Weak Interaction

Outline

Introduction to weak interactions
- Charged current (CC) interactions
- Neutral current (NC)
- Weak vector bosons $W^{\pm}$ and Z^0

Weak charged interactions
- Beta decay
- Flavour changing charged current
- $W^{\pm}$ boson propagator
- Fermi coupling constant
- Parity violation

Muon decay
- Decay rate /lifetime

Lepton Universality
- $W^{\pm}$ boson couplings for leptons
- Tau decays

Weak quark decays
- $W^{\pm}$ boson couplings for quarks
- Cabibbo angle, CKM mechanism
- Spectator model

Introduction

Weak Interactions

Account for large variety of physical processes

Muon and Tau decays, Neutrino interactions

Decays of lightest mesons and baryons

Z^0 and $W^\pm$ boson production at $\sqrt{s} \sim O(100\ \text{GeV})$

Natural radioactivity, fission, fusion (sun)

Major Characteristics

Long lifetimes

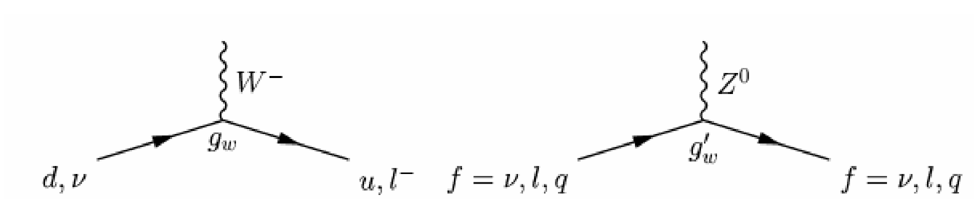
Small cross sections (not always)

"Quantum Flavour Dynamics"

Charged Current (CC) Neutral Current (NC)

mediated by exchange of

$W^\pm$ boson **Z^0 boson**



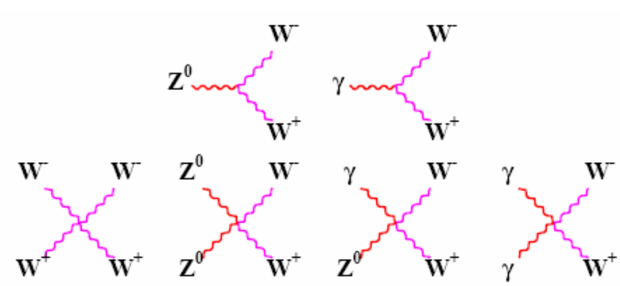
Intermediate vector bosons $M_W = 80.4$ GeV

$W^\pm$ and Z^0 have mass $M_Z = 91.2$ GeV

Self Interactions

of $W^\pm$ and Z^0

also $W^\pm$ and γ



Beta Decay

Weak Nuclear Decays

See also Nuclear Physics

Recall $\beta^+ = e^+$

$\beta^- = e^-$

β^-	$(A,Z) \rightarrow (A,\ Z+1) + e^- + \underline{\nu}$
β^+	$(A,Z) \rightarrow (A,\ Z-1) + e^+ + \nu$

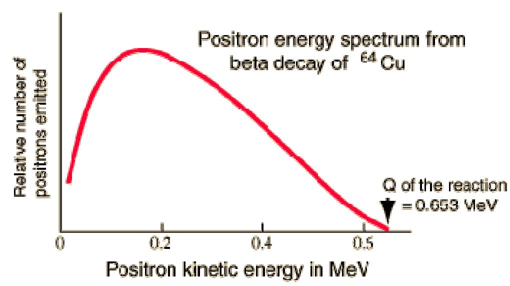
Continuous energy spectrum of e^- or e^+

➔ 3-body decay, Pauli postulates **neutrino**, 1930

Interpretation

Fermi, 1932

Bound n or p decay



$$n \rightarrow p e^- \bar{\nu}_e$$

$$N(t) = N(0)\exp\left(\frac{-t}{\tau_n}\right)$$

$$\tau_n = 885.7 \pm 0.8\,\mathrm{s}$$

$$\tau_{1/2} = \tau_n \ln 2 = 613.9 \pm 0.6\,\mathrm{s}$$

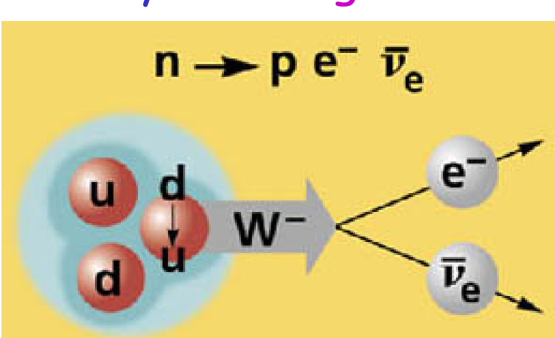
$$p \rightarrow n e^+ \nu_e \qquad \text{(bound p)}$$

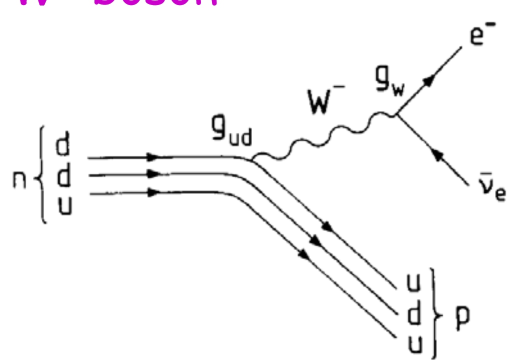
$$\tau_p > 10^{32}\ \mathrm{y} \qquad \text{(p stable)}$$

Modern quark level picture

Weak charged current mediated

by exchange of virtual $W^{\pm}$ boson

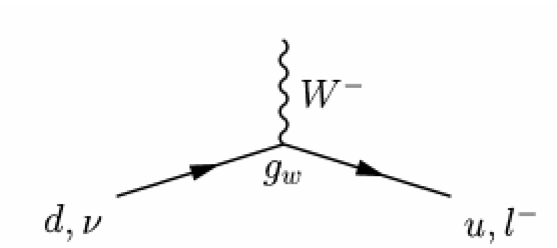


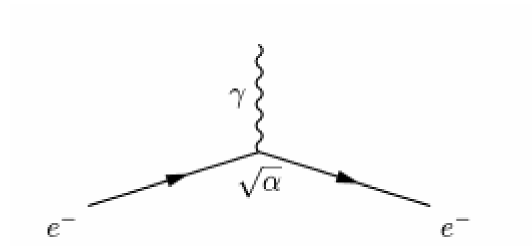


Weak Charged Interactions

Fundamental Vertex

Weak Charged Current   QED





Flavour Changing

Weak charged current changes lepton and quark flavours

$W^\pm$ boson transforms ν_e into e^-, u into d-quark

Only weak interaction is flavour changing

Weak Charged Current

$W^\pm$ boson couples to weak charge g_W of neutrino/electron or up/down quark pair

Coupling strength $\propto g_W$

➔ For each vertex add factor g_W to matrix element

Probability for weak vertex

cross section or decay rate $\propto g_W^2$

$W^\pm$ Boson Propagator

W boson has mass M_W

Recall photons and gluons are massless

➔ Add propagator term $1/(q^2 - M_W^2)$ to matrix element/amplitude

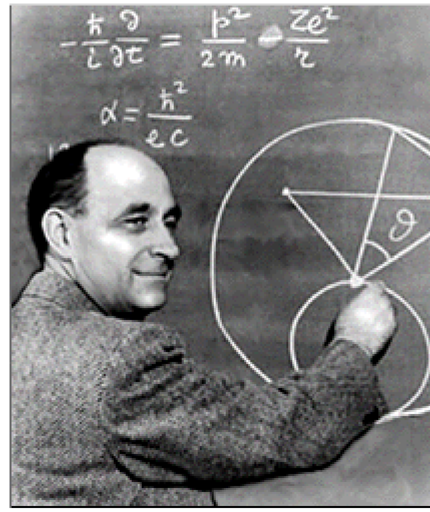
Fermi Theory of Weak Interaction

$W^{\pm}$ Boson Propagator

for small $q^2 \ll M_W^2$ ($q^2 \to 0$)

$1/(q^2 - M_W^2) \to 1/M_W^2$

Propagator is constant



Fermi

Fermi Coupling Constant G_F

Weak coupling constant

includes propagator

$$G_F \propto \frac{g_W^2}{M_W^2} \qquad \text{exact} \qquad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

different from QED & QCD Perkins, p 151&210

g_W dimensionless → units of G_F are GeV^{-2}

$G_F/(\hbar c)^3 = 1.16637(1) \cdot 10^{-5}$ GeV^{-2}

Range of Weak Interaction

Massive exchange boson ↔ short range

$$R_{weak} = \frac{\hbar c}{M_W c^2} \approx 2 \cdot 10^{-18}\ \mathrm{m} = 0.002\ \mathrm{fm} \ll 0.1\ \mathrm{fm}$$

Analogous to Yukawa interaction

Strength of Weak Interaction

$$G_F \ \&\ M_W \quad \Rightarrow \quad g_W = 0.66 \quad \Rightarrow \quad \alpha_W = \frac{g_W^2}{4\pi} = \frac{1}{29} > \alpha_{em} = \frac{1}{137}$$

Not intrinsically weak

at low q^2 weak due to large M_W

$\alpha_S \approx 0.2 > \alpha_W \approx 0.03 > \alpha_{em} \approx 0.008$ all running

Will these meet at high energy? – unification?

Parity Violation

Parity P

Intrinsic quantum number of particles

Conserved in strong and electromagnetic interactions

Kaon Decays

Observe $K^+ \to \pi^+ \pi^0$ and $K^+ \to \pi^+ \pi^- \pi^+$ decays

Parity of K^+ and π^+ $\quad P(K^+) = P(\pi^+) = -1$

Parity of final states $\quad P(\pi^+\pi^0) = P_\pi P_\pi (-1)^{L=0} = 1$

➔ Puzzle $\quad P(\pi^+\pi^-\pi^+) = P_\pi P_\pi P_\pi (-1)^{L=0} = -1$

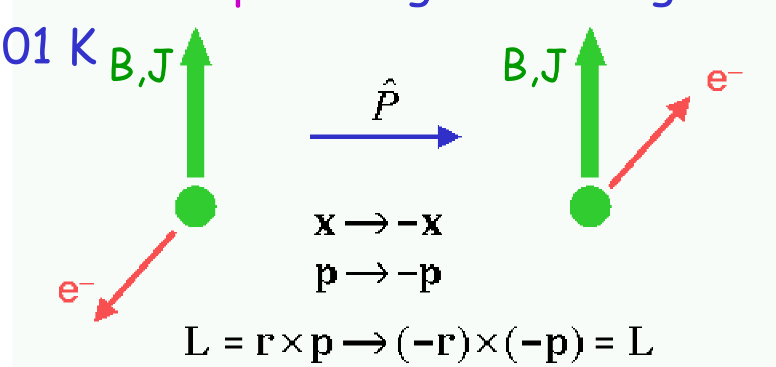
Parity Violation in Weak Interactions

1956 proposed by Lee and Yang

1957 experimentally confirmed by Wu et al

Measure e- from ^{60}Co beta decay $\quad ^{60}Co \to {}^{60}Ni^* e^- \bar{\nu}_e$

Polarised ^{60}Co spin J aligned to magnetic field B at 0.01 K



If parity conserved expect equal e- rates parallel and antiparallel to B-field and spin

Observe asymmetry ➔ **Parity violated**

Muon Decay

Muon

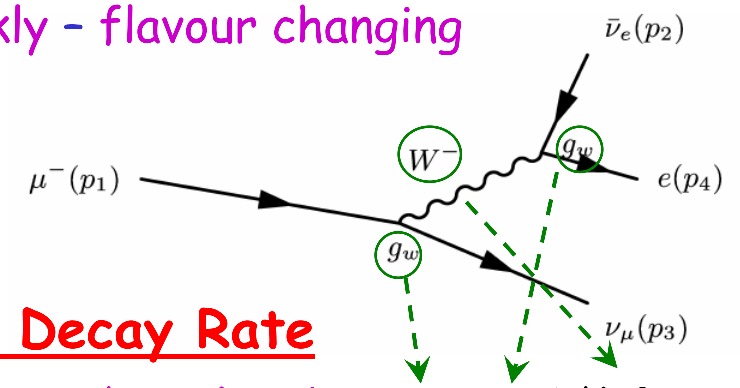
Fundamental lepton with mass m_μ = 105.7 MeV

does not interact strongly

Electromagnetic decay $\mu^+ \rightarrow e^+\gamma$ is forbidden

Decays weakly – flavour changing

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$
$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$



Amplitude and Decay Rate

Matrix element / amplitude $\propto g_W g_W \, 1/(q^2 - M_W^2)$

$q^2 \ll M_W^2 \rightarrow$ decay rate $\Gamma_\mu \propto G_F^2$

dimensions $\rightarrow \Gamma_\mu \propto G_F^2 m_\mu^5$

Total decay rate or lifetime τ_μ

$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \qquad (\hbar = c = 1)$$

Measurements

Lifetime τ_μ = 2.19703(4) 10^{-6} s

Mass m_μ = 105.658369(9) MeV

➔ G_F = 1.16637(1) $\cdot 10^{-5}$ GeV^{-2}

weak coupling constant G_F is measured in muon decay

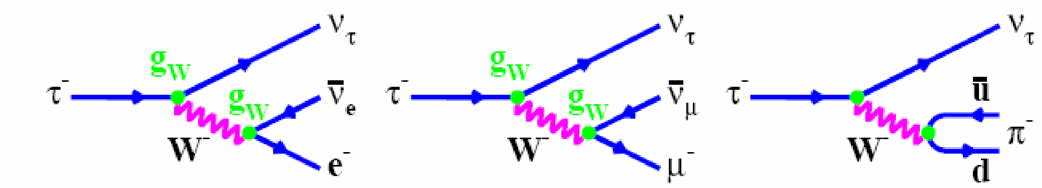
see 4th year projects for measuring τ_μ and m_μ

Lepton Universality in Weak Interaction

Tau Decays

m_τ = 1.777 GeV > m_μ, m_π, m_ρ, …

Several weak decay modes possible



Branching Fractions

$$BF(\tau^- \to X) = \Gamma(\tau^- \to X)/\Gamma(\tau^- \to all)$$

$\tau^- \to e^-\bar{\nu}_e\nu_\tau$ $\quad BF = 17.8 \pm 0.1\%$

$\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$ $\quad BF = 17.4 \pm 0.1\%$

$\tau^- \to \text{hadrons}\,\nu_\tau$ $\quad BF = 64.7 \pm 0.2\%$

Tau Decay Rates

Investigate decay $\tau^- \to e^-\bar{\nu}_e\nu_\tau$

$$\frac{1}{\tau_\tau} = \Gamma_{\tau\to all} = \frac{\Gamma_{\tau\to all}}{\Gamma_{\tau\to e\bar{\nu}_e\nu_\upsilon}}\Gamma_{\tau\to e\bar{\nu}_e\nu_\upsilon} = \frac{1}{BF_{\tau\to e\bar{\nu}_e\nu_\upsilon}}\Gamma_{\tau\to e\bar{\nu}_e\nu_\upsilon} = \frac{1}{0.178}\frac{G_F^2 m_\tau^5}{192\pi^3}$$

compare with muon decay

Expect lifetime $\tau_\tau = \tau_\mu\, BF_{\tau\to e\bar{\nu}_e\nu_\tau}\dfrac{m_\mu^5}{m_\tau^5} = (2.91 \pm 0.01)\times 10^{-13}$ s

Measure $\tau_\tau = (2.906 \pm 0.011)\times 10^{-13}$ s

➔ Weak coupling of τ and μ identical

Lepton Universality in Standard Model

$W^\pm$ boson couples identically to all leptons

$$e^- \to W^-\nu_e \text{ or } \nu_e \to e^- W^+ \quad \mu^- \to W^-\nu_\mu \text{ or } \nu_\mu \to \mu^- W^+ \quad \tau^- \to W^-\nu_\tau \text{ or } \nu_\tau \to \tau^- W^+$$

Charged weak current

Couples within lepton doublets $\begin{pmatrix}\nu_e \\ e^-\end{pmatrix} \begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix} \begin{pmatrix}\nu_\tau \\ \tau^-\end{pmatrix}$

Weak Interaction of Quarks

Weak charged quark currents

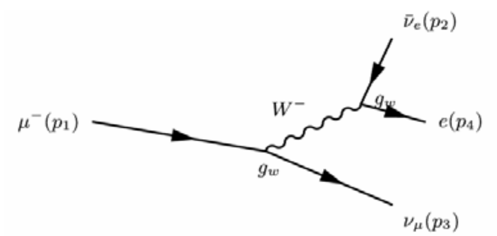
Are weak charged quark currents also universal?

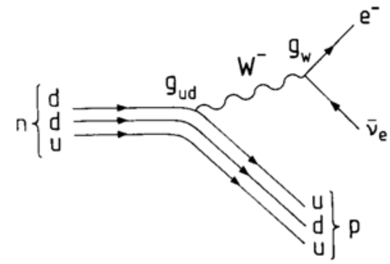
e.g. is $M(d \to uW^-) = M(\mu^- \to \nu_\mu W^-)$?

Nature is not quite that simple

Comparison

Measure G_F in muon and nuclear beta decay



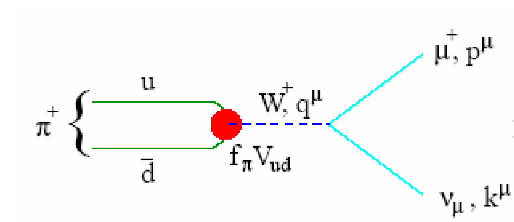


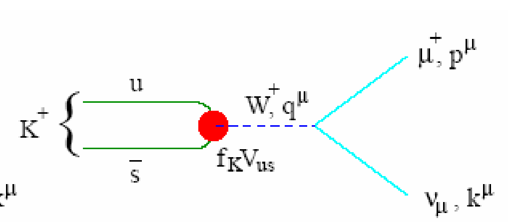
Obtain ratio $G_F(\text{beta})/ G_F(\text{muon}) = 0.974(3)$

➔ Charged weak current almost equal for leptons and up/down quarks

Leptonic Pion and Kaon Decays

Observe $\pi^+ \to \mu^+ \nu_\mu$ and $K^+ \to \mu^+ \nu_\mu$





Evidence for flavour changing coupling $u \to sW^+$ between quark generations

Rate $\Gamma(K^+ \to \mu^+ \nu_\mu) \approx 5\%$ expected $\Gamma(\pi^+ \to \mu^+ \nu_\mu)$

Matrix element $M(u \to sW^+) \sim \sin\theta_C$

$M(u \to dW^+) \sim \cos\theta_C$

Cabibbo Mixing Angle

Quark Flavour Mixing

Two generations & four quark flavours (u, d, c, s)

weak eigenstates not equal to mass eigenstates

are linear combination of mass eigenstates

Cabibbo Hypothesis

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

W boson couples to weak doublet (u,d')

Cabbibo angle θ_C rotates weak eigenstates

Coupling strength to W boson within doublet

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \to g_W \quad \begin{pmatrix} u \\ d \end{pmatrix} \to g_W \cos\theta_C = g_W V_{ud} \quad \begin{pmatrix} u \\ s \end{pmatrix} \to g_W \sin\theta_C = g_W V_{us}$$

Experimental Results

Muon decay $\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = G_F^2 m_\mu^5 / 192\pi^3 = \Gamma_\mu$

Beta decay $\Gamma(d \to u \bar{\nu}_e \nu_\mu) \propto G_F^2 \cos^2\theta_C = G_F^2 |V_{ud}|^2$

Pion decay $\Gamma(\pi^+ \to \mu^+ \nu_\mu) \propto G_F^2 \cos^2\theta_C = G_F^2 |V_{ud}|^2$

Kaon decay $\Gamma(K^+ \to \mu^+ \nu_\mu) \propto G_F^2 \sin^2\theta_C = G_F^2 |V_{us}|^2$

Cabibbo angle $\theta_C = 12.96(9)^0 \qquad \sin\theta_C = 0.2243(16)$

Cabibbo-Kobayashi-Maskawa Mechanism

W couples quarks of different generations

For 3 generations expand to

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Hadron Decays

Lightest Mesons and Baryons

only flavour changing weak decay possible for Pions (π^+ = u dbar), kaons (K^+ = u sbar), charmed (D) and B mesons, neutron, Λ, Σ, Ξ, Ω

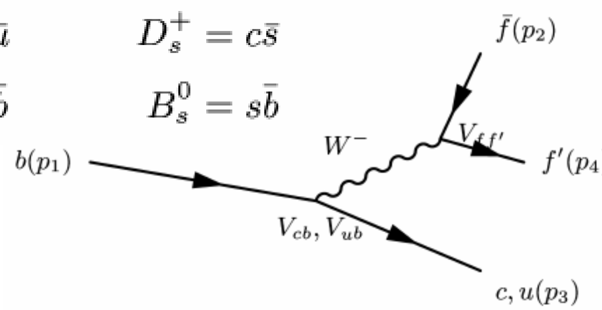
Exceptions: zero net quantum numbers allow electromagnetic decay, e.g. $\pi^0 \to \gamma\gamma$

Spectator Model

Weak decay of heavy B or D meson (b or c quark)

$D^+ = c\bar{d}$ $\quad D^0 = c\bar{u}$ $\quad D_s^+ = c\bar{s}$

$B^+ = u\bar{b}$ $\quad B^0 = d\bar{b}$ $\quad B_s^0 = s\bar{b}$



Decay rate of B or D meson equal to decay rate of heavy b or c quark, light quark is only spectator

Predictions of Spectator Model

Semileptonic decay rate $B \to X_{c,u}\, e^- \nu\text{bar}$

$$\Gamma\left(B \to X_{c,u} e^- \bar{\nu}_e\right) = \frac{G_F^2 m_b^5}{192\pi^3} \left|V_{cb(ub)}\right|^2$$

Equal total decay rates/lifetimes for D^+, D^0, D_s^+ and B^+, B^0, B_s^+

D meson lifetimes $\tau_{D^+} \approx 2.5\, \tau_{D^0} \approx 2.5\, \tau_{D_s^+}$

B mesons lifetimes are equal within 10%

Spectator model works well for B mesons