Particle Physics - Measurements and Theory

Outline

Natural Units

Relativistic Kinematics

Particle Physics Measurements

- Lifetimes
- Resonances and Widths
- Scattering
- Cross section
- Collider and Fixed Target Experiments

Conservation Laws

- Charge, Lepton and Baryon number,
- Parity, Quark flavours

Theoretical Concepts

- Quantum Field Theory
- Klein-Gordon Equation
- Anti-particles
- Yukawa Potential
- Scattering Amplitude - Fermi's Golden Rule
- Matrix elements

Particle Physics Units

Particle Physics

is relativistic and quantum mechanical

➔ $c = 299\,792\,458$ m/s

➔ $\hbar = h/2\pi = 1.055 \cdot 10^{-34}$ Js

Length

size of proton: 1 fm = 10^{-15} m

Lifetimes

as short as 10^{-23} s

Charge

1 e = $-1.60 \cdot 10^{-19}$ C

Energy

Units: 1 GeV = 10^9 eV -- 1 eV = $1.60 \cdot 10^{-19}$ J

use also MeV, keV

Mass

in GeV/c^2, rest mass is $E = mc^2$

Natural Units

Set $\hbar = c = 1$

➔ Mass [GeV/c^2], energy [GeV] and momentum [GeV/c] in GeV

➔ Time [$(\text{GeV}/\hbar)^{-1}$], Length [$(\text{GeV}/\hbar c)^{-1}$] in 1/GeV, area [$(\text{GeV}/\hbar c)^{-2}$]

Useful relations

$\hbar c = 197$ MeV fm

$\hbar = 6.582 \cdot 10^{-22}$ MeV s

Particle Physics Measurements

How do we measure particle properties and interaction strengths?

Static properties

Mass — How do you weigh an electron?

Magnetic moment — couples to magnetic field

Spin, Parity

Particle decays

Lifetimes

Resonances & Widths

Allowed/forbidden

Decays

Conservation laws

Force	Lifetimes
Strong	10^{-23} -- 10^{-20} s
El.mag.	10^{-20} -- 10^{-16} s
Weak	10^{-13} -- 10^{3} s

Scattering

Elastic scattering $e^- p \rightarrow e^- p$

Inelastic annihilation $e^+ e^- \rightarrow \mu^+ \mu^-$

Cross section

total σ

Differential $d\sigma/d\Omega$

Luminosity L

Particle flux

Event rate N

Force	Cross sections
Strong	O(10 mb)
El.mag.	O(10^{-1} mb)
Weak	O(10^{-1} pb)

Relativistic Kinematics

Basics

4-momentum

Invariant mass

Four-vector notation

$$p^{\mu} = \left(\frac{E}{c}, p_x, p_y, p_z\right)$$

$$p^2 = p^{\mu} \cdot p_{\mu} = (E/c)^2 - \vec{p}^{\,2} = m^2c^2$$

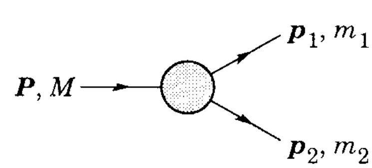
Useful Lorentz boosts relations

set $\hbar = c = 1$	invariant mass
$\gamma = E/mc^2 = E/m$	$m^2 = E^2 - p^2$
$\gamma\beta = pc/mc^2 = p/m$	$\gamma = 1/\sqrt{(1-\beta^2)}$
$\beta = pc/E = p/E$	$\beta = \sqrt{(1 - 1/\gamma^2)}$

2-body decays

$P_0 \rightarrow P_1\, P_2$

work in P_0 rest frame



$$p^{\mu} = (m_0, \vec{0}) \quad p_1^{\mu} = (E_1, \vec{p}_1) \quad p_2^{\mu} = (E_2, \vec{p}_2)$$

$$p_2^2 = (p^{\mu} - p_1^{\mu})^2 = p^2 + p_1^2 - 2p \cdot p_1$$

$$m_2^2 = m_0^2 + m_1^2 - 2m_0E_1$$

$$E_1 = \frac{m_0^2 + m_1^2 - m_2^2}{2m_0} \qquad |\vec{p}_1| = |\vec{p}_2|$$

Example: $\pi^+ \rightarrow \mu^+ \nu_\mu$

work in π^+ rest frame

use $m_\nu^2 = 0$

$$p^{\mu} = (m_\pi, \vec{0}) \quad p_1^{\mu} = (E_\mu, \vec{p}_\mu) \quad p_2^{\mu} = (E_\nu, \vec{p}_\nu)$$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = 109.8\ \text{MeV}$$

$$|\vec{p}_\mu| = \sqrt{E_\mu^2 - m_\mu^2} = 29.8\ \text{MeV/c}$$

Lifetimes

Decay time distribution

Mean lifetime

$\tau = \langle d\Gamma/dt \rangle$

aka proper time, eigen-time

of a particle

$$\frac{d\Gamma}{dt} = \Gamma \exp\left(-\frac{t}{\tau}\right) \qquad \Gamma = \frac{1}{\tau}$$

Lifetime measurements

In laboratory frame

Decay Length

$L = \gamma\beta c\tau$

Example: $B_d \rightarrow \pi^+\pi^-$

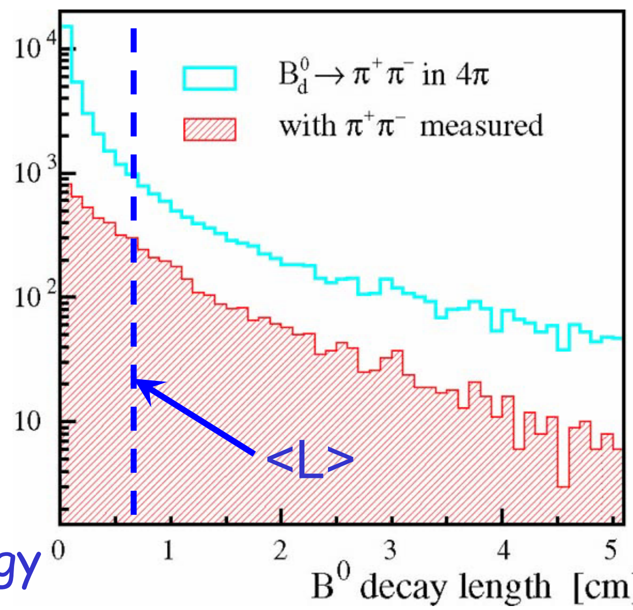
in LHCb experiment

$\langle L \rangle \approx 7$ mm

Average B meson energy

$\langle E_B \rangle \approx 80$ GeV

➔ $\tau = 1.54$ ps



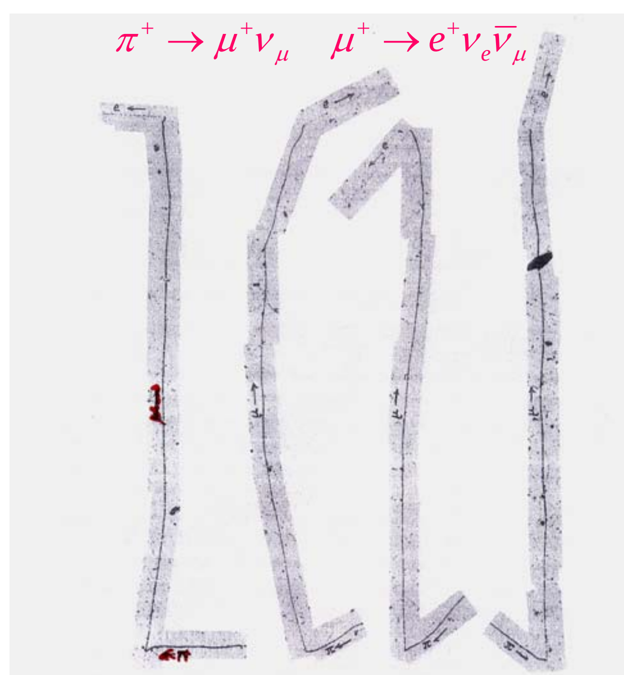
Example: π^+ discovery

Decay sequence

$\pi^+ \rightarrow \mu^+\nu_\mu \quad \mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$

Emulsions exposed to

Cosmic rays



Resonances and Widths

Strong Interactions

Production and decay of particles

Lifetime $\tau \sim 10^{-23}$ s $\quad c\tau \sim O(10^{-15}$ m)

unmeasurable

Heisenberg's Uncertainty Principle

$$\Delta E \Delta t \approx \hbar$$

Time and energy measurements are related

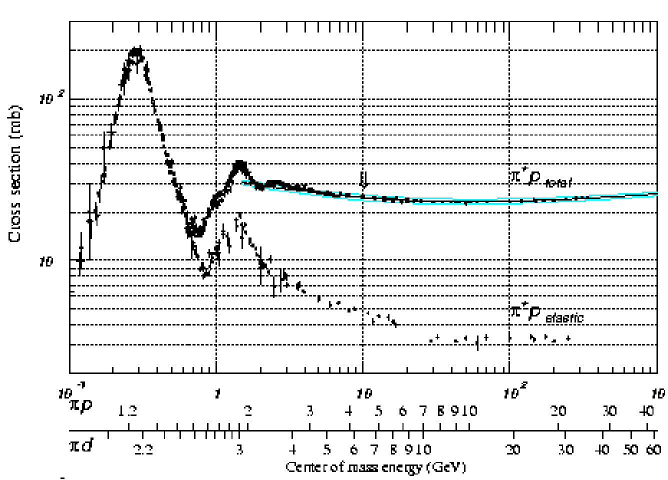
Natural width

Energy width Γ and lifetime τ of a particle

$\Gamma = \hbar/\tau \quad \rightarrow \quad$ Width $\Gamma = O(100$ MeV)

measurable

Example - Delta(1232) Resonance



Production

$$\pi^+ p \rightarrow \Delta^{++} \rightarrow \pi^+ p$$

Peak at Energy

E = 1.23 GeV

(Centre-of-Mass)

Width Γ = 120 MeV

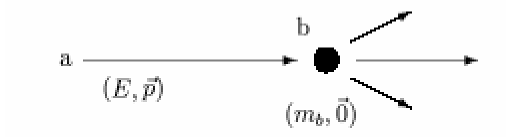
Lifetime

$\tau = \hbar/\Gamma \approx 5 \cdot 10^{-24}$ s

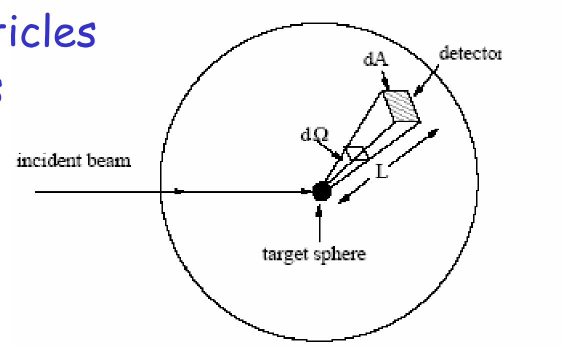
Scattering

Fixed Target Experiments

$a + b \rightarrow c + d + \ldots$



n_a # of beam particles
v_a velocity of beam particles
n_b # of target particles per unit area
Incident flux $F = n_a v_a$



Cross Section

effective area of any scattering happening
normalised per unit of incident flux
depends on underlying physics

What you want to study

dN # of scattered particles in solid angle $d\Omega$
$d\sigma/d\Omega$ differential cross section in solid angle $d\Omega$
σ total cross section

$$dN = n_a v_a n_b d\sigma = F n_b d\sigma = L d\sigma \quad \Rightarrow$$

L Luminosity
N Event rate

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega \quad \Rightarrow$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{L}\frac{dN}{d\Omega}$$

$$N = L\sigma$$

Event Rate N

Luminosity

Incident flux times number of targets

Depends on your experimental setup

$$1\,\text{barn} = 1\,\text{b} = 10^{-24}\,\text{cm}^2 \qquad \text{Luminosity } [L] = 10^{30 \ldots 34}\,\text{cm}^{-2}\text{s}^{-1}$$

Event Rate = Luminosity times Cross Section

Scattering

Centre-of-Mass Energy

a + b → c + d + … Collision of two particles

s is invariant quantity Mandelstam variable

$$s = \left(p_1^\mu + p_2^\mu\right)^2 = \left(E_1 + E_2\right)^2 - \left(\vec{p}_1 + \vec{p}_2\right)^2$$
$$= p_1^2 + p_2^2 + 2\,p_1 \cdot p_2$$
$$= m_1^2 + m_2^2 + 2\left(E_1 E_2 - |\vec{p}_1||\vec{p}_2|\cos\theta\right)$$

$$E_{\mathrm{CoM}} = \sqrt{s}$$ centre-of-mass energy

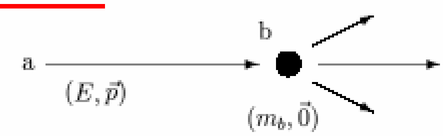
Total available energy in centre-of-mass frame

E_{CoM} is invariant in any frame, e.g. laboratory

Energy Threshold

for particle production $E_{\mathrm{CoM}} = \sqrt{s} \geq \sum_{j=c,d,\ldots} m_j$

Fixed Target Experiments



$$p_1^\mu = (E_{\mathrm{lab}}, \vec{p}_1) \quad p_2^\mu = (m_2, \vec{0})$$

$$E_{\mathrm{CoM}} = \sqrt{s} = \sqrt{m_1^2 + m_2^2 + 2E_{\mathrm{lab}} m_2} \quad \Rightarrow \quad E_{\mathrm{CoM}} \cong \sqrt{2E_{\mathrm{lab}} m_2} \text{ if } E_{\mathrm{lab}} >> m_i$$

Example:

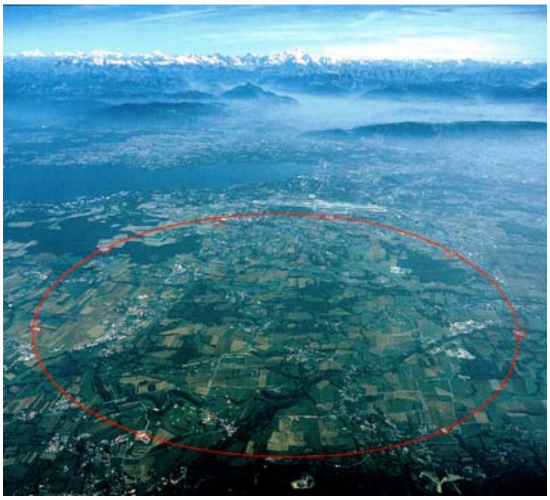
100 GeV proton onto proton at rest

$E_{CoM} = \sqrt{s} = \sqrt{(2E_p m_p)} = 14$ GeV

Most of beam energy goes into CoM momentum and is not available for interactions

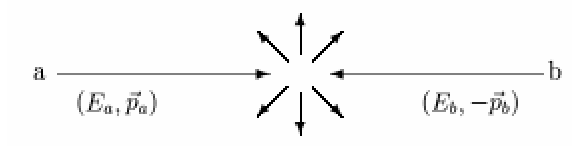
Scattering

Collider Experiments



Head-on collisions

of two particles



$\theta = 180^0$

$$s = m_1^2 + m_2^2 + 2\left(E_1 E_2 - |\vec{p}_1||\vec{p}_2|\cos\theta\right)$$

$$E_{\mathrm{CoM}} = \sqrt{m_1^2 + m_2^2 + 2\left(E_1 E_2 + |\vec{p}_1||\vec{p}_2|\right)} \quad \Rightarrow \quad E_{\mathrm{CoM}} \cong \sqrt{4E_1E_2} \;\text{ if } E_i >> m_i$$

All of beam energy available for particle production

Example

LEP - Large Electron Positron Collider at CERN

100 GeV e- onto 100 GeV e+

Centre-of-mass energy

$E_{CoM} = \sqrt{s} = 2E = 200$ GeV

Cross section	$\sigma(e^+ e^- \rightarrow \mu^+ \mu^-) = 2.2$ pb
Luminosity	$\int L dt = 400\ \mathrm{pb}^{-1}$

Number of recorded events $N = \sigma \int L dt = 870$

Conservation Laws

Noether's Theorem

Every symmetry has associated with it
a conservation law and vice-versa

Energy and Momentum, Angular Momentum

conserved in all interactions
Symmetries – translations in space and time, rotations in space

Charge conservation

Well established
$|q_p + q_e| < 1.60 \cdot 10^{-21}\, e$
Valid for all processes
Symmetry – gauge transformation

Lepton and Baryon number (L and B)

|L+B| conservation = matter conservation
Proton decay not observed (B violation)
Lepton family numbers L_e, L_μ, L_τ conserved
Symmetry – mystery

Quark Flavours, Isospin, Parity

conserved in strong and electromagn processes
Violated in weak interactions
Symmetry – unknown

Theoretical Concepts

Standard Model of Particle Physics

Quantum Field Theory (QFT)

Describes fundamental interactions of Elementary particles

Combines quantum mechanics and special relativity

Very small
$\Delta x \, \Delta p \approx \hbar c$

	Classical Physics	Quantum mechanics
Very fast $v \rightarrow c$	Special relativity	Quantum field theory

Natural explanation for antiparticles and for Pauli exclusion principle

Full QFT is beyond scope of this course

Introduction to Major QFT concepts

- Transition Rate
- Matrix elements
- Feynman Diagrams
- Force mediated by exchange of bosons

Klein-Gordon Equation

Schroedinger Equation

For free particle

non-relativistic

1st order in time derivative

2nd order in space derivatives

not Lorentz-invariant

$$\frac{\hat{p}^2}{2m}\psi = \hat{E}\,\psi$$

$$\frac{-\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial}{\partial t}\psi$$

Klein-Gordon (K-G) Equation

Start with relativistic equation

$E^2 = p^2 + m^2$ ($\hbar = c = 1$)

$$E \to i\hbar\frac{\partial}{\partial t} \qquad \vec{p} \to -i\hbar\vec{\nabla}$$

Apply quantum mechanical operators

$$\left(-\frac{\partial^2}{\partial t^2} + \vec{\nabla}^2\right)\psi = m^2\psi \quad \text{or} \quad \left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 + m^2\right)\psi = 0$$

2nd order in space and time derivatives

Lorentz invariant

Plane wave solutions of K-G equation

$$\psi(x^\mu) = N\exp(-ip_\nu x^\nu) \quad \Rightarrow \quad E = \pm\sqrt{p^2 + m^2}$$

➔ negative energies ($E < 0$)

also negative probability densities ($|\psi|^2 < 0$)

Negative Energy solutions

➔ Dirac Equation, but -ve energies remain

➔ Antimatter

Klein-Gordon Equation

Interpretation

K-G Equation is for spinless particles

Solutions are wave-functions for bosons

Time-Independent Solution

Consider static case, i.e. no time derivative

$$\nabla^2\psi = m\psi$$

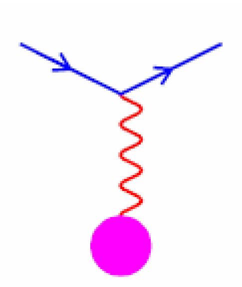
Solution is spherically symmetric

$$\psi(r) = -\frac{g^2}{4\pi r}\exp(-mr)$$

Interpretation - Potential

analogous to Coulomb potential

Force is mediated by exchange of massive bosons



Yukawa Potential

Introduced to explain nuclear force

$$V(r) = -\frac{g^2}{4\pi r}\exp\left(-\frac{r}{R}\right) \qquad R = \frac{\hbar}{mc}$$

g strength of force – "strong nuclear charge"

m mass of boson

R Range of force see also nuclear physics

For m = 0 and g = e → Coulomb Potential

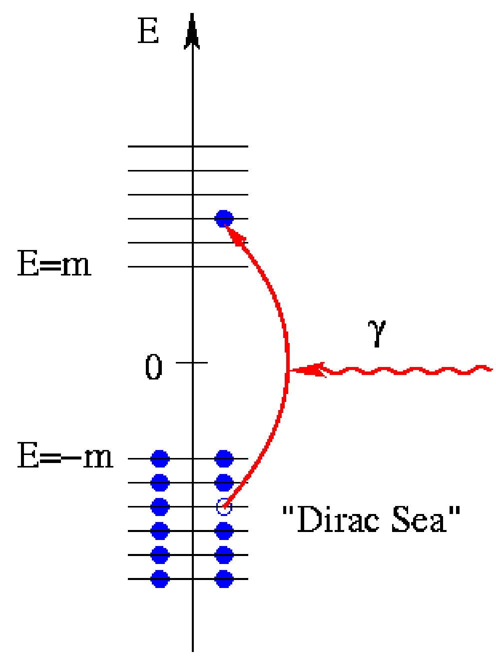
Antiparticles

Klein-Gordon & Dirac Equations

predict negative energy solutions

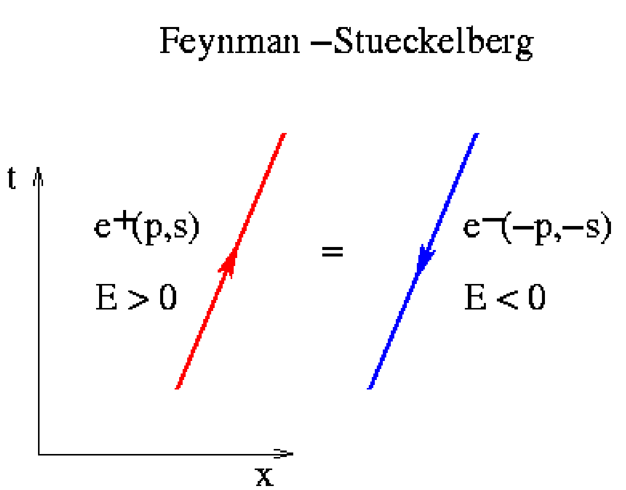
Interpretation - Dirac

Vacuum filled with $E < 0$ electrons
2 electrons with opposite spins
per energy state - "Dirac Sea"
Hole of $E < 0$, -ve charge
in Dirac sea -> antiparticle
$E > 0$, +ve charge
-> positron, e^+ discovery (1931)
Predicts e^+e^- pair production
and annihilation



Modern Interpretation – Feynman-Stueckelberg

$E < 0$ solutions: Negative energy particle
moving backwards in space and time correspond to



Antiparticles

Positive energy,
opposite charge
moving forward
in space and time

$$\exp[-i((-E)(-t)-(-\vec{p})\cdot(-\vec{x}))]$$
$$=\exp[-i((Et-\vec{p}\cdot\vec{x})]$$

Scattering Amplitude

Transition Rate W

Scattering reaction a + b → c + d

$$W = \sigma F$$

Interaction rate per target particle

related to physics of reaction

Fermi's Golden Rule

$$W = \frac{2\pi}{\hbar} |M_{fi}|^2 \rho_f$$

Matrix Element M_{fi}

scattering amplitude

Density ρ_f

# of possible final states

"phase space"

non-relativistic

1st order time-dependent perturbation theory

see e.g. Halzen&Martin, p. 80, Quantum Physics

Matrix Element

Contains all physics of the interaction

$$M_{fi} = \langle \psi_f | \hat{H} | \psi_i \rangle$$

Hamiltonian H is perturbation – 1st order

Incoming and outgoing plane waves

works if perturbation is small

Born Approximation

Matrix Element

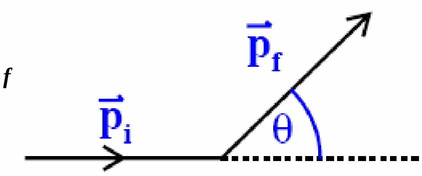
Scattering in Potential

Example: e- p → e- p

Incoming and outgoing plane waves

Matrix element

Momentum transfer $\vec{q} = \vec{p}_i - \vec{p}_f$



$$M_{fi} = \int \psi_f^* V(\vec{r}) \psi_i d^3\vec{r}$$

$$= \frac{1}{N^2} \int \exp(-i\vec{p}_f \cdot \vec{r}) V(\vec{r}) \exp(i\vec{p}_i \cdot \vec{r}) d^3\vec{r}$$

$$= \frac{1}{N^2} \int \exp(i\vec{q} \cdot \vec{r}) V(\vec{r}) d^3\vec{r} \qquad \vec{q} = \vec{p}_i - \vec{p}_f$$

M_{fi} (q) is Fourier transform of Potential V(r)

Scattering in Yukawa Potential

$$V(r) = -\frac{g^2}{4\pi r} \exp(-mr)$$

$$M_{fi} = -\frac{g^2}{4\pi} \int_0^\infty \int_0^\pi \int_0^{2\pi} \exp(i|\vec{q}|r\cos\theta) \frac{\exp(-mr)}{r} r^2 dr \sin\theta d\theta d\phi$$

$$= -\frac{g^2}{2i|\vec{q}|} \int_0^\infty \left(\exp(i|\vec{q}|r) - \exp(-i|\vec{q}|r)\right) \exp(-mr) dr$$

$$M_{fi} = -\frac{g^2}{\left(m^2 + |\vec{q}|^2\right)}$$

Propagator

term in M_{fi} $1/(m^2 + q^2)$

Cross section $\frac{d\sigma}{d\Omega} \propto |M|^2 \propto \frac{1}{\left(m^2 + |\vec{q}|^2\right)^2} \Rightarrow \frac{d\sigma}{d\Omega} \propto \frac{1}{q^4} \quad m = 0$

Result still holds relativistically

4-momentum transfer $q^\mu = \left(E_i - E_f, \vec{p}_i - \vec{p}_f\right)$