Quantum Electrodynamics

Outline

Classical versus Quantum Theory
- Force/interaction mediated by exchange of field quanta

Virtual Particles
- Propagator

Feynman Diagrams
- Feynman Rules
- Matrix Elements
- Cross sections

Electromagnetic vertex
- Coupling strength
- Coupling constant
- Conservation laws

QED processes
- Electron-proton scattering
- Electron-positron annihilation

Higher order Diagrams
- Precision QED tests
- Running of alpha

Dirac Equation
- Appendix

Quantum Electrodynamics

Quantum Theory (QED)

of Electromagnetic Interactions

Classical Electromagnetism

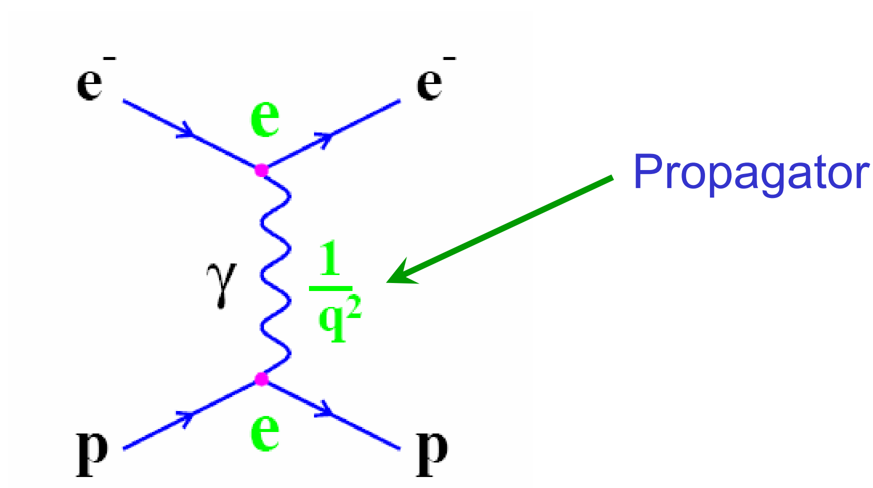
Forces arise from Potentials V(r)

act instantaneously at a distance

QED Picture

Forces described by exchange of

virtual field quanta - photons



Matrix element

Full derivation in 2nd order perturbation theory

Gives propagator term $1/(q^2 - m^2)$

for exchange boson

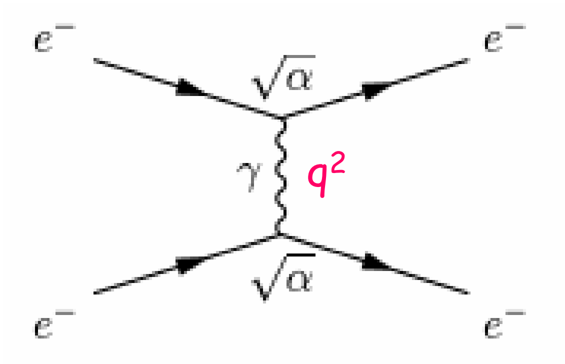
$$M_{fi} = \frac{g^2}{(q^2 - m^2)}$$

Equivalent to scattering in Yukawa potential

Virtual Particles

Electromagnetic Interaction

Forces between two charged particles are due to exchange of virtual photons



Example: electron-electron scattering: e- e- → e- e-

"Photon mediates electromagnetic interaction"

No action at a distance required!

Virtual Particles

Do not have mass of physical particle

$$m_X^2 \neq E_X^2 - \vec{p}_X^{\,2}$$

known as "Off mass-shell"

e.g. non-zero for photon

4-momentum of virtual particle $q^\mu = (E_q, \vec{q})$

is energy and momentum transfer

between scattered particles

Virtual mass-squared $q^2 = E_X^2 - p_X^2$

$q^2 > 0$ and $q^2 < 0$ possible

Propagator - how far particle is off mass-shell

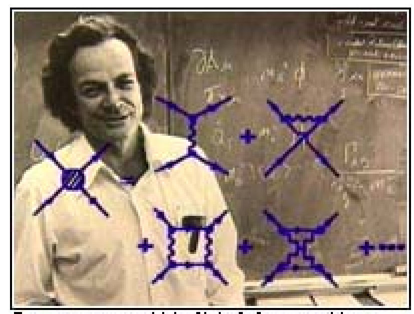
Internal lines, not observable must observe $\Delta E \, \Delta t \approx \hbar$

Feynman Diagrams

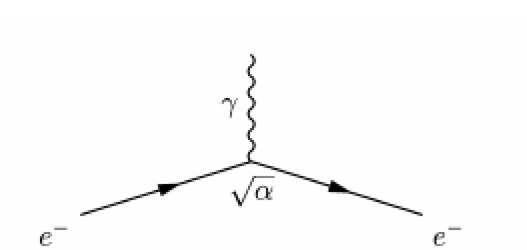
A Feynman diagram is a pictorial representation of a process corresponding to a particular transition amplitude   Aitchison & Hey
"Gauge Theories in Particle Physics"

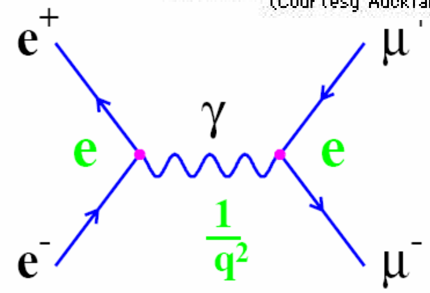
Basic Principle

Transition amplitude for all processes - scattering, decay, absorption, emission - is described by **Feynman Diagrams**



Feynman earned his Nobel for creating these diagrams
(Courtesy Auckland University)

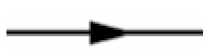




Feynman diagrams a most useful tool in modern particle physics and will be used a lot in this course!

Conventions

Time flows from left to right (sometimes upwards)

 Fermions are solid lines with arrows

 Bosons are wavy or dashed lines

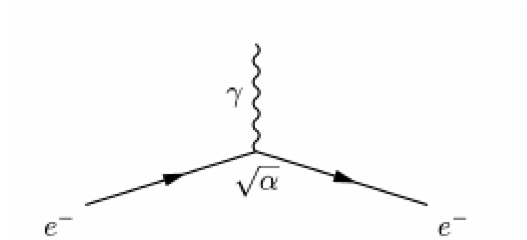
Feynman Rules

Allow to calculate quantitative results of transition
Derived from quantum field theory (QFT)

Electromagnetic Vertex

QED Feynman Diagram

- Initial state fermion
- Absorption or emission of photon
- Final state fermion



Examples: $e^- \to e^- \gamma$ Bremsstrahlung

$e^- + \gamma \to e^-$ Photoelectric effect

All electromagnetic interactions are described by vertex and photon propagator

Coupling Strength

Transition amplitude proportional to fermion charge $M_{fi} \propto e$

Probability/rate of γ emission or absorption

$$\text{rate} \propto |M_{fi}|^2$$

Rate proportional to coupling constant α

Coupling Constant

Fine structure constant

$\alpha \propto e^2$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cong \frac{1}{137} \quad \text{SI}$$

QED Conservation Laws

Momentum and energy conserved at all vertices

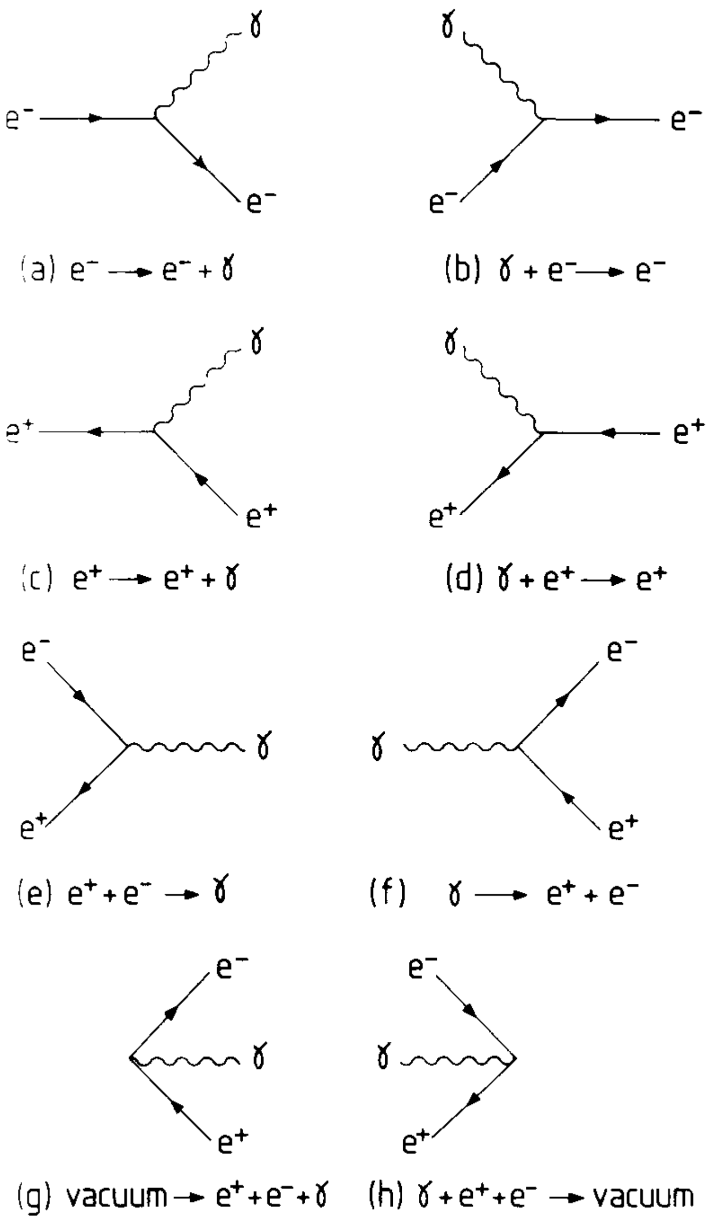
Charge is conserved in all QED vertices

Fermion flavour is conserved

$e^- \to e^- \gamma$ exists, but not $c \to u\,\gamma$

QED vertex also conserves parity

Basic QED Processes



Initial and final state particle satisfy relativistic four-momentum conservation $m^2 = E^2 - p^2$

In free space - Energy conservation violated for above diagrams if all particles are real

Feynman Rules for QED

Each line and vertex in **Feynman diagram**
corresponds to a term in the matrix element

Initial and final state fermions
Fermion wave function Ψ (spinor when including spin)

Initial and final state bosons
Boson wave function includes polarisation

Internal virtual photons
Photon propagator $1/(q^2 - m^2) = 1/q^2$

Internal fermions
Spinor propagator exchanged between charged particles similar in structure to photon propagator

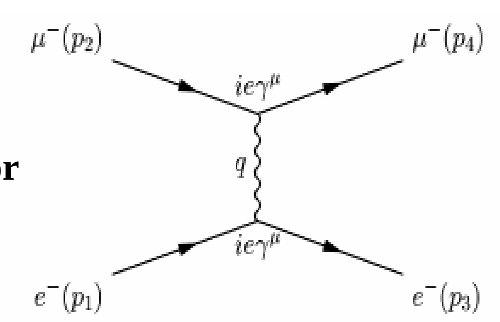
Vertex
Coupling constant $\sqrt{\alpha} \sim e$

Example:
electron-muon scattering: $e^- \mu^- \rightarrow e^- \mu^-$
Transition amplitude

$$-iM_{fi} = \langle \bar{u}_4 | ie\gamma^{\nu} | u_2 \rangle \quad \text{muon current}$$

$$\frac{-ig^{\mu\nu}}{q^2} \quad \text{photon propagator}$$

$$\langle \bar{u}_3 | ie\gamma^{\mu} | u_1 \rangle \quad \text{electron current}$$



γ^{μ} and $g^{\mu\nu}$ are 4x4 matrices account for spin-structure of electromagnetic interaction

Electron-Proton Scattering

Matrix Element

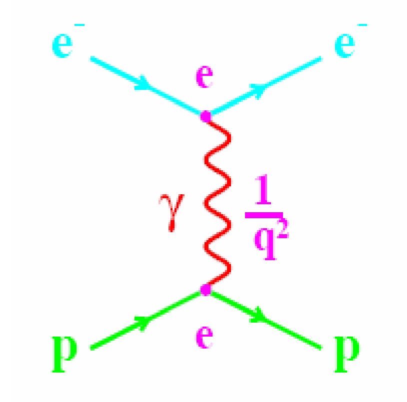
Transition Amplitude

use Feynman rules

simple if neglecting spins

$$M \propto e \frac{1}{q^2} e = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

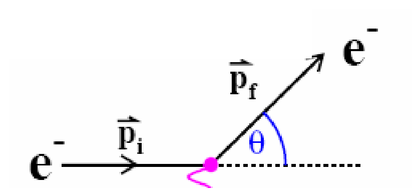
e- p → e- p



Cross section

Probability for scattering

$$\frac{d\sigma}{d\Omega} \propto |M|^2 \propto \frac{e^4}{q^4} = \frac{16\pi^2\alpha^2}{q^4}$$



4-momentum transfer

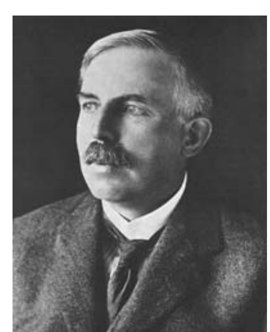
$$q^2 = q^\mu q_\mu = \left(p_f^\mu - p_i^\mu\right)^2 = p_f^2 + p_i^2 - 2p_f \cdot p_i$$
$$= 2m_e^2 - 2\left(E_f E_i - |\vec{p}_f||\vec{p}_i|\cos\theta\right)$$
$$= -4E_f E_i \sin^2\left(\frac{\theta}{2}\right)$$

$$p_f^\mu = \left(E_f, \vec{p}_f\right)$$
$$p_i^\mu = \left(E_i, \vec{p}_i\right)$$

when neglecting m_e

Rutherford Scattering

Elastic $E_f = E_i = E$, neglect proton recoil



$$\left.\frac{d\sigma}{d\Omega}\right|_{Lab} = \frac{\alpha^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{Lab} = \frac{\alpha^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)} \frac{E_f}{E_i}\left(\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M_p^2}\sin^2\left(\frac{\theta}{2}\right)\right)$$

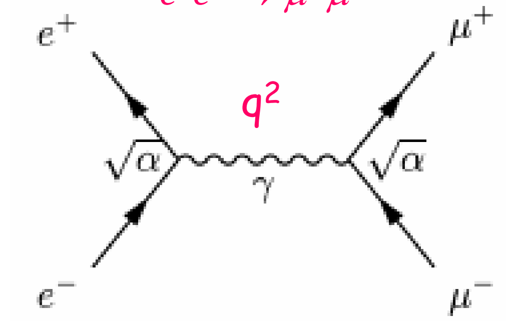
e+e- Annihilation

Matrix element

Neglecting spin effects

$$M \propto e \frac{1}{q^2} e = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

$e^+e^- \rightarrow \mu^+\mu^-$



Cross section

Work in centre-of-mass frame

4-momentum transfer

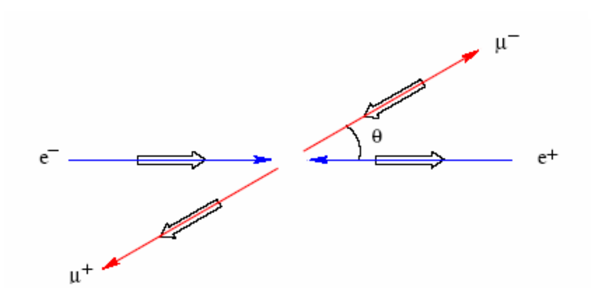
$$q^2 = q^\mu q_\mu = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 = (2E)^2 = s$$

Use Fermi's Golden Rule, density of final state

normalisation of wave function

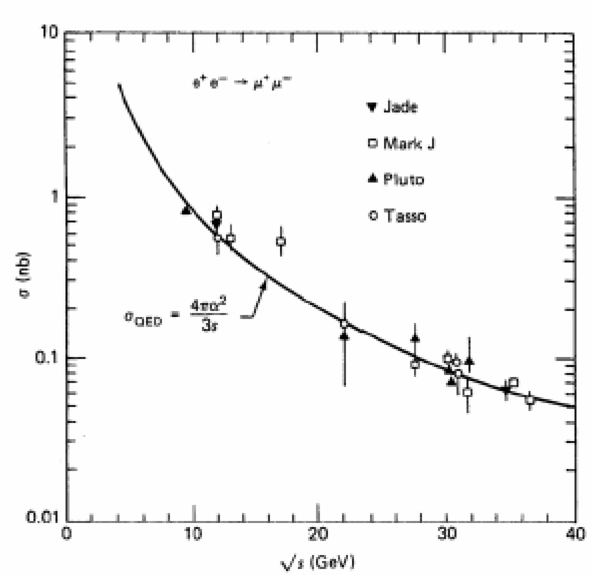
$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{CoM}} = 2\pi |M|^2 \frac{E^2}{(2\pi)^3} = \frac{\alpha^2}{s}$$

Correct treatment of spins $\quad \left\langle |M|^2 \right\rangle = 2e^4 \frac{t^2 + u^2}{s^2}$



$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{CoM}} = \frac{\alpha^2}{4s}\left(1 + \cos^2\theta\right)$$

$$\sigma\left(e^+e^- \rightarrow \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3s} = \frac{87\ nb}{s\left[\text{GeV}^2\right]}$$



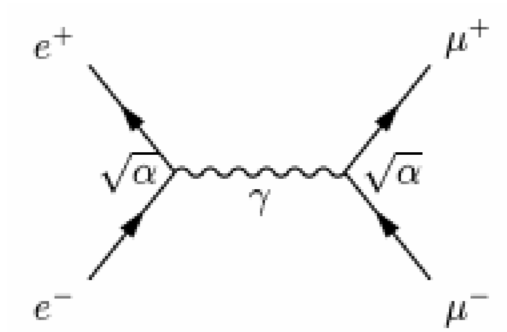
Higher Order Diagrams

QED

time dependent perturbation theory

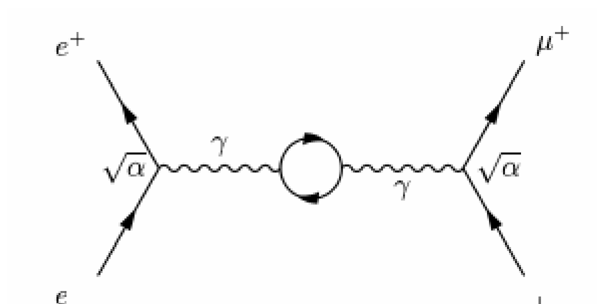
Lowest order

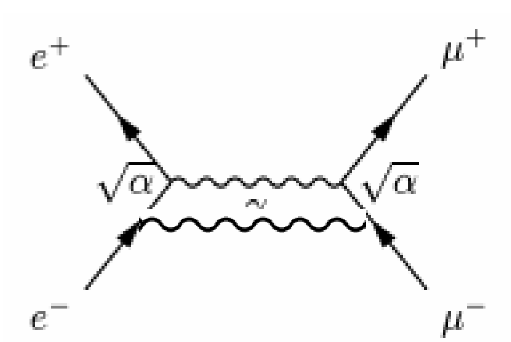
$$\sigma \propto |M|^2 \propto \alpha^2 \approx \frac{1}{137^2}$$

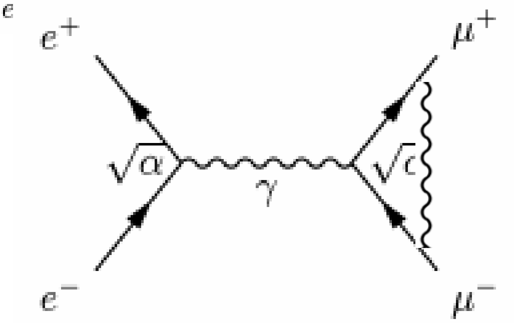


Second order

$$\sigma \propto |M|^2 \propto \alpha^4 \approx \frac{1}{137^4}$$







Higher orders

Order n suppressed by α = 1/137^{2n}

Lowest order dominates if coupling constant α is small

QED converges rapidly

QED Precision Tests

Magnetic moment

Couples to spin of electron

$$\vec{\mu} = g\mu_B \vec{S} \quad \text{where} \quad \mu_B = \frac{e\hbar}{2m_e c}$$

Dirac Equation predicts gyromagnetic ratio $g = 2$ for point-like particles

Anomalous magnetic moment

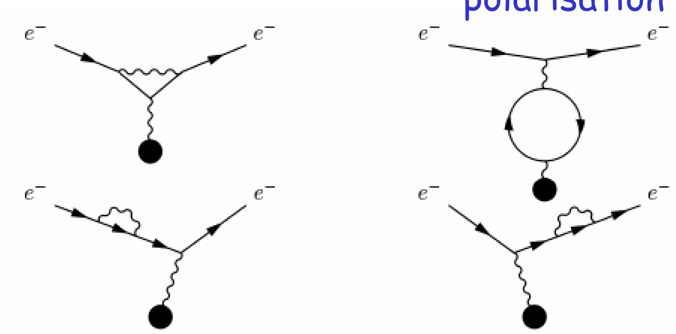
$a = (g-2)/2$

Higher order corrections

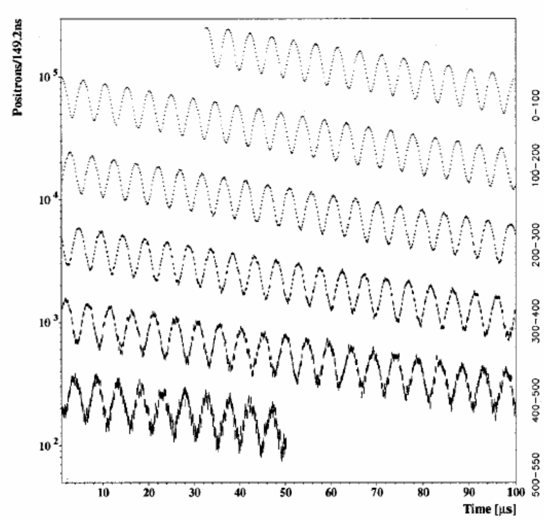
Bullet represents external electromagnetic field

vertex

vacuum polarisation



muon spin-precession in magnetic field



$$\text{Vertex } a_e = a_\mu = \frac{\alpha}{2\pi} = 1.1617 \cdot 10^{-3}$$

2 loops – 7 Feynman diagrams

3 loops – 72 Feynman diagram

4 loops – 891 Feynman diagra

QED is most precise theory in physics

Experiment $a_e = (11596521.869 \pm 0.041) \cdot 10^{-10}$

Theory $a_e = (11596521.3 \pm 0.3) \cdot 10^{-10}$ electrons

$$a_\mu(\text{exp}) = 11\,659\,208(6) \times 10^{-10} \quad (0.5\,\text{ppm}),$$

$$a_\mu(\text{SM}) = 11\,659\,181(8) \times 10^{-10} \quad (0.7\,\text{ppm})$$

muons

Running of α

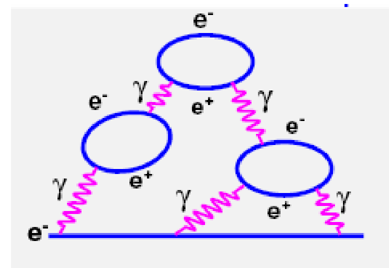
Coupling constant α

Strength of electromagnetic interaction

$\alpha = e^2/4\pi$ is not a constant at all distances

Vacuum

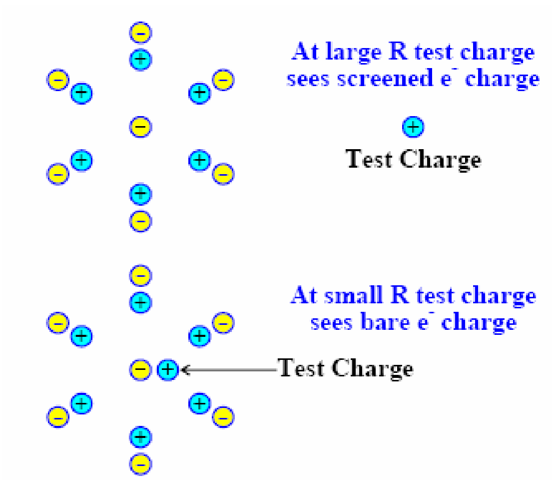
Not empty, around free electron creation and annihilation of virtual electron-positron pairs



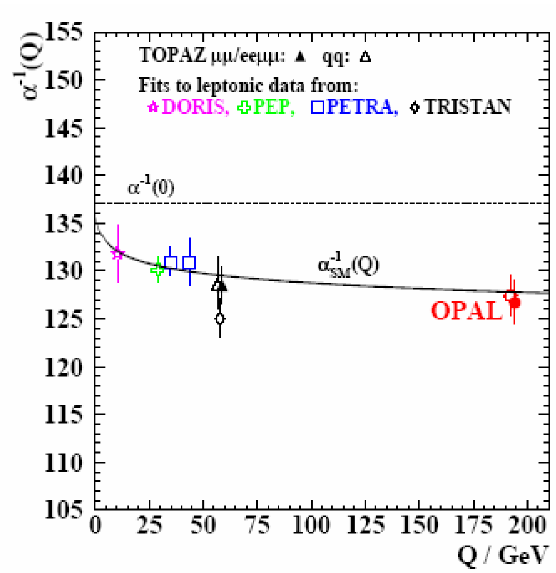
Screening

Bare charge and mass of electron only visible at very short distances

α increases with with larger momentum transfer



$e^+e^- \rightarrow \mu^+\mu^-$



Classical limit

$q^2 = 0$ ➔ $\alpha = 1/137$

At short distances

$q^2 = (90\ \text{GeV})^2$ ➔ $\alpha = 1/128$

Dirac Equation

Schroedinger equation

1st order in time derivative

2nd order in space derivatives

Klein-Gordon equation

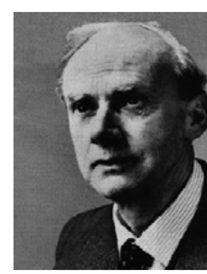
2nd order in space and time derivatives

$$\left(-\frac{\partial^2}{\partial t^2}+\vec{\nabla}^2\right)\psi = m^2\psi \quad \text{or} \quad \left(\frac{\partial^2}{\partial t^2}-\vec{\nabla}^2+m^2\right)\psi = 0$$

negative energies ($E < 0$)

and negative probability densities ($|\Psi|^2 < 0$)

Dirac Equation



1st order in time and space derivatives

$$\left(i\gamma^0\frac{\partial}{\partial t}+i\vec{\gamma}\cdot\vec{\nabla}-m\right)\Psi = 0 \quad \text{or} \quad \left(i\gamma^\mu\partial_\mu - m\right)\Psi = 0$$

γ^μ are 4x4 matrices

Solutions of Dirac equation

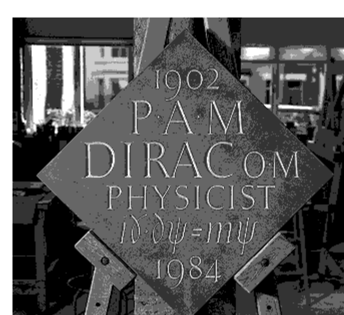
Wave function with 4-component spinor

$$\Psi = \begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\\ \Psi_4\end{pmatrix}$$

$$\Psi(\vec{x},t) = N\,u(p)\exp\left(-ip_\nu x^\nu\right) \quad \Rightarrow \quad E = \pm\sqrt{p^2+m^2}$$

2 positive energy solutions, $E > 0$

2 negative energy solutions, $E < 0$



Dirac Equation not examinable