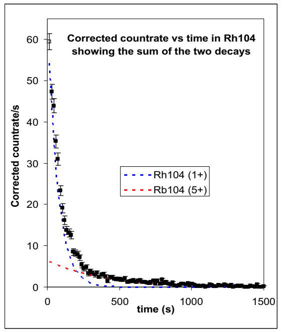

Figure 3: Corrected decay data for Rh105, showing the contributions from the Rh105(1+) and Rh105(5+) states. The calculated contributions are determined from the measured half-lives.

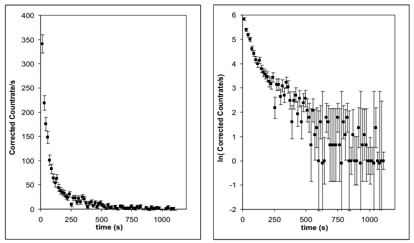

Figure 4: (a) The corrected count rate from element X, and (b) the natural log of the countrate.

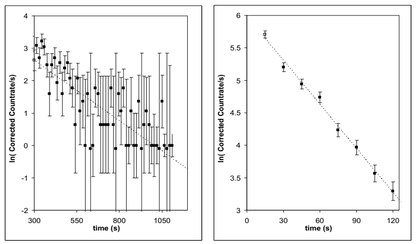

Figure 5: (a) The high-time data for element X. The dashed line through the data points is the best-fitting straight line from least-squares fit. (B) The least-squares fit to the low-time data after the contribution from the longer-lived isotope has been subtracted.

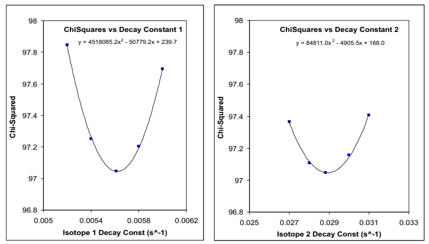

Figure 6: Plots of χ^2 vesus the two decay constants in element X. The lines through the data points are second-order polynomial fits, the equations of which are given on the graphs.

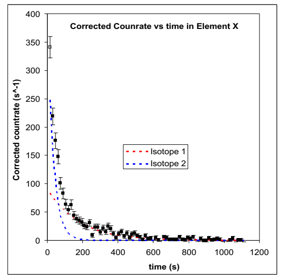

Figure 7: Non-linear fit to the decay data from element X showing the contributions from the two isotopes.

Half-life of Element X: Finally, the half-lives of the unknown element X were determined. A trial run suggested that, as in Rh, there were two half lives, and that after 20mins, the count rate dropped to background.

The decay data from element X were collected for 20 minutes, and the raw counts corrected for the background and detector deadtime. The time dependence of the corrected count rate is shown in Fig. 4(a), and the log of the count rate in Fig. 4(b).

From a non-weighted least-squares fit to the high-t (>300s) data using LINEST (see Fig. 5(a)), the decay constant of the longer lived element X isotope was determined to be 186(22)seconds. The contribution of this longer-lived state to the low-t data was determined and subtracted from the measured count rate. The subsequent least-squares fit to the low-t (<120s) data is shown in Fig. 5(b), from which the half-life of the shorter lived isotope was determined to be 31(1)s.

The element X decay curve was also fitted using non-linear fitting techniques. Using the SOLVER function in Excel, the full decay curve was modeled as a sum of two exponential decays

$$N = N_1 e^{-\lambda_1 t} + N_2 e^{-\lambda_2 t}$$

with initial values for the decay constants taken from the linear fits above. The best fitting decay constants were determined by minimising χ^2, and their uncertainties were determined by finding the values where χ^2 increased by 1. Plots of χ^2 versus the two decay constants in the vicinity of their best fitting values are shown in Figure 6. The half lives were determined to be 124(10)s and 24(3)s. The final fit to the total decay curve is shown in Figure 7.