Quark Model

Outline

Hadrons
- Isospin, Strangeness

Quark Model
- 3 Flavours u, d, s

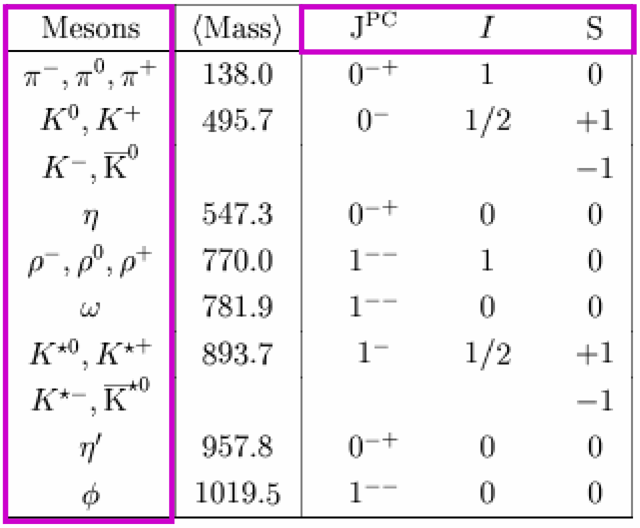
Mesons
- Pseudoscalar
- and vector mesons

Baryons
- Decuplet, octet

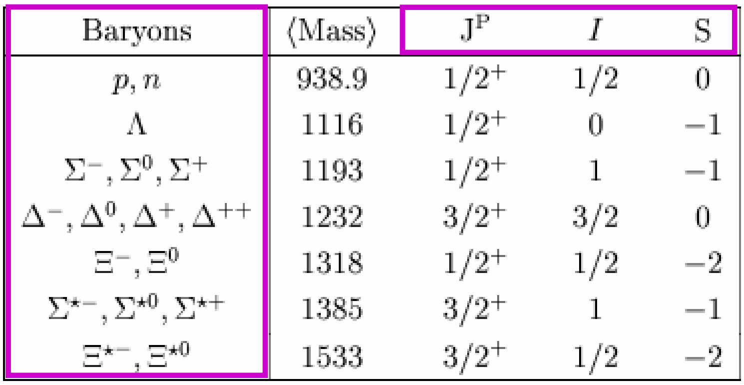
Hadron Masses
- Spin-spin coupling

Heavy Quarks
- Charm, bottom,
- Heavy quark
- Mesons
- Top quark

Hadrons known in 1960



Mesons	⟨Mass⟩	J^{PC}	I	S
π^-, π^0, π^+	138.0	0^{-+}	1	0
K^0, K^+	495.7	0^-	1/2	+1
$K^-, \overline{K}^0$				−1
η	547.3	0^{-+}	0	0
ρ^-, ρ^0, ρ^+	770.0	1^{--}	1	0
ω	781.9	1^{--}	0	0
$K^{\star 0}, K^{\star +}$	893.7	1^-	1/2	+1
$K^{\star -}, \overline{K}^{\star 0}$				−1
η'	957.8	0^{-+}	0	0
ϕ	1019.5	1^{--}	0	0



Baryons	⟨Mass⟩	J^P	I	S
p, n	938.9	$1/2^+$	1/2	0
Λ	1116	$1/2^+$	0	−1
$\Sigma^-, \Sigma^0, \Sigma^+$	1193	$1/2^+$	1	−1
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	1232	$3/2^+$	3/2	0
Ξ^-, Ξ^0	1318	$1/2^+$	1/2	−2
$\Sigma^{\star -}, \Sigma^{\star 0}, \Sigma^{\star +}$	1385	$3/2^+$	1	−1
$\Xi^{\star -}, \Xi^{\star 0}$	1533	$3/2^+$	1/2	−2

Motivation for Quark Model
- Particle "Zoo" proliferates
- " … the finder of a new particle used to be rewarded by a Nobel prize, but such a discovery ought to be punished by a $10000 fine" Lamb, 1955

Isospin

Nucleons

Proton and neutron have almost equal mass

Strong nuclear force is charge independent

$V_{pp} \approx V_{pn} \approx V_{nn}$

Isospin

p and n form part of single entity with

isospin ½ analogous to ↑ and ↓ of spin ½

Isospin I is conserved in strong interactions

Addition by rules of angular momentum

Isospin Multiplets

Useful for classification of hadrons, see slide 1

2I+1 states in a isospin muliplet $|I, I_3>$

$$\eta = |0,0\rangle$$

$$p = |\tfrac{1}{2}, \tfrac{1}{2}\rangle \qquad n = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$$\pi^+ = |1,1\rangle \qquad \pi^0 = |1,0\rangle \qquad \pi^- = |1,-1\rangle$$

$$\Delta^{++} = |\tfrac{3}{2}, \tfrac{3}{2}\rangle \qquad \Delta^{+} = |\tfrac{3}{2}, \tfrac{1}{2}\rangle \qquad \Delta^{0} = |\tfrac{3}{2}, -\tfrac{1}{2}\rangle \qquad \Delta^{-} = |\tfrac{3}{2}, -\tfrac{3}{2}\rangle$$

Quark Model

Gives natural explanation for Isospin

$I_3 = \tfrac{1}{2}\left(n_u - n_d + n_{\bar{d}} - n_{\bar{u}}\right)$ n_i number of i quarks

Isospin works well

Masses of u and d quark are almost equal

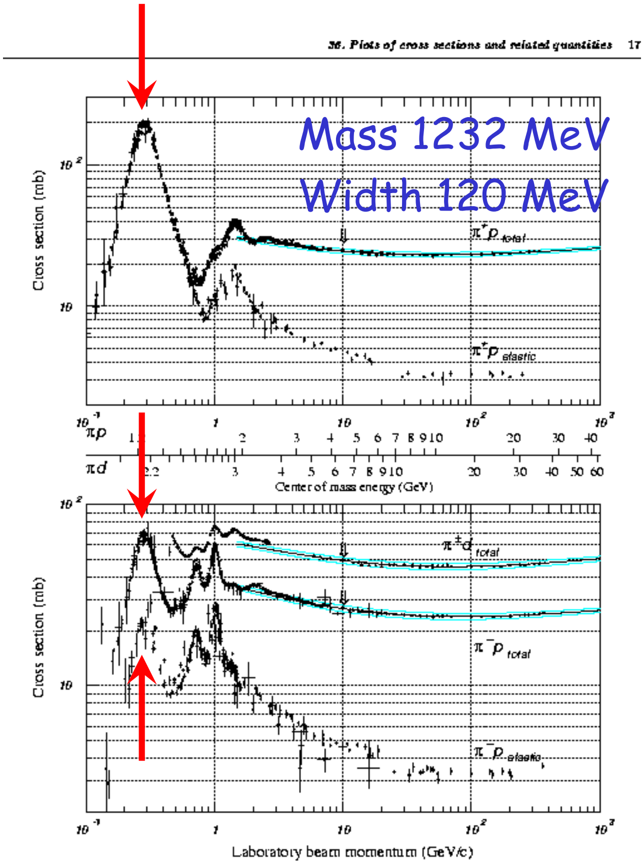
Isospin Conservation

Conservation Law

Isospin I is conserved in strong interactions

Allows to calculate ratios of cross sections and branching fractions in strong interactions

Delta(1232) Resonance



Mass 1232 MeV

Width 120 MeV

Figure 36.32: Total and elastic cross sections for $\pi^+ p$, $\pi^+ d$ (total only), and $\pi^- p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html (Courtesy of the COMPAS Group, IHEP, Protvino, Russia, 1996.)

Production

$$\pi^+ p \to \Delta^{++} \to \pi^+ p$$
$$\pi^- p \to \Delta^0 \to \pi^- p$$
$$\pi^- p \to \Delta^0 \to \pi^0 n$$

Isospin addition

$$\pi^+ p: \quad |1,1\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle$$
$$\pi^- p: \quad |1,-1\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$
$$\pi^0 n: \quad |1,0\rangle \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

Matrix element

depends on I, not I_3

$$M_3 = \left\langle \tfrac{3}{2} \middle| H_3 \middle| \tfrac{3}{2} \right\rangle$$
$$M_1 = \left\langle \tfrac{1}{2} \middle| H_1 \middle| \tfrac{1}{2} \right\rangle$$

$$M\left(\pi^+ p \to \Delta^{++} \to \pi^+ p\right) = M_3$$
$$M\left(\pi^- p \to \Delta^0 \to \pi^- p\right) = \tfrac{1}{3} M_3 + \tfrac{2}{3} M_1$$
$$M\left(\pi^- p \to \Delta^0 \to \pi^0 n\right) = \tfrac{\sqrt{2}}{3} M_3 - \tfrac{\sqrt{2}}{3} M_1$$

Cross sections

$$\sigma \propto |M|^2$$

In agreement with

I=3/2 Isospin prediction

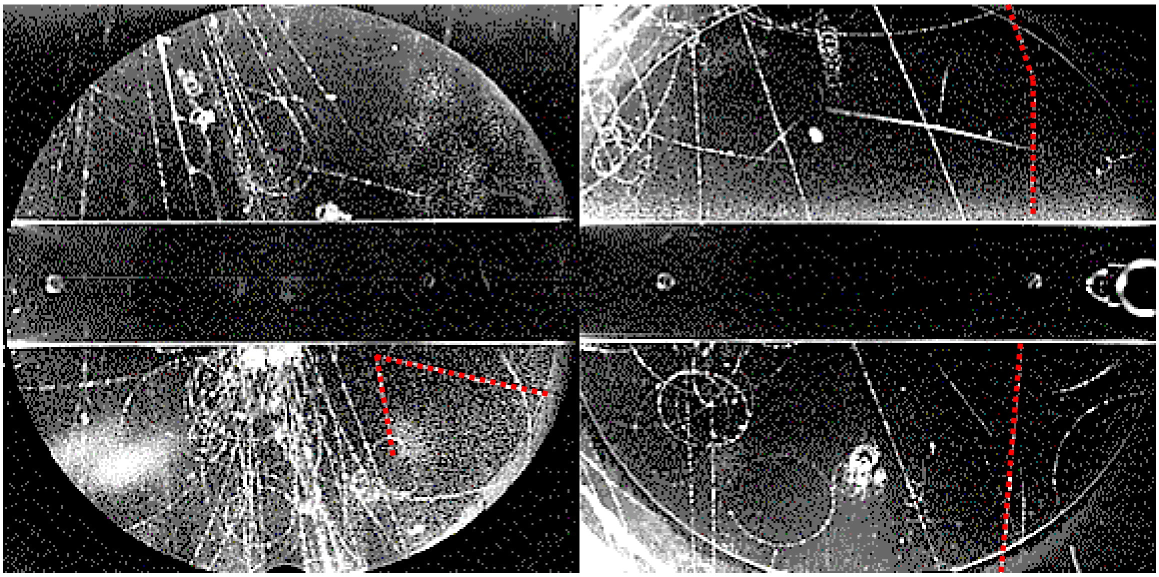
$$\sigma\left(\pi^+ p \to \Delta^{++} \to \pi^+ p\right) \approx 200\,\text{mb} \approx 9\text{x}$$
$$\sigma\left(\pi^- p \to \Delta^0 \to \text{all}\right) \approx 70\,\text{mb} \approx 3\text{x}$$
$$\sigma\left(\pi^- p \to \Delta^0 \to \pi^- p\right) \approx 23\,\text{mb} \approx 1\text{x}$$

Strangeness

Strange Particles

Discovered in 1947 Rochester and Butler

V, "fork", and K, "kink"



Production of V(K^0, Λ) and $K^{\pm}$
via strong interaction,
weak decay

$$\pi^- p \to K^0 \Lambda \quad \tau = O\left(10^{-23} s\right)$$

$$K^0 \to \pi^+ \pi^- \quad \tau_{K^0} = 0.89 \times 10^{-10} s$$

$$\Lambda \to \pi^- p \quad \tau_\Lambda = 2.63 \times 10^{-10} s$$

Associated Production

Strange particles produced in pairs Pais

Strangeness S

Additive quantum number Gell-Mann Nishijima

Conserved in strong and electromagnetic interactions

Violated in weak decays

Non-zero for Kaons $S = 0: \quad \pi, p, n, \Delta, \ldots \qquad S = 1: \quad K^+, K^0$

and hyperons $S = -1: \quad K^-, \overline{K}^0, \Lambda, \Sigma, \ldots \qquad S = -2: \quad \Xi$

Naturally explained in quark model $S = n_{\bar{s}} - n_s$

Quark Model

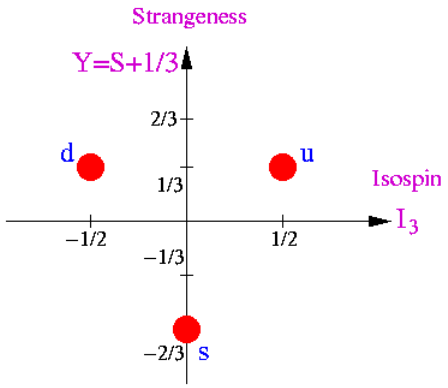
3 Quark Flavours u, d, s

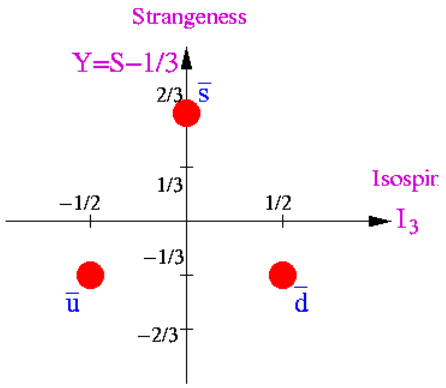
1964 - introduced by Gell-Mann & Zweig

Gell-Mann

Zweig

Quark	Charge Q [e]	Isospin $\lvert I, I_3 \rangle$	Strange-ness S
up (u)	+2/3	$\lvert \frac{1}{2}, +\frac{1}{2} \rangle$	0
down (d)	-1/3	$\lvert \frac{1}{2}, -\frac{1}{2} \rangle$	0
strange (s)	-1/3	$\lvert 0,0 \rangle$	-1





Charge, Isospin and Strangeness

Additive quark quantum numbers are related

$$\boxed{Q = I_3 + \tfrac{1}{2}(S + B)}$$

not all independent

Gell-Mann Nishijima predates quark model
valid also for hadrons

Baryon number B quarks $B = +1/3$
anti-quarks $B = -1/3$

Hypercharge $Y = S + B$ is useful quantum number

Quark model gives natural explanation for Isospin and Strangeness

Mesons

Bound $q\bar{q}$ States

Zero net colour charge

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left|r\bar{r} + g\bar{g} + b\bar{b}\right\rangle$$

Zero net baryon number

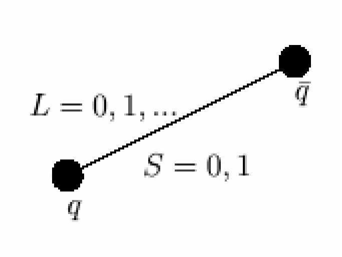
B = +1/3 +(-1/3) = 0

Angular Momentum $\vec{L}$

For lightest mesons

Ground state

L = 0 between quarks



Parity P

Intrinsic quantum number of quarks and leptons

P=+1 for fermions P=-1 for anti-fermions

$$P(q\bar{q}) = P_q P_{\bar{q}} (-1)^L$$

$$= (+1)(-1)(-1)^L = -1 \quad \text{for} \quad L = 0$$

Total Angular Momentum $\vec{J}$

$\vec{J} = \vec{L} + \vec{S}$ include quark spins

S = 0 $q\bar{q}$ spins anti-aligned ↑↓ or ↓↑

➔ $J^P = 0^-$ **Pseudo-scalar mesons**

S = 1 $q\bar{q}$ spins aligned ↑↑ or ↓↓

➔ $J^P = 1^-$ **Vector mesons**

Quark flavours

$u\bar{d}, u\bar{s}, d\bar{u}, d\bar{s}, s\bar{u}, s\bar{d}$ non-zero flavour states

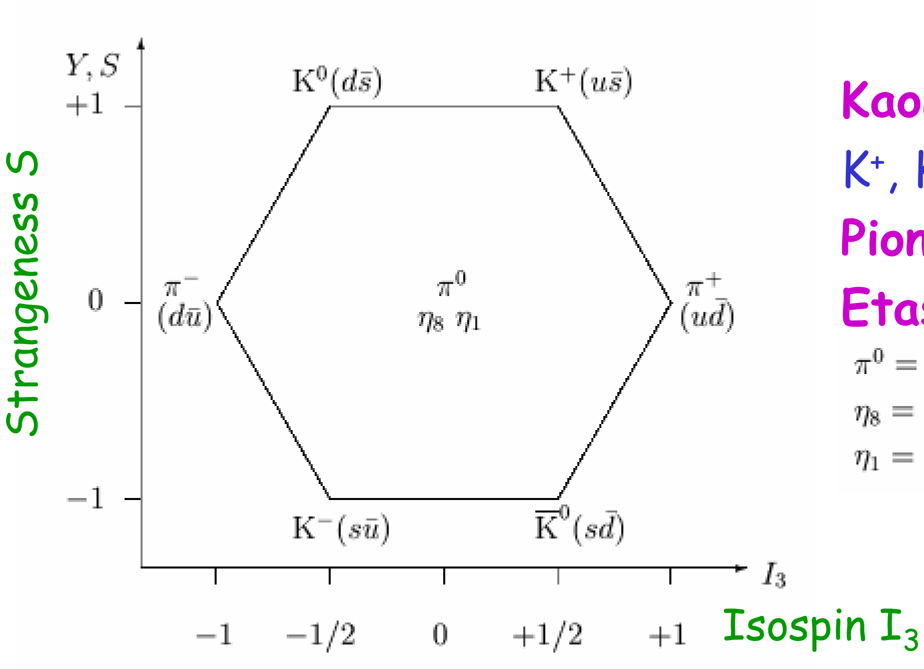
$u\bar{u}, d\bar{d}, s\bar{s}$ zero net flavour states

have identical additive quantum numbers

Physical states are mixtures

Mesons

Pseudoscalar Mesons $J^P = 0^-$



Kaons:

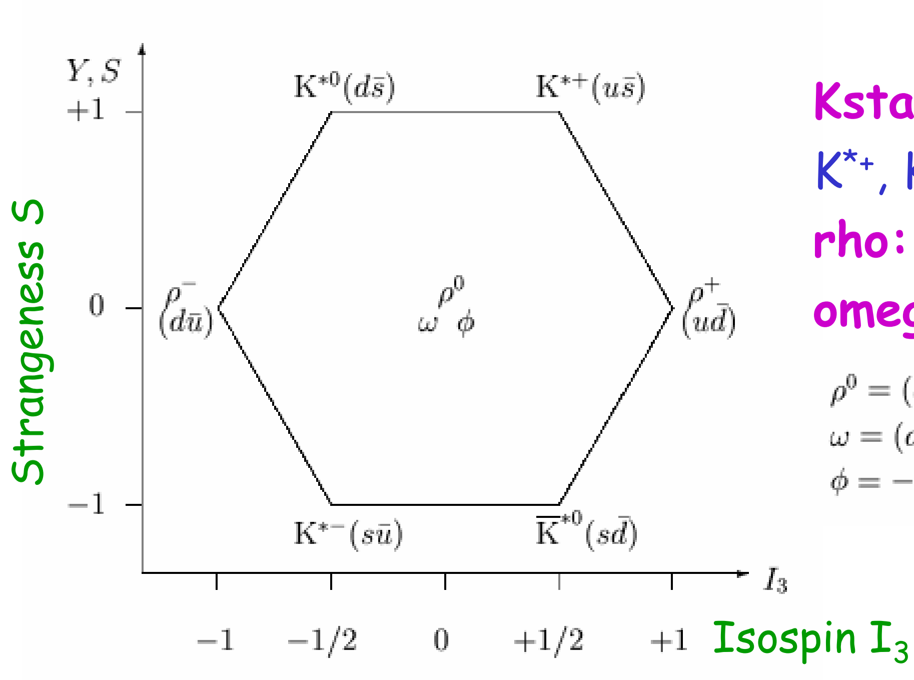
K^+, K^0, anti-K^0, K^-

Pions: π^+, π^0, π^-

Etas: η, η'

$$\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$$
$$\eta_8 = (d\bar{d} + u\bar{u} - 2s\bar{s})/\sqrt{6}$$
$$\eta_1 = (d\bar{d} + u\bar{u} + s\bar{s})/\sqrt{3}$$

Vector Mesons $J^P = 1^-$



Kstar:

K^{*+}, K^{*0}, anti-K^{*0}, K^{*-}

rho: ρ^+, ρ^0, ρ^-

omega/phi: ω, ϕ

$$\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$$
$$\omega = (d\bar{d} + u\bar{u})/\sqrt{2}$$
$$\phi = -s\bar{s}$$

Baryon Decuplet

Baryon Wavefunction

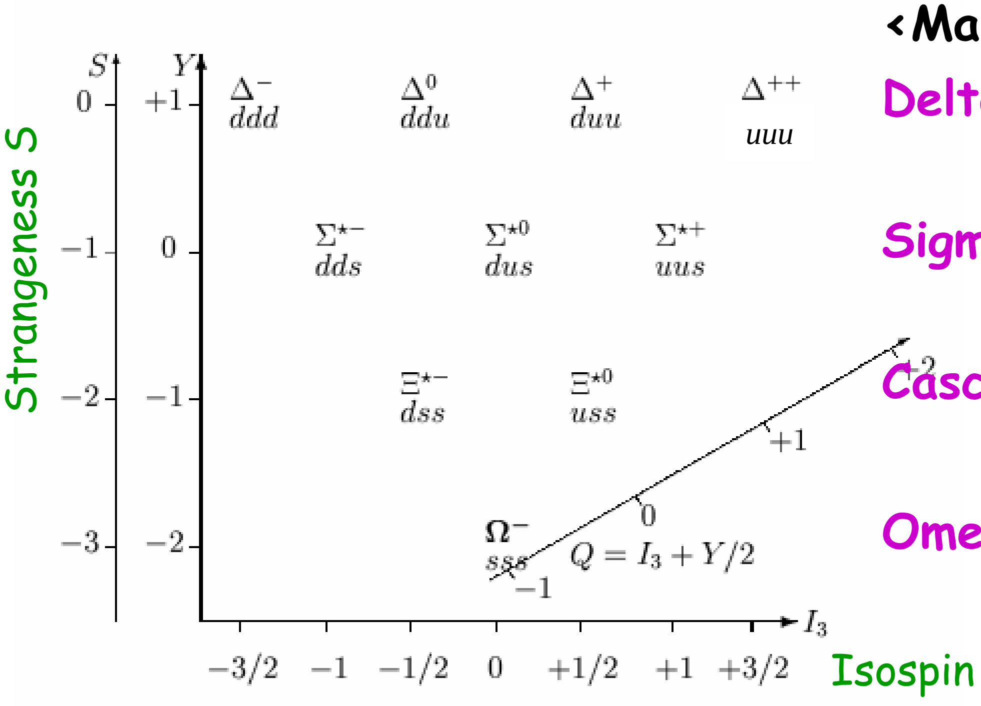
Ψ(total) = Ψ(space) Ψ(spin) Ψ(flavour) Ψ(colour)

Space	symmetric - L = 0
Flavour	symmetric, e.g. uuu, (udu+duu+uud)/√3
Spin	symmetric all 3 quarks aligned → S = 3/2
Colour	antisymmetric

$$(rgb - rbg + gbr - grb + brg - bgr)/\sqrt{6}\,.$$

Total antisymmetric - obeys Pauli Exclusion Principle

Baryon Decuplet $J^P = 3/2^+$



	<Mass>
Delta	1232 MeV
Sigma*	1385 MeV
Cascade*	1533 MeV
Omega$^-$	1672 MeV

Quark model predicted unobserved state Ω^- (sss)

Baryon Octet

Baryon Wavefunction

Ψ(space) symmetric (L = 0) Ψ(colour) antisymmetric

Mixed symmetric Ψ(spin, flavour)

Flavour mixed symmetric: e.g. (ud - du) u/√2

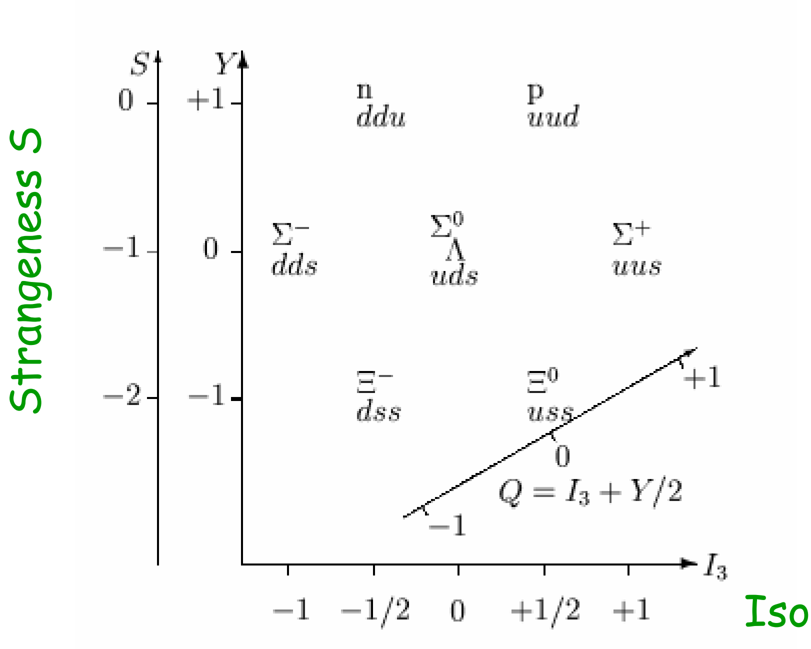
Spin as flavour: e.g. (↑↓ - ↑↓) ↑/√2

Spin-flavour e.g. (u↑d↓ - d↑u↓ - u↓d↑ + d↓u↑) u↑/√6

Symmetrisation by cyclic permutations

Ψ(proton, $s=+\frac{1}{2}$) = (2u↑u↑d↓ - u↑u↓d↑- u↓u↑d↑
+2d↓u↑u↑ - d↑u↑u↓- d↑u↓u↑
+2u↑d↑u↓ - u↑d↓u↑- u↓d↑u↑) /√18

Baryon Octet $J^P = \frac{1}{2}^+$



<Mass>

p,n	938.9 MeV
Sigma	1193 MeV
Lambda	1116 MeV
Cascade (Xi)	1318 MeV

Lightest baryons stable or long-lived

Antibaryons $(\bar{p}, \bar{n}, ...)$ also form Octet and Decuplet

Discovery of Ω^-

Ω^- (sss) Hyperon

Hyperon - baryon with at least one s quark

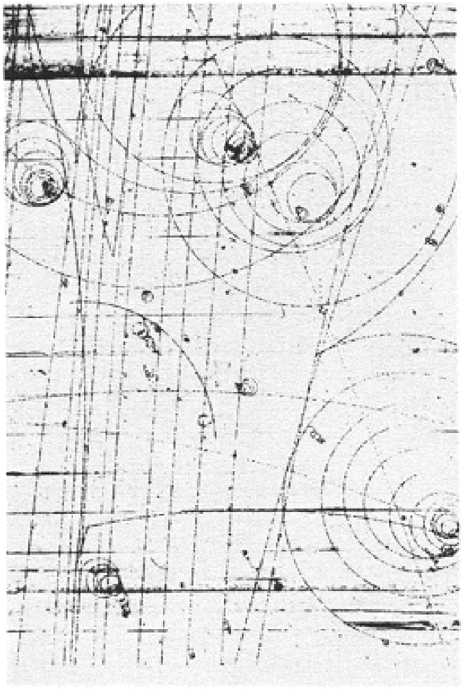
Quark model predicted existence and mass

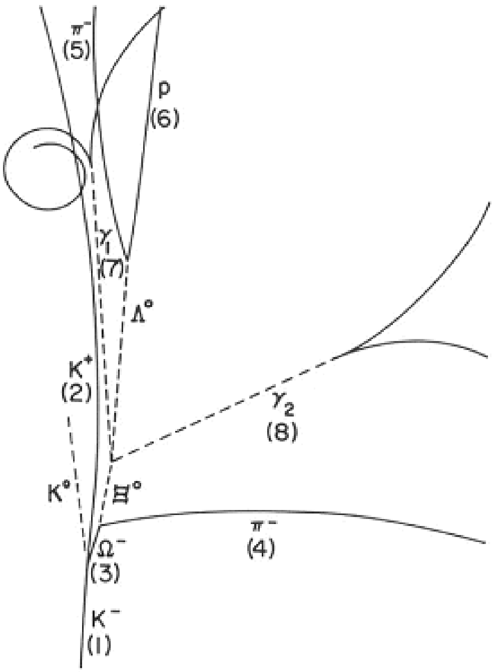
Missing member of baryon decuplet $J^P = 3/2^+$

discovered 1964 at Brookhaven

K^- beam onto hydrogen target

Bubble Chamber detector





$$K^- + p \to .\Omega^- + K^- + K^0$$

$$\mapsto \Xi^0 + \pi^+$$

$$\mapsto \Lambda^0 + \pi^0$$

$$\mapsto \gamma + \gamma$$

$$\mapsto e^+ e^-$$

$$\mapsto e^+ e^-$$

$$\mapsto \pi^- p$$

Hadron Masses

Quark Masses

u, d & s quark masses light at short distance

$q^2 > 1$ GeV^2 $m_u < m_d \sim 5$ MeV $m_s \sim 100$ MeV

Constituent mass is relevant for quark model

$q^2 < 1$ GeV^2 $m_u = m_d \sim 300$ MeV $m_s \sim 500$ MeV

Meson Masses

$m(K) > m(\pi)$ due to $m_s > m_u, m_d$

$m(\rho) > m(\pi)$ same quark content e.g. ρ^+, π^+: (u-dbar)

Mass difference is due to quark spins

Chromomagnetic Mass Splitting

Spin-spin coupling of quarks $S_1 = S_2 = 1/2$

analogous to hyperfine splitting in el. mag. interaction

$$\Delta E \propto \alpha_S \frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2} \qquad \boxed{m(q\bar{q}) = m_1 + m_2 + A\frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2}}$$

$$\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}\left(\vec{S}^2 - \vec{S}_1^{\,2} - \vec{S}_2^{\,2}\right) = \frac{1}{2}\left(S(S+1) - S_1(S_1+1) - S_2(S_2+1)\right)$$

$$= \begin{cases} 1 - \frac{3}{4} = \frac{1}{4} & S = 1 \\ 0 - \frac{3}{4} = -\frac{3}{4} & S = 0 \end{cases}$$

Meson Masses

$m_u = m_d = 310$ MeV

$m_s = 483$ MeV

$A = (2m_u)^2 \cdot 160$ MeV

Excellent agreement

What about eta(')?

	Mass [MeV]	
Meson	Prediction	Experiment
π	140	138
K	484	496
ρ	780	770
ω	780	782
K*	896	894
ϕ	1032	1019

Heavy Quarks

Charm and bottom quarks

Charmonium (c-cbar) --- see QCD lecture

1977 Discovery of Upsilon States

Interpretation is

Bottomonium (b-bar)

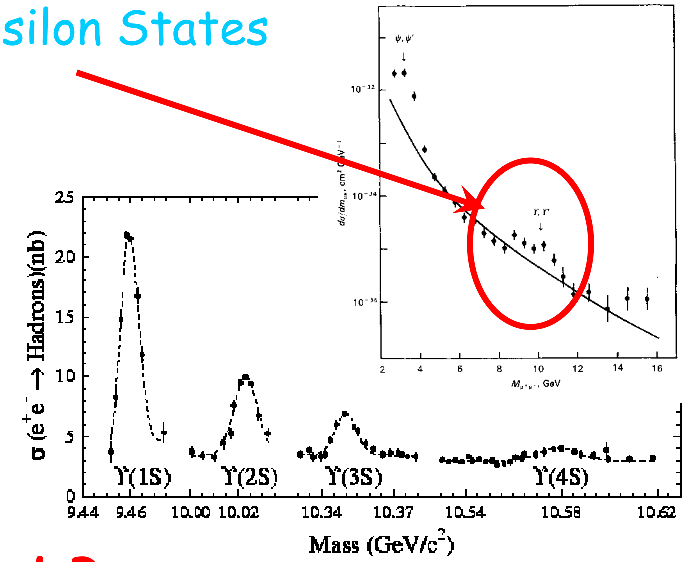
Spectroscopy

Charmonium

and Upsilon →

$m_c \sim 1.1 \ldots 1.4$ GeV

$m_b \sim 4.1 \ldots 4.5$ GeV



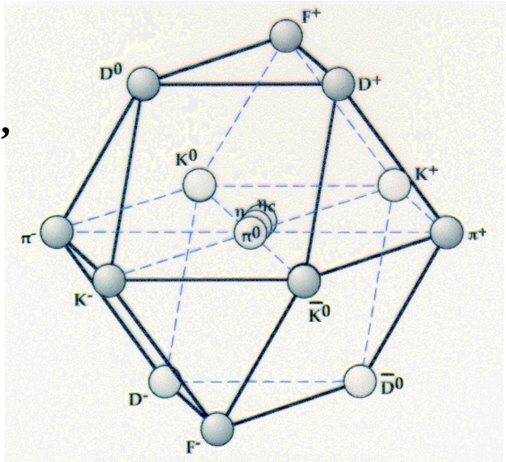
Heavy-light Mesons and Baryons

Charmed (c-quark) hadrons

$$J^P = 0^- \quad D^0 = c\bar{u}, \quad D^+ = c\bar{d}, \quad D_s^+ = c\bar{s},$$

$$J^P = 1^- \quad D^{*0} = c\bar{u}, \quad D^{*+} = c\bar{d}, \quad D_s^{*+} = c\bar{s},$$

$$J^P = \frac{1}{2}^- \quad \Lambda_c^+ = cud$$



SU(4)-15-plett der einfachsten Mesonen

Bottom-quark hadrons

$$J^P = 0^- \quad B^+ = u\bar{b}, \quad B^0 = d\bar{b}, \quad B_s^0 = s\bar{b},$$

$$J^P = 1^- \quad B^{*+} = u\bar{b}, \quad B^{*0} = d\bar{b}, \quad B_s^{*0} = s\bar{b},$$

$$J^P = \frac{1}{2}^- \quad \Lambda_b^0 = bud$$

Top quark

Decays before forming bound states

$m_t \sim 174$ GeV discovered in 1995 at Fermilab